Enhancing Diversity and Multiplexing Gains in Multi-User Wireless Relay Systems

Chao Wang

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Abstract

The demand for higher transmission rates and better quality of service in modern wireless communications is endless. The use of multiple transmit or/and receive antennas has been considered as one of the most powerful approaches to facilitate high-speed and high-quality communications. However, in practical cellular systems, mobile terminals may not be able to support a multiple-antenna setup. Thus an emerging technique called *cooperative diversity* is under consideration to utilize the multi-hop relay concept to realize the advantages of multiple-antenna systems in multi-user single-antenna networks. Cooperative diversity has attracted much interest in recent years as a very promising direction for future wireless communication evolution.

Due to the fact that in practice terminals cannot transmit and receive simultaneously (i.e. the half-duplex limitation), the diversity improvement brought by the standard cooperative diversity transmission protocols is in general accompanied by a multiplexing loss (equivalent to a reduction in transmission data rate in high signal-to-nose ratio (SNR)). The purpose of this thesis is to use advanced transmission protocols to provide both good diversity and multiplexing performance when using the practical repetition-coded decode-and-forward (DF) relaying strategy in uplink mobile-to-base station transmission of cellular systems.

The task is fulfilled by relaxing the orthogonal channel allocation requirement of the standard protocols and by using two relays to take turns forwarding source information to destination. We start our analysis from an M-source two-relay one-destination network. Through diversity-multiplexing tradeoff (DMT) analysis, we prove that for an isolated-relay scenario and a strong-interference scenario, the considered approach effectively recovers the multiplexing loss induced by the standard protocols while still obtaining diversity improvement over direct source-destination transmission without considering relaying.

In addition, since the optimal multiplexing gain of the considered system can be achieved by the above approach, we study further improving diversity performance for a two-source network. We analyze taking full advantage of the multiple-source structure, multiple-relay structure, and the capability of affording complex signal processing at the destination (base station). For all three cases, we prove that the diversity performance of the above approach can be enhanced without a significant loss of multiplexing performance or using complex coding strategies at relays. Since the good DMT performance is not affected by source-relay channel conditions, the protocols discussed in this thesis make relaying more beneficial.

Declaration of originality

The work presented in this thesis was a collaboration work between Dr. Yijia Fan and myself. More specifically, the initial idea of using two relays to successively assist a single source was proposed by Dr. Yijia Fan in [1] and [2]. Dr. Fan's focus was mainly on a transmission rate upper bound analysis and a V-BLAST structure design on assuming perfect source-relay transmissions. The theoretical diversity-multiplexing tradeoff (DMT) analysis on the single-source protocol, proposing the adaptive protocol under general source-relay channel conditions, extending the single-source network to multiple-source networks, and designing advanced transmission protocols to further improve diversity performance without losing multiplexing performance were my contribution [3–7].

I hereby declare that Section 3.3.1, Section 3.3.2, and the rate upper bound analysis in Section 3.3.3 are based on Dr. Yijia Fan's work. The research recorded in all the remaining part of Chapter 3, Chapter 4, and the thesis itself were composed and originated entirely by myself in the School of Engineering at The University of Edinburgh.

Chao Wang

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Acronyms and abbreviations

AF	Amplify-and-forward
AWGN	Additive white Gaussian noise
BPCU	Bits per channel use
CSI	Channel state information
CTS	Clear-to-send
DAF	Decode-amplify-forward
DDF	Dynamic decode-and-forward
DF	Decode-and-forward
DMT	Diversity-multiplexing tradeoff
i.i.d.	Independent identically distributed
ISI	Intersymbol interference
LHS	Left hand side
MIMO	Multiple-input multiple-output
MISO	Multiple-input single-output
ML	Maximal likelihood
MMSE	Minimum mean-square error
NAF	Nonorthogonal amplify-and-forward
OFDM	Orthogonal frequency division multiplexing
QAM	Quadrature amplitude modulation
RHS	Right hand side
RTS	Ready-to-send
SDMA	Space-division-multiple-access
SIC	Successive interference cancellation
SINR	Signal-to-interference-plus-noise ratio
SIMO	Single-input multiple-output
SISO	Single-input single-output
SNR	Signal-to-noise ratio
TDMA	Time-division-multiple-access
WLAN	Wireless local-area network

Nomenclature

[1]

M	Number of sources
K	Number of relays
N	Number of antennas at destination
$\mathcal{S}_i$	Source i
$\mathcal{R}_j$	Relay j
$\mathcal{D}$	Destination
$x_i[j]$	The jth codeword from source i
$y[j]$	Received signal at destination during the jth time slot
$h_{\mathcal{S}_i}$	Channel fading coefficient between source i and destination
$h_{\mathcal{R}_j}$	Channel fading coefficient between relay j and destination
$h_{\mathcal{S}_i,\mathcal{R}_j}$	Channel fading coefficient between source i and relay j
$\mathbf{h}$	Channel fading vector
$\mathbf{H}$	Channel fading matrix
n	Additive white Gaussian noise (AWGN) at receiver
$\mathbf{n}$	AWGN vector
ρ	Signal-to-noise ratio (SNR)
$\bar{R}_i$	Source i's transmission rate in bits per codeword
R_i	Source i's transmission rate in bits per channel use (BPCU)
d	Diversity gain
r	Multiplexing gain
L	Source transmit frame length
T_L	Total time used to transmit L codewords from each source
$\mathbf{I}$	Identity matrix
P_{out}	Outage probability
$P(A)$	Probability of event A
$\log(\cdot)$	Base-2 logarithm
$\doteq$	Exponential equality

[1]Only the notations that are frequently used throughout the thesis are listed. Others will be given and explained specifically in certain parts of the thesis.

Chapter 1
Introduction

1.1 Introduction

The beginning of wireless communications can be traced back to 1896 when Guglielmo Marconi first demonstrated the wireless telegraph and was awarded a patent for it in the next year. Over the last one hundred and ten years, with fast developing technologies and emerging services, wireless communications have rapidly and successfully penetrated through everyone's daily life and have been treated as one of the most important inventions that influence various aspects of the world's politics, economy, and culture.

Compared with communications conducted within wired lines, wireless communications have the major advantage of user mobility. The movement is not limited by the length of transport medium so that ideally information can always be delivered from its source (transmitter) to its destination (receiver) at any time and any location. In addition, adding in or removing devices from a network can be done without additional cost or delay of rewiring. However, the most challenging (but also interesting) problems in wireless communications are two-fold. Firstly, wireless transmission is affected by the *fading* phenomenon. The transmitted signal power decays with increasing distance and randomly varies due to large objects in the environment (generally termed *large-scale fading*). The signal amplitudes and phases also randomly and rapidly fluctuate because of the multiple signal paths between the transmitter and the receiver (termed *small-scale fading*). The fading effects dramatically raise the difficulty for information recovery process at the receiver. Secondly, because there are radio channels between each individual transmitter-receiver pair and multiple users share a finite amount of useful radio spectrum, *interference* among different users may severely reduce the communication capability.

For example, the initial mobile phone service introduced in the US in 1946 used a central transmitter to cover an entire metropolitan area [11]. Due to the aforementioned two major drawbacks along with the technologies of that time, the capacity of that system was very limited so that only a small number of users could be served. A practical solution to this problem is the *cellular* concept developed at AT&T Bell Labs during the 1950s and 1960s [11]. The basic idea is to divide the service area into small cells, each of which is covered by a fixed base station that provides service to the wireless subscribers within the cell. In this way, low-power transmitters can be adopted to provide high-rate and high-quality communications. In addition, services in geographically separated cells may operate in the same frequency band (i.e. frequency reuse [12]) without causing severe interference to each other. The radio spectrum is efficiently used and a lot more customers can be accommodated. In fact, the cellular telephone system is the most commercially successful wireless communication application nowadays.

During the last three decades, cellular systems have rapidly evolved from the first generation analog systems to the second generation digital systems, and to the third generation (3G) that provides high-speed data and/or voice services [13, 14]. Retrospecting this process and envisioning the beyond 3G or the fourth generation (4G) cellular systems [15] to be implemented in the future, it is clear that the demand for higher transmission rates and better quality of service in wireless communications is endless. The developments of multiple-input multiple-output (MIMO) techniques and multi-hop relaying techniques have been treated as two of the most powerful approaches to facilitate meeting the demand.

1.2 Multiple-antenna systems and multi-hop systems

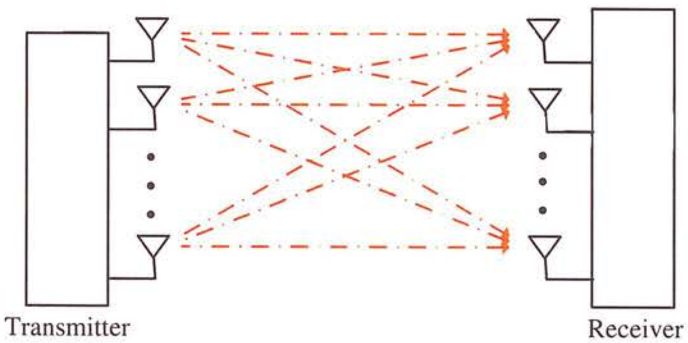

Figure 1.1: *A point-to-point MIMO system.*

Conventionally, wireless communications are conducted within single-antenna systems, i.e. both the transmitter and the receiver of each wireless link are equipped with only one antenna. However, research has shown that a multiple-antenna setup at either side or both sides of a link (i.e. a MIMO system displayed in Figure 1.1) can provide a spatial *diversity gain* to mitigate small-scale fading effects (e.g. [16–20]). Since the use of multiple antennas permits the transmitted information to be conveyed to the receiver through more than a single path, the probability that the communication quality is limited by deep fading is much smaller than that for single-antenna systems. Error performance at the receiver can be significantly improved. On the other hand, MIMO systems can offer a spatial *multiplexing gain*, which results in a substantial increase of transmission data rate without additional power or bandwidth consumption [21, 22]. With these advantages, MIMO techniques have been adopted into various current wireless communication standards.

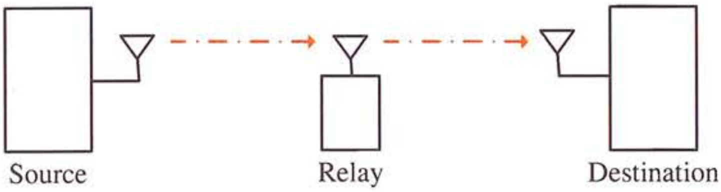

Figure 1.2: *A two-hop relay system.*

Another approach to obtain better communication performance over conventional means is through multi-hop relaying [23, 24]. Figure 1.2 displays a simple two-hop relay system, where one relay terminal is used to assist in the communication between a source and its intended destination. In this system, the source's transmission firstly reaches the relay. Then the relay can act as a linear amplifier or a regenerative repeater to forward its received source information to the destination [25]. By placing the relay in an advantageous location (e.g. where both source-relay and relay-destination links have smaller distances than the direct source-destination link, or where communication through the source-relay-destination link can pass around an obstruction that blocks the direct source-destination link), the impact of large-scale fading can be notably reduced.

Applying multi-hop relaying into cellular networks usually considers placing fixed relay stations near cell boundaries. Many investigations (e.g. [26–28]) have been performed to show that the system throughput can be dramatically enhanced. These results imply that the use of relays can act as a simple solution to increase cell coverage instead of increasing the number of relatively complex and expensive base stations. In addition, decreased transmit power of mobiles or base stations reduces battery consumption and co-channel interference among different cells.

1.3 Cooperative diversity

Although MIMO systems can provide dramatic spectral efficiency and link reliability improvements over conventional single-antenna systems, they also demand high complexity at transmitters and receivers. Additionally, to guarantee good performance, the multiple antennas are required to be placed sufficiently far apart. For cellular systems, due to size, cost or hardware limitations, mobile terminals thus may not be able to support such a multiple-antenna structure and the advantages of multiple-antenna systems are difficult to exploit. Therefore, the relaying concept has been considered as a way to (at least partially) solve these problems. The new approach is generally called *cooperative diversity* [29, 30] and is very promising in future wireless communications.

The basic idea behind cooperative diversity is that single-antenna mobiles in a multi-user scenario assist in each other's transmission by acting as relays. In this way, all the cooperative mobiles pool their antennas to create a virtual antenna array to mimic a multiple-antenna system. For instance, Figure 1.3 displays a simple cooperative diversity system where two mobiles help each other to communicate with the base station. Due to the broadcast nature of wireless communications, when one mobile (say, mobile 1) sends information to the base station, the other mobile (mobile 2) can also overhear the transmission. After that, mobile 2 processes its received signal and retransmits it to the base station. By this means, it is as if mobile 1's information is transmitted from a two-antenna transmitter. Higher diversity gain is thus achieved without actually adding additional antennas to mobiles. Compared with the original multi-hop relaying concept, where there is no direct communication between the source and the destination, the destination receives signals conveyed through both the relay link (i.e. the two-hop source-relay-destination link) and the direct link (i.e. the one-hop source-destination link). Although the relay can also be chosen to mitigate large-scale fading effects, diversity gain to combat small-scale fading is of more concern in cooperative diversity.

1.4 Contributions

Although ideally the cooperative diversity concept exhibits a bright prospect of using distributed single-antenna terminals to realize the advantages of multiple-antenna systems, in practice the price of establishing such virtual multiple-antenna systems is always nontrivial. One major issue is that due to the fact that terminals generally cannot transmit and receive simul-

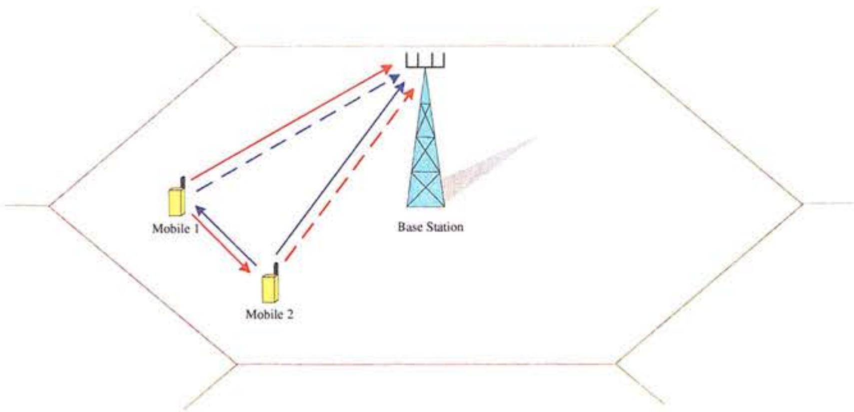

Figure 1.3: *A simple cooperative diversity system model. The solid lines denote that each mobile broadcasts its transmit information to the other mobile and the base station. The dashed lines denote that the mobiles relay each other's information to the base station to realize cooperative diversity.*

taneously in the same frequency band (i.e. the *half-duplex limitation*), standard cooperative diversity transmission protocols [31, 32] normally divide the available channel into orthogonal parts and allocate only a fraction to the source to send information. As a result, the communication reliability improvement provided by the standard protocols is accompanied by a sacrifice of transmission rate, especially in the high signal-to-noise ratio (SNR) region (i.e. a reduction of multiplexing gain, termed *multiplexing loss*).

Many investigations aiming at using advanced transmission protocols to recover the multiplexing loss have been performed in recent years. For digital relaying systems (i.e. decode-and-forward (DF) relaying systems, where relays decode, re-encode, and retransmit the source information after receiving it), those protocols in general demand feedback from destination to source (e.g. [31, 33]) or using complex coding strategies at relays (e.g. [34, 35]). However, in this thesis, we try to avoid these requirements to minimize the system complexity. More specifically, the purpose of this thesis is to obtain both good multiplexing performance (i.e. better than that of the standard protocols and approaching the optimal multiplexing gain the system can provide) and good diversity performance (i.e. better than that of direct source-destination transmission without relaying) in cooperative networks where the sources cannot exploit feedback from their intended destinations and relays only utilize a simple repetition-coding strategy.

The task is fulfilled by relaxing the orthogonal channel allocation requirement and using two relays to take turns helping the source-destination communications. The way that multiple relays are used in this thesis is different from conventional approaches, where adding more relays is considered as a means to achieve only a higher diversity gain (e.g. [32, 34]). We start our analysis from the simplest single-source two-relay one-destination network[1]. We use a performance metric named diversity-multiplexing tradeoff (DMT) [36] to provide a theoretical proof that when the inter-relay link is sufficiently weak (i.e. an isolated-relay scenario) or sufficiently strong (i.e. a strong-interference scenario), the considered protocol effectively recovers the multiplexing loss induced by the standard protocols while still obtaining diversity improvement over direct source-destination transmission. Although in general the diversity performance of DF relaying protocols is limited by the quality of source-relay links, by permitting the relays to work adaptively, we show that the good diversity performance of the considered protocol can still be achieved under general source-relay channel conditions. After this, we extend the protocol to a two-source network and further to a general network with M sources.

As the optimal multiplexing gain the system can afford is actually able to be achieved now, in the next step, we consider further improving the diversity performance of these protocols, especially for a two-source network. For the strong-interference scenario, we make use of the inter-relay interference. For the isolated-relay scenario, we take advantage of the multiple-source structure of the two-source network. In addition, we also study the impact of using multiple antennas at the destination (base station). For all the three approaches, we prove that the achievable diversity performance of the aforementioned protocols can be enhanced without a significant loss of multiplexing gain. Since these new protocols still use the simple repetition-coding strategy at relays and the good diversity and multiplexing performance is not affected by source-relay channel conditions, the protocols discussed in this thesis make relaying more beneficial.

1.5 Structure of thesis

The rest of the thesis is organized as follows. In Chapter 2, we give the background and motivation of the thesis by providing a detailed introduction to fading phenomenon, single-antenna and multiple-antenna systems, single-source and multiple-source systems, and the advantages and disadvantages of existing cooperative diversity transmission protocols. In Chapter 3 we present our approach to recover the multiplexing loss induced by the standard protocols. Protocols aiming at further improving diversity performance of the protocol analyzed in Chapter 3 are introduced in Chapter 4. Finally, in Chapter 5, we summarize the thesis and discuss future

[1]The protocol for such a single-source network was initially proposed by Dr. Yijia Fan in [1]

work.

Chapter 2
Background

2.1 Introduction

In this chapter, we provide some background for the thesis. We will start our introduction from the most challenging problem confronted in wireless communications: the fading phenomenon. It is well known that the high-SNR error performance in a fading environment is much worse than that when fading is not considered. This is because the high-SNR error event in a fading channel occurs mostly due to the channel is in deep fade. This is in contrast to the additive noise sample being large, which is the reason that error events occur when there are no fading effects. In a fast fading environment, where the transmit codeword experiences many fading realizations, as long as the codeword is sufficiently long and its transmission rate is below a certain positive threshold (i.e. the ergodic capacity of fast fading channels), it is possible to drive the error probability at the receiver to be arbitrarily close to zero.

However, if the transmit codeword can only experience a single fading realization (i.e. in a slow fading environment), there is no positive transmission rate that can guarantee reliable communication with no errors. In addition, for any transmission rate, the probability that the channel cannot support the rate (i.e. the outage probability) decays slowly with increasing SNR since the signal transmission highly relies on the quality of a single transport path. This fact has led to many investigations on *diversity* techniques to improve communication reliability in slow fading environments. The basic idea behind diversity is to convey multiple replicas of the transmit information to the receiver through independent fading paths so that the probability that all channels suffer from deep fading is much lower than that when only one path is considered. Diversity can be exploited across the time domain by sending information during different fading blocks. A more spectrally efficient solution is to use multiple antennas at the transmitter and/or the receiver to obtain spatial diversity. Further, using multiple antennas at both the transmitter and receiver sides can attain substantial gains with respect to both spectral efficiency and channel reliability over a single-antenna system. Such gains can be measured by a performance metric named the diversity-multiplexing tradeoff.

Although multiple-antenna systems provide dramatic advantages over single-antenna systems, in current cellular systems, the complexity of mobile terminals is limited so that they may not be able to support a multiple-antenna setup. Therefore, *cooperative diversity* has been considered as a novel approach to provide spatial diversity by demanding distributed single-antenna terminals to work as relays for each other. With another practical limitation that terminals cannot transmit and receive at the same time in the same frequency band (i.e. the half-duplex limitation), standard relaying transmission protocols provide higher communication reliability over direct transmission without considering relaying but with a sacrifice of transmission rate, especially in the high SNR region. We will present the standard protocols and highlight their diversity advantage as well as their high-SNR spectral inefficiency. Diverse advanced transmission protocols aiming at improving the spectral efficiency of the standard protocols have been proposed by many researchers. However, when considering a simple system setup in which there is no feedback from destination to source and complex coding strategies at relays are not permitted, those protocols may not be applicable. This observation triggers our work which will be presented in the next two chapters.

2.2 Wireless fading channels

When we talk about "communications", we generally refer to an information delivery process between information transmitters and receivers. Unlike wired channels, communications conducted *wirelessly* are always affected by signal refraction, diffraction, and scattering induced by various objects in the environment, which form the phenomenon of *fading*, as well as additive noise at receivers. Therefore, as a beginning of this thesis, the purpose of this section is to introduce the wireless fading channels, more details of which can be found in some classic textbooks (e.g. [11, 13, 37]). In the following, we present basic system models on which the analysis throughout the thesis is based. Although typical wireless communications occur in a passband, major processing (e.g. coding/decoding and modulation/demodulation) is done at the baseband [13]. We thus always assume a discrete complex baseband model.

2.2.1 Discrete complex baseband model

Although wireless communications occur in various setups such as between a single source and multiple destinations, between multiple sources and a single destination, between multiple

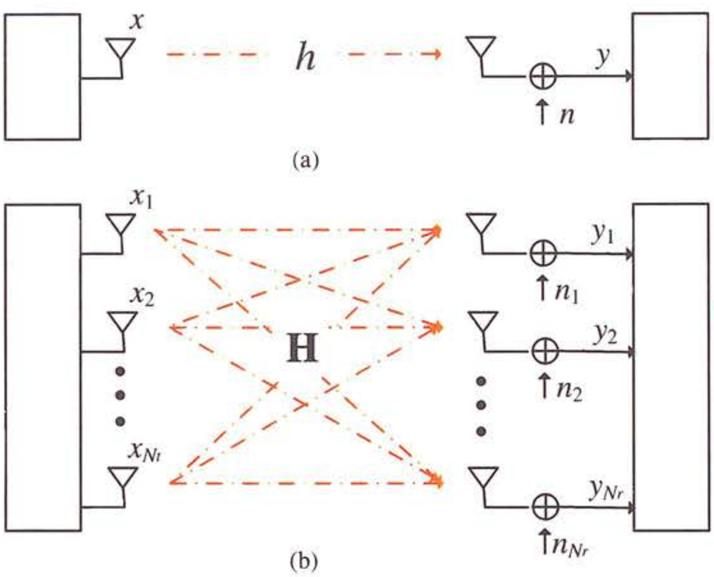

Figure 2.1: *System models of (a) a point-to-point single-antenna system, and (b) a point-to-point multiple-antenna system with N_t transmit antennas and N_r receive antennas.*

sources and multiple destinations, or through multiple hops, the basic system is always a point-to-point transmission model in which a single transmitter intends to send information to a single receiver. For such a point-to-point transmission model, the simplest setup is that both the transmitter and the receiver are equipped with a single antenna as displayed in Figure 2.1 (a). In general, the transmit information is coded into *codewords*. We assume the transmit codeword x is chosen from an independent identically distributed (i.i.d.) Gaussian random codebook[1] with zero mean and unit average power, i.e. $x \sim \mathcal{CN}(0, 1)$. Assuming x is transmitted by the source with average transmit power P, the transmit SNR ρ is defined as $\rho = \frac{P}{N_0}$, in which N_0 denotes the average noise power. Without loss of generality, we normalize N_0 to be 1. The discrete-time complex baseband input-output relation between the transmitter and the receiver can be expressed by

$$y = \sqrt{\rho} h x + n, \tag{2.1}$$

in which y denotes the received signal at the receiver, the channel fading coefficient h represents the attenuation due to the impact of fading, and n is the additive white Gaussian noise (AWGN) with $n \sim \mathcal{CN}(0, 1)$. The task for the receiver is thus to correctly recover the transmitted information according to its received signal.

[1]Information theory shows that the use of i.i.d. Gaussian random code maximizes the mutual information between the transmitted and received signals and thus obtains the system capacity. More details about the i.i.d. Gaussian random code can be found in [13, 38].

For a more general system in which N_t ($N_t \geq 1$) antennas are equipped at the transmitter and N_r ($N_r \geq 1$) antennas are equipped at the receiver as shown in Figure 2.1 (b), the input-output relation of the channel can be expressed by [13]

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{N_r} \end{bmatrix} = \underbrace{\begin{bmatrix} h_{1,1} & h_{2,1} & \cdots & h_{N_t,1} \\ h_{1,2} & h_{2,2} & \cdots & h_{N_t,2} \\ \vdots & \vdots & \ddots & \vdots \\ h_{1,N_r} & h_{2,N_r} & \cdots & h_{N_t,N_r} \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} \sqrt{\rho_1} & 0 & \cdots & 0 \\ 0 & \sqrt{\rho_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{\rho_{N_t}} \end{bmatrix}}_{\mathbf{P}} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_t} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{N_t} \end{bmatrix} \tag{2.2}$$

where y_i and n_i denote the received signal and AWGN at the ith receive antenna, x_i ($x_i \sim \mathcal{CN}(0,1)$) is the transmitted signal from the ith transmit antenna, the element $h_{i,j}$ in the $N_r \times N_t$ channel matrix $\mathbf{H}$ denotes the fading coefficient between the ith transmit antenna and the jth receive antenna, and the matrix $\mathbf{P}$ is an $N_t \times N_t$ diagonal matrix with the ith main diagonal element $\sqrt{\rho_i}$ ($i = 1, \ldots, N_t$) denoting the transmit power allocated to the ith transmit antenna. For fair comparison with the single-antenna system, we assume the average transmit power remains as ρ, i.e. $\sum_{i=1}^{N_t} \rho_i = \rho$. Such a system is called an $N_t \times N_r$ system. When $N_t = N_r = 1$, the system is a single-input single-output (SISO) system. When $N_t = 1$ and $N_r > 1$, the system is called a single-input multiple-output (SIMO) system. When $N_t > 1$ and $N_r = 1$, the system is called a multiple-input single-output (MISO) system. Otherwise, when both N_t and N_r are larger than 1, the system is an $N_t \times N_r$ MIMO system.

In the following, based on the input-output relations (2.1) and (2.2), we will discuss the impact of fading on the receiver's decoding of the transmit information, i.e., on the communication reliability establishment at the receiver.

2.2.2 Fading phenomenon

In wireless communications, the average receive signal power in general changes with respect to the changing distance between the transmitter and the receiver. In addition, large objects in the environment cause the average receive signal power to vary randomly. Two such effects, normally referred to as *path loss* and *shadowing* respectively, form a *large-scale fading*. In cellular systems, large-scale fading occurs when the mobiles move through a distance of the order of the cell size and is more relevant to issues such as cell-site planning [13]. Hence, in this thesis, we only consider another type of fading which is more related to reliable and efficient communication systems design: *small-scale fading* [13].

Small-scale fading is caused by the constructive and destructive interference of the multiple signal paths between the transmitter and the receiver and leads to rapid fluctuations of the amplitudes and phases of received signal. Unlike large-scale fading, which is frequency independent, small-scale fading is normally frequency dependent. An important characteristic of small-scale fading is the coherence bandwidth B_c, which denotes the range of frequencies over which two unit-amplitude frequency components have a high amplitude correlation at the receiver [37]. Denoting B_s as the transmit signal bandwidth, if $B_c \gg B_s$, the received signal is said to undergo *frequency-flat fading*. For frequency-flat fading, only the strength of the transmit signal is changed due to the channel gain but the spectrum characteristics can be preserved at the receiver. On the other hand, if $B_c \ll B_s$, the channel fading is referred to as *frequency-selective*. In this scenario, the received signal is distorted by intersymbol interference (ISI) since the receiver simultaneously receives signals coming from different propagation paths. By the use of signal processing techniques such as Orthogonal Frequency Division Multiplexing (OFDM) [39, 40], a frequency-selective fading channel can be converted to a frequency-flat fading channel. Therefore, in this thesis, we only consider frequency-flat fading channels.

In mobile radio channels, when the number of reflectors is large, *Rayleigh fading* is commonly used to model a frequency-flat fading environment. In this model, the received signal envelope varies randomly with respect to time following a Rayleigh distribution. More specifically, the channel fading coefficient h in the discrete complex baseband model (2.1) (also each $h_{i,j}$ in (2.2)) is a circular symmetric complex Gaussian random variable with zero mean and variance σ^2, i.e.

$$h = h_r + h_i j, \tag{2.3}$$

where the real and imaginary parts h_r and h_i are i.i.d. Gaussian random variables with zero mean and variance $\frac{\sigma^2}{2}$. Hence, the magnitude $|h|$ is Rayleigh distributed with density function

$$f(r) = \frac{r}{\sigma^2} \exp\left\{-\frac{r^2}{2\sigma^2}\right\}, \quad r \geq 0. \tag{2.4}$$

And the squared magnitude $|h|^2$ is exponentially distributed with density function

$$f(r) = \frac{1}{\sigma^2} \exp\left\{-\frac{r}{\sigma^2}\right\}, \quad r \geq 0. \tag{2.5}$$

Throughout the thesis, we assume the value of σ is 1 so that ρ also denotes the average receive

SNR.

Another important characteristic for small-scale fading is the channel coherence time T_c. Corresponding to the channel coherence bandwidth B_c, T_c is the time duration over which two received signals have a strong potential for amplitude correlation [37]. In other words, the channel fading coefficient within T_c (usually termed a *coherence interval*) remains roughly the same. The fading channel can be modeled in a simple *block fading* model, in which the channel fading coefficient h (or $\mathbf{H}$ for a multiple-antenna system) is assumed to remain static for a coherence interval (i.e. a fading block) and changes identically and independently in different coherence intervals. In addition, if the source's transmitted codeword spreads over multiple fading blocks (i.e. the delay requirement of the application is much larger than the coherence time), as displayed in Figure 2.2 (a), it is said the channel is in *fast fading*. On the contrary, the channel is categorized as *slow fading* if the transmitted codeword only experiences a single fading block or fading realization (i.e. the delay requirement is less than the coherence time), as displayed in Figure 2.2 (b).

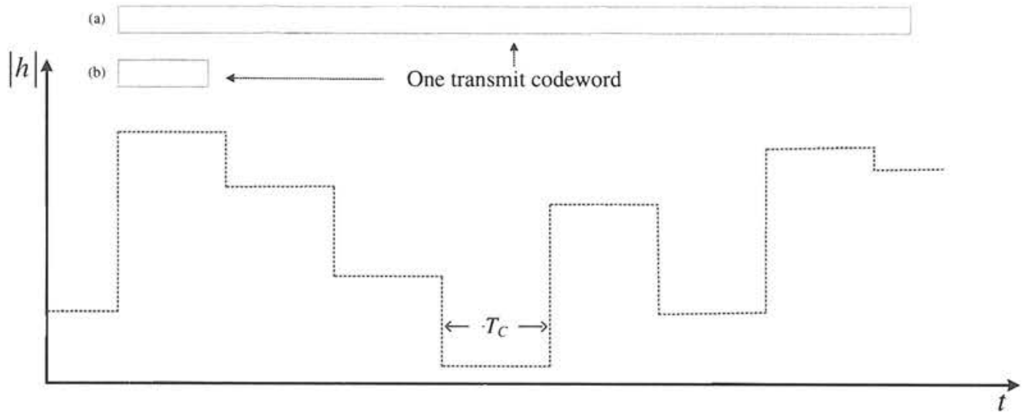

Figure 2.2: *Block fading models: (a) fast fading and (b) slow fading.*

2.2.3 Channel state information

Throughout the thesis, we assume that the receiver always has full channel state information (CSI). In other words, each realization of the channel fading coefficients (both amplitudes and phases) is perfectly known at the receiver of each link. To attain the receiver side full CSI in practice, the transmitter sends a training sequence (known to the receiver) during each coherence interval. Then the receiver can estimate the channel characteristics based on its received signal [41]. The full CSI may also be obtained at the transmitter through feedback sent by the receiver. With transmitter side CSI, the transmitter can adjust its transmission according to the channel characteristics to improve the communication performance. For example, in multiple-antenna systems, the waterfilling algorithm allocates transmit power to different transmit antennas to achieve the optimal system capacity [13] [42]. In multi-user systems, a multi-user diversity gain can be obtained by letting only the user, whose channel is well above its average value, to transmit [13]. Another example is that in multi-hop relay systems, power and/or bandwidth allocations based on the transmitter side CSI can be adopted to minimize decoding errors at the destination [43–45]. In general, attaining the transmitter side full CSI requires additional resource (e.g. a high-rate feedback channel) along with large signalling overhead. Partial CSI containing, for example, only the amplitudes [46] or even a scalar quantized version of the amplitudes [47] of the channel fading coefficients may serve as an optional substitution. However, to keep the system simple, we do not consider transmitter side CSI in this thesis. As a result, for a system with multiple transmit antennas, the transmit power is evenly assigned to each antenna. The matrix $\mathbf{P}$ in (2.2) can thus be simplified as a single scalar $\sqrt{\frac{\rho}{N_t}}$.

2.3 Fast fading environment

As mentioned in the above section, if the codeword length spans many fading blocks, the transmission is in a fast fading environment. In this section, we introduce the concept of ergodic capacity, which implies that it is always possible to establish reliable communication in fast fading channels. We start from the time-invariant channels where the channel fading does not change with respect to time.

2.3.1 Time-invariant channels

Considering the input-output relation (2.1) for a single-antenna system, if the channel fading coefficient h is fixed, the mutual information[2] $\mathcal{I}$ between the transmitted signal x and the received signal y is expressed as

$$\mathcal{I} = \log\left(1 + \rho|h|^2\right) \tag{2.6}$$

in bits per channel use (BPCU). If the transmission rate R is below $\mathcal{I}$, reliable communication between the transmitter and the receiver can always be established (i.e. the decoding error at

[2]Since we assume the transmitted codewords are chosen from i.i.d. Gaussian random codebooks, the mutual information actually represents the maximal mutual information of the channel.

the receiver can be made arbitrarily small). Equation (2.6) is termed the *Shannon capacity* of the deterministic channel.

For the multiple-antenna system (2.2), the mutual information (i.e. the Shannon capacity) is given by [21]

$$\mathcal{I} = \log\det\left(\mathbf{I} + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^H\right). \tag{2.7}$$

If the transmission rate is below the mutual information (2.7), the channel realization can support the transmission so that correct decoding at the destination is always possible.

If the channel is random, the situation is more complicated. However, in fast fading channels, correct decoding can still be guaranteed as long as the transmission obeys certain conditions. Furthermore, the use of multiple antennas at both the transmitter and the receiver leads to a substantial gain in terms of transmission rate.

2.3.2 Fast fading channels

In fast fading channels, information theory shows that by choosing a long codeword length such that the codeword experiences a sufficiently large number of independent fades, the decoding error can be made arbitrarily small if the transmission rate is below the *average* mutual information between the transmitted and received signals. More specifically, for a single-antenna system, the transmission rate constraint is expressed by

$$R \leq \mathcal{E}_h\left\{\log\left(1 + \rho|h|^2\right)\right\}, \tag{2.8}$$

where $\mathcal{E}_h$ denotes expectation with respect to the random channel. For a more general $N_t \times N_r$ system expressed in (2.2), correct decoding at the destination can be guaranteed if the following inequality is met

$$R \leq \mathcal{E}_{\mathbf{H}}\left\{\log\det\left(\mathbf{I} + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^H\right)\right\}. \tag{2.9}$$

The right hand side (RHS) of (2.9) defines the upper bound of the reliable transmission rate the channel can afford. Such an upper bound is called the *ergodic capacity* (or simply *capacity*) of the fast fading channel.

To illustrate the capacity difference between single-antenna and multiple-antenna systems, we display the ergodic capacity of a 1×1 SISO system, a 1×4 SIMO system, a 4×1 MISO system, and a 2×2 MIMO system in Figure 2.3. It can be seen from the figure that at high

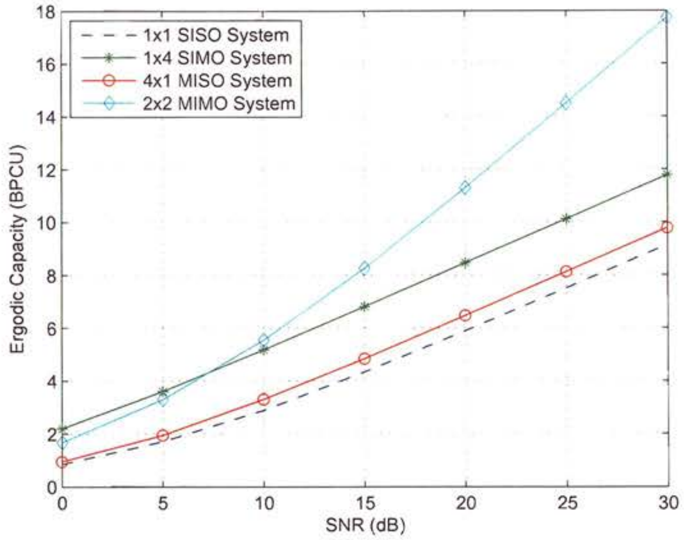

Figure 2.3: *Ergodic capacity comparison of multiple-antenna configurations.*

SNR, the use of multiple antennas at either the transmitter or the receiver only leads to a small capacity improvement over a SISO system. However, the capacity dramatically increases when equipping multiple antennas at both sides of the link. This is because at high SNR, the ergodic capacity of an $N_t \times N_r$ system can be approximated as

$$\mathcal{E}_{\mathbf{H}}(\mathcal{I}) \approx \min\{N_t, N_r\} \log \frac{\rho}{N_t} + \mathcal{O}(1), \tag{2.10}$$

where $\mathcal{O}(1) > -\infty$ [13]. Roughly, at high SNR, the ergodic capacity of a MIMO system grows like $\min\{N_t, N_r\} \log \rho$ instead of $\log \rho$ for a system in which one side or both sides of the link are equipped with a single antenna. The gain $\min\{N_t, N_r\}$ (i.e. the minimum number of transmit and receiver antennas) is called *spatial multiplexing gain* and leads to a substantial capacity improvement.

The MIMO capacity (2.9) can be achieved by sending N_t independently coded data streams through the N_t transmit antennas and performing joint maximal likelihood (ML) decoding of the data streams at the receiver. However, the complexity of ML decoding grows exponentially with the number of transmit antennas [13]. Even though the complexity can be decreased by the use of reduced-complexity decoding schemes such as sphere decoding (e.g. [48, 49]), it may still be intolerable when large number of antennas and high order modulation are used. A practical solution of this problem is to use a V-BLAST algorithm [50, 51] at the destination. Instead of jointly decoding all the data streams simultaneously, the V-BLAST algorithm conducts a successive interference cancellation (SIC) process. Specifically, the destination first detects the "best" stream which has the highest signal-to-interference-plus-noise ratio (SINR) while treating other streams as Gaussian noise. Afterwards, this stream is subtracted from the received signal and the "second best" data stream is detected. The process continues until the detection of the last data stream is finished. With a linear minimum mean-square error (MMSE) nulling, the so-called V-BLAST-MMSE receiver also achieves the MIMO capacity [13].

2.4 Slow fading environment

For fast fading channels, the ergodic capacity is defined as the maximal transmission rate at which reliable transmission can be guaranteed. However, for slow fading channels, such a positive upper bound of reliable transmission rate does not exist. This is because the transmit codeword spans only a single fading block such that no matter how small the transmission rate is, there is always a nonzero probability that the mutual information $\mathcal{I}$ is below the transmission rate R (i.e. the channel realization does not support the rate). When $R > \mathcal{I}$, the system is said to be in *outage* and it is impossible to drive the error probability to zero. Corresponding to the ergodic capacity for fast fading channels, the $\epsilon\%$ *outage capacity* C_ϵ for slow fading channels is defined as the transmission rate at which $(1-\epsilon)\%$ of the channel realizations can be guaranteed [52], i.e.

$$P\left(\mathcal{I} \leq C_\epsilon\right) = \epsilon\%. \tag{2.11}$$

Another important performance measurement for slow fading channels is the *outage probability*, which is defined as the probability that an outage event occurs at transmission rate R, i.e.

$$P_{\text{out}} = P(R > \mathcal{I}). \tag{2.12}$$

In what follows, we will first look at the outage probability performance of a single-antenna system and then introduce the concept of diversity that aims at improving the outage probability performance.

2.4.1 Single-antenna system

For a SISO system, to guarantee correct decoding at the destination, the transmission rate should be constrained as the following inequality

$$R \leq \log\left(1 + \rho|h|^2\right). \tag{2.13}$$

Otherwise, outage occurs. The outage probability thus is expressed by

$$P_{\text{out}} = P\left(R > \log\left(1 + \rho|h|^2\right)\right) = P\left(|h|^2 < \frac{2^R - 1}{\rho}\right). \tag{2.14}$$

Because the source codeword is conveyed to the destination through only a single path, the communication reliability highly depends on the single fading realization. If the channel is in deep fade such that it cannot support the transmission rate, the destination cannot correctly decode the source codeword and the transmission fails. To gain a deeper understanding, recall that the squared magnitude $|h|^2$ for Rayleigh fading environments follows an exponential distribution with density function (2.5). As a result, at high SNR, the outage probability (2.14) can be approximated as:

$$P_{\text{out}} \approx \frac{2^R - 1}{\rho}. \tag{2.15}$$

Clearly, with a fixed transmission rate R, the outage probability decays only as $\frac{1}{\rho}$.

The performance can be improved by so-called *diversity* techniques, the basic idea behind which is to transit information through multiple independently-faded paths. By this means, as long as one of the paths is strong enough, reliable communication can be established so that the outage performance is substantially improved. In general, diversity can be exploited by sending the same information over time, frequency, and/or space domains. Since we consider frequency-flat fading environments, in the following, we introduce the time diversity and spatial diversity respectively.

2.4.2 Time diversity

Time diversity can be obtained via sending information to the destination over different fading blocks (coherence intervals) to experience independent fading realizations. The simplest approach is a repetition-coding scheme, in which the source repeats the same codeword in different fading blocks. Assuming that the transmission takes N_l time slots (each spans a single block), the input-output relation can be expressed by

$$\begin{bmatrix} y[1] \\ y[2] \\ \vdots \\ y[N_l] \end{bmatrix} = \sqrt{\rho} \underbrace{\begin{bmatrix} h[1] \\ h[2] \\ \vdots \\ h[N_l] \end{bmatrix}}_{\mathbf{h}} x + \begin{bmatrix} n[1] \\ n[2] \\ \vdots \\ n[N_l] \end{bmatrix}, \tag{2.16}$$

where $y[i]$ denotes the signal received at the destination during the ith time slot, and $h[i]$ is the associated channel fading coefficient. We assume the codeword x is transmitted with rate $\bar{R}$ bits per codeword. Since N_l time slots (i.e. N_l channels) are used to transmit x, the average transmission rate R BPCU is calculated as

$$R = \frac{1}{N_l}\bar{R}. \tag{2.17}$$

The destination combines the signals it received in all N_l time slots and performs joint decoding to recover the transmitted codeword. Therefore, the destination can correctly decode the source codeword if the following inequality is met

$$\bar{R} \leq \log\left(1 + \rho||\mathbf{h}||^2\right), \tag{2.18}$$

in which the left hand side (LHS) denotes the information the source sends and the RHS denotes the overall mutual information between the transmitted and received signals for all the N_l time slots. Substituting (2.17) into (2.18), the outage probability can be expressed by

$$P_{\text{out}} = P\left(R > \frac{1}{N_l}\log\left(1 + \rho||\mathbf{h}||^2\right)\right) = P\left(\sum_{i=1}^{N_l} |h[i]|^2 < \frac{2^{N_l R} - 1}{\rho}\right). \tag{2.19}$$

For Rayleigh fading environments, the sum squared magnitudes $\sum_{i=1}^{N_l} |h[i]|^2$ follows a Chi-square distribution with $2N_l$ degrees of freedom, the density function of which is

$$f(r) = \frac{1}{(N_l - 1)!} r^{N_l - 1} e^{-r}, \quad r \geq 0. \tag{2.20}$$

At high SNR, the outage probability can be approximated by [13]

$$P_{\text{out}} \approx \frac{\left(2^{N_l R} - 1\right)^{N_l}}{N_l! \rho^{N_l}}. \tag{2.21}$$

We can see with a fixed transmission rate R, the outage probability now decays as $\frac{1}{\rho^{N_l}}$, which is much faster than $\frac{1}{\rho}$ in (2.15). We refer to the value of N_l as the *maximal diversity gain* (the precise definition of diversity gain will be introduced later).

The repetition-coding scheme requires the source to repetitively transmit a short codeword, each of which spans a single fading block. On the other hand, one can encode the information into a long codeword such that the single codeword spans N_l independent fading blocks. In such a case, reliable communication can be guaranteed as long as the following inequality is met

$$R \leq \frac{1}{N_l} \sum_{i=1}^{N_l} \log\left(1 + \rho |h[i]|^2\right). \tag{2.22}$$

For sufficiently large N_l, due to the law of large numbers, the RHS of (2.22) approaches a positive value $\mathcal{E}_h \left\{\log\left(1 + \rho |h|^2\right)\right\}$, which is the ergodic capacity of fast fading channels we discussed in Section 2.3.2. This confirms that by coding over a large number of coherence intervals (i.e. a fast fading environment), a positive reliable rate of transmission can be guaranteed.

2.4.3 Spatial diversity

When equipping multiple antennas at the transmitter and/or the receiver, as long as the multiple antennas are located sufficiently far apart, the fading between each transmit and receive antenna pair can be considered as independent. Therefore, diversity can be realized over the space domain.

2.4.3.1 SIMO systems

We first consider the simplest case in which the transmitter is equipped with a single antenna while the receiver has N_r antennas. The input-output relation of this SIMO system is expressed by

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{N_r} \end{bmatrix} = \sqrt{\rho} \underbrace{\begin{bmatrix} h_{1,1} \\ h_{1,2} \\ \vdots \\ h_{1,N_r} \end{bmatrix}}_{\mathbf{h}} x + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{N_r} \end{bmatrix}. \tag{2.23}$$

To guarantee correct decoding at the destination, the transmission rate constraint is expressed as

$$R \leq \log\left(1 + \rho||\mathbf{h}||^2\right) = \log\left(1 + \rho \sum_{i=1}^{N_r} |h_{1,i}|^2\right). \tag{2.24}$$

Following the analysis in Section 2.4.2, the approximation of outage probability at high SNR can be expressed by

$$P_{\text{out}} \approx \frac{\left(2^R - 1\right)^{N_r}}{N_r! \rho^{N_r}}. \tag{2.25}$$

The fact that with fixed transmission rate the outage probability decays as $\frac{1}{\rho^{N_r}}$ shows the maximal diversity gain is N_r.

2.4.3.2 MISO systems

When the transmitter is equipped with N_t antennas but the receiver is a single-antenna terminal, the input-output relation of such a MISO system can be expressed as

$$y = \sqrt{\frac{\rho}{N_t}} \underbrace{\begin{bmatrix} h_{1,1} & h_{2,1} & \cdots & h_{N_t,1} \end{bmatrix}}_{\mathbf{h}} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_t} \end{bmatrix} + n. \tag{2.26}$$

The reliable transmission rate is limited by

$$R \leq \log\det\left(\mathbf{I} + \frac{\rho}{N_t}\mathbf{h}\mathbf{h}^H\right) = \log\left(1 + \frac{\rho}{N_t}\sum_{i=1}^{N_t} |h_{i,1}|^2\right). \tag{2.27}$$

The high SNR outage probability approximation can be written as

$$P_{\text{out}} \approx \frac{\left(2^R - 1\right)^{N_t}}{N_t! \left(\frac{\rho}{N_t}\right)^{N_t}}. \tag{2.28}$$

Clearly, with a fixed R, the outage probability decays as $\frac{1}{\rho^{N_t}}$ so that the maximal diversity gain is N_t.

In practice, the maximal diversity gain can be achieved by a simple method in which the transmission spans N_t time slots and during each time slot a single transmit antenna is activated to send information to the destination. Time diversity coding schemes we discussed in the last subsection can be directly adopted in this case. Specifically, the signals transmitted from different antennas can be the same (i.e. the repetition-coding scheme) or coding can be done across all the transmitted signals. Another approach is to use codes specifically designed for systems with multiple transmit antennas: space-time codes [16–19], where the transmitted information is encoded across both space domain (i.e. across different antennas at the same time) and time domain (i.e. across different time at the same antenna).

2.4.3.3 MIMO systems

If both the transmitter and the receiver are equipped with multiple antennas, the transmission rate at which reliable communication can be guaranteed is limited by

$$R \leq \log\det\left(\mathbf{I} + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^H\right). \tag{2.29}$$

At high SNR, it can be proved that the maximal diversity gain is $N_t N_r$ (i.e. the outage probability decays as a function of $\frac{1}{\rho^{N_t N_r}}$ for fixed R). The maximal diversity gain can be practically achieved by using a single transmit antenna to communicate with the destination at each of N_t time slots or using all the N_t antennas to send information simultaneously through space-time codes.

In Figure 2.4 and Figure 2.5, we display the 10% outage capacity and outage probability ($R = 1$ BPCU) performance for some configurations. For spatial diversity systems, the behavior of the 10% outage capacity for slow fading channels is almost identical to that of the ergodic capacity for fast fading channels displayed in Figure 2.3. At high SNR, a MIMO system can dramatically enhance the outage capacity performance over systems without using multiple antennas at both sides. From Figure 2.5, we can see that the 1×4 SIMO system, 4×1 MISO system, and the 2×2 MIMO system have the same high-SNR outage probability slopes because all the three systems achieve maximal diversity gain 4. The same diversity performance can also be obtained by the repetition-coding time diversity scheme with $N_l = 4$. However, the repetition-coding

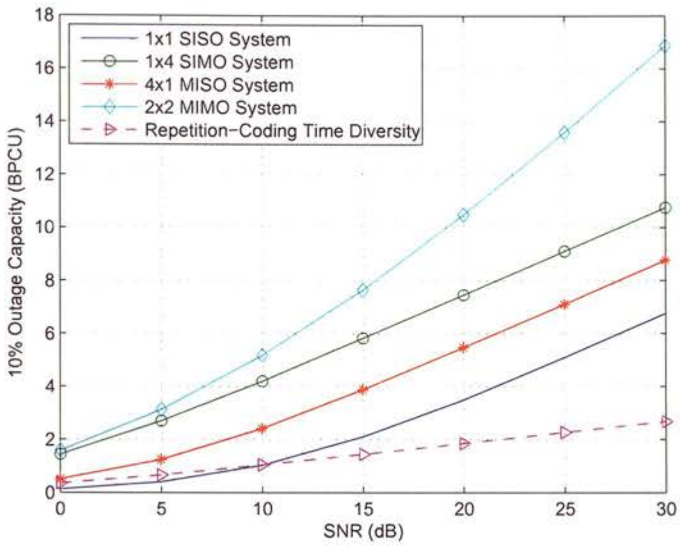

Figure 2.4: 10% *outage capacity comparison of multiple-antenna configurations. For the repetition-coding time diversity scheme, the transmission takes $N_L = 4$ time slots and both the transmitter and the receiver are equipped with a single antenna.*

scheme has a much smaller outage capacity performance. This can be simply explained by, for example, comparing the repetition-coding time diversity scheme with the 1×4 SIMO system. The two systems have similar input-output relations (2.16) and (2.23) except that independent fading realizations are obtained in either the time domain or the space domain. To send the same codeword to the destination, the repetition-coding time diversity scheme uses four time slots (channels) while the 1×4 SIMO system uses only one. Clearly, although both systems have a same diversity performance, the spatial diversity is more spectrally efficient.

In the following subsection, we will provide a more theoretical explanation of the inefficiency of the repetition-coding time diversity scheme compared with the use of multiple antennas to realize spatial diversity. We will also show the substantial advantages of using multiple antennas at both the transmitter and the receiver sides. These will be done through a performance metric named diversity-multiplexing tradeoff.

2.4.4 Diversity-multiplexing tradeoff

For slow fading environments, when the SNR approaches infinity, the diversity gain d is defined as the rate at which the outage probability decays with increasing SNR. The multiplexing gain r

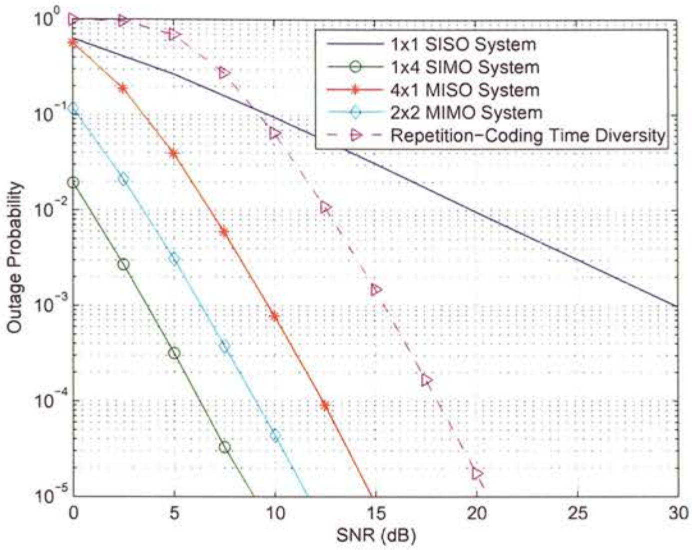

Figure 2.5: *Outage probability comparison for $R = 1$ BPCU.*

is defined as the rate at which the transmission rate scales with respect to SNR. More precisely, d and r are defined as [36]

$$d = -\lim_{\rho \to \infty} \frac{\log(P_{\text{out}}(\rho))}{\log \rho}, \tag{2.30}$$

$$r = \lim_{\rho \to \infty} \frac{\log(R(\rho))}{\log \rho}. \tag{2.31}$$

The curve $d(r)$ is termed *diversity-multiplexing tradeoff* (DMT). With a fixed multiplexing gain, higher diversity gain implies that the outage probability decays faster with increasing SNR (i.e. the outage probability has a deeper high-SNR slope). With a fixed diversity gain, higher multiplexing gain implies that increasing the SNR by the same amount can lead to a faster increase of transmission rate while still maintaining the same high-SNR outage probability slope. In addition, for any system, if the multiplexing gain $r = 0$ (i.e. the transmission rate is fixed), the diversity gain d achieves its maximal value, which represents the maximal protection of the transmitted information provided by the system. On the other hand, if the diversity gain $d = 0$ (i.e. the high-SNR outage probability does not vary with changing SNR), the multiplexing gain r achieves its maximal value, which represents the maximal amount of transmit information rate afforded by the system. Therefore, through DMT analysis, any transmission scheme can be simultaneously measured from both spectral efficiency and communication reliability aspects.

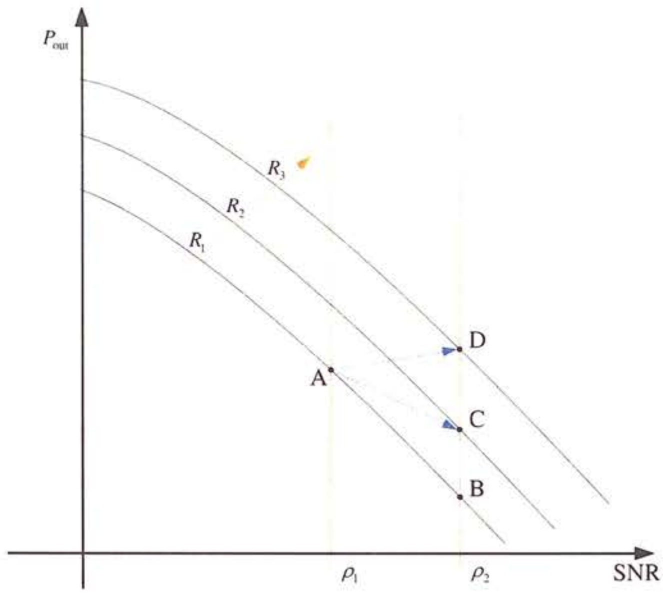

Figure 2.6: *Outage probabilities versus SNR for three different transmission rates* $R_1 < R_2 < R3$.

To more explicitly explain the idea of DMT, we sketch the outage probability performance versus SNR for three different transmission rates in Figure 2.6. For a large value of SNR ρ_1, the outage probability for rate R_1 is displayed as point "A". When the SNR is increased to ρ_2, if the transmission rate is fixed at R_1 (the outage probability is displayed as point "B"), the multiplexing gain $r = 0$ and the maximal diversity gain is achieved. When the transmission rate R_1 is also increased with increasing SNR (i.e. $r \neq 0$) and becomes R_2 at ρ_2, the outage probability is displayed as point "C". If the rate does not increase too fast (the multiplexing gain is less than the maximal multiplexing gain), a positive diversity gain can still be obtained and the outage probability still decreases with increasing SNR. Clearly, the slope of the line segment "AB" is steeper than the line segment "AC". On the other hand, when the rate R_1 is increased too fast (the multiplexing gain is higher than the maximal multiplexing gain) and it reaches R_3 at ρ_2, the outage probability (displayed as point "D") is even higher than that for R_1 at a relatively smaller value of SNR ρ_1. This means the system can no longer provide a positive diversity gain.

Now, assuming that when the SNR approaches infinity, the transmission rate R scales like $R = r \log \rho$ (i.e. (2.31)) and considering (2.18), it is not difficult to have the achievable DMT for the repetition-coding time diversity scheme as

$$d(r) = N_l\,(1 - N_l r)\,, \qquad 0 \le r \le \frac{1}{N_l}. \tag{2.32}$$

Similarly, using (2.24), the achievable DMT for the SIMO channel is expressed by

$$d(r) = N_r\,(1 - r)\,, \qquad 0 \le r \le 1. \tag{2.33}$$

Assuming $N_l = N_r = N$, it can be seen that although both schemes achieve the same maximal diversity gain N, the repetition-coding time diversity only obtains maximal multiplexing gain $\frac{1}{N}$, which is much worse than 1, the maximal achievable multiplexing gain of a $1 \times N$ SIMO system. These results more fundamentally explain the reason that the repetition-coding time diversity scheme has a much worse outage capacity performance compared with the SIMO system as displayed in Figure 2.4, although in Figure 2.5 both schemes have the same high-SNR outage probability slopes.

For a more general system with N_t transmit antennas and N_r receive antennas, the optimal achievable DMT is a piecewise linear curve connecting the points [36]

$$(k, (N_t - k)(N_r - k))\,,\ \ k = 0, 1, \ldots, \min\{N_t, N_r\}. \tag{2.34}$$

Clearly, the maximal achievable diversity gain is the maximal number of independent fading links through which a codeword can be conveyed (i.e. $N_t N_r$, as mentioned in Section 2.4.3.3). The maximal achievable multiplexing gain is the minimum number of transmit and receive antennas (i.e. $\min\{N_t, N_r\}$, also known as the maximal degree of freedom the link can provide [13]).

The DMT comparison of the configurations considered in Figure 2.4 and Figure 2.5 is displayed in Figure 2.7. The maximal diversity gain of a single-antenna system can be significantly improved by both the time diversity and spatial diversity schemes. However, the repetition-coding time diversity scheme only attains maximal multiplexing gain $\frac{1}{4}$ since it uses 4 time slots (channels) to transmit only one codeword. The cost of the diversity improvement is a sacrifice of transmission rate. On the other hand, both the 1×4 SIMO and 4×1 MISO systems obtain maximal multiplexing gain 1, which matches the result for a SISO system. The diversity is improved in a very efficient way. In addition, when equipping 2 antennas at both the transmitter and the receiver, a 2×2 MIMO system can achieve maximal diversity gain 4 and

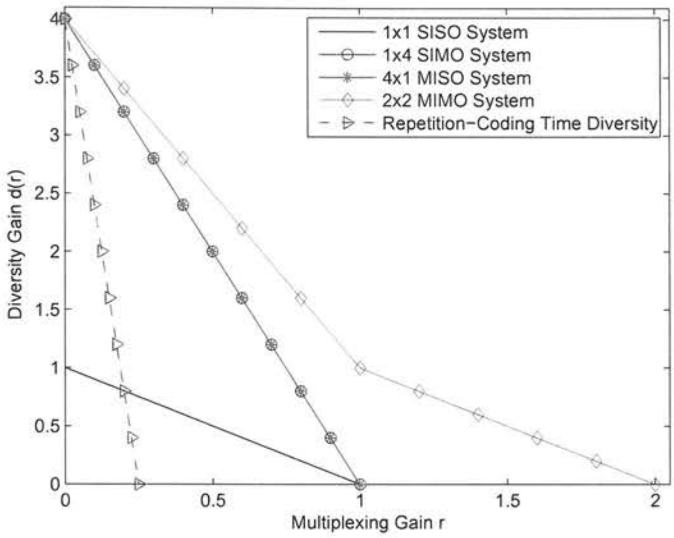

Figure 2.7: *DMT comparison of multiple-antenna configurations.*

an even higher maximal multiplexing gain 2. Such a multiplexing gain improvement results in an important advantage in terms of spectral efficiency and explains the good outage capacity performance of the 2×2 MIMO system in Figure 2.4.

2.5 Multi-user systems

In the last section, we discussed the achievable DMT performance for point-to-point communication systems where a single terminal sends information to another terminal. In practical cellular systems, wireless communications generally consider a network setup where multiple terminals (e.g. mobile users) intend to communicate with a common terminal (e.g. base station). For such scenarios, communication techniques have to deal with not only the fading effects but also the interference generated among terminals. Multiple access is referred to as the techniques that allow multiple users to share the available radio spectrum [13]. For a narrowband system (the user transmissions are restricted to separate narrowband channels), the channel is normally divided into many orthogonal subchannels across time or frequency and each mobile user individually uses one of the subchannels to transmit or receive. The orthogonal multiple access techniques aim to eliminate inter-user interference. However, from an information theory viewpoint, they are not DMT optimal. In this thesis, we consider a narrowband uplink transmission in which M source terminals (mobile users) communicate with a common destination (base station), which is displayed in Figure 2.8 (a). In the following, we provide the achievable DMT performance for an orthogonal multiple access technique, time-division-multiple-access (TDMA), and the optimal DMT performance the multiple-source system can support. For simplicity, we assume each source $\mathcal{S}_i$ ($i = 1, \dots, M$) intends to transmit one independent codeword x_i to the destination $\mathcal{D}$. In addition, it is assumed that the system is symmetric, which means each source has the same number of transmit antennas N_t, the same average transmit power ρ, and the same multiplexing gain r.

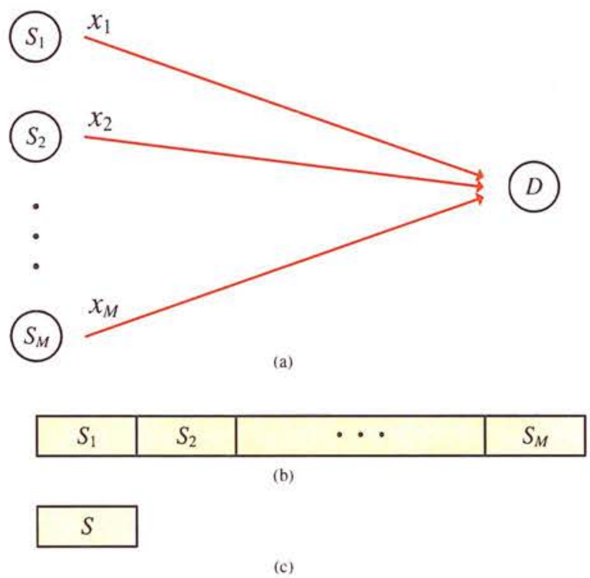

Figure 2.8: *(a) System model of an M-source network, and time-division channel allocations for (b) TDMA, and (c) multiple-access channel. The source displayed in each time slot denotes the transmitter during that time slot. S means all the sources transmit simultaneously.*

2.5.1 Time-division-multiple-access

TDMA divides the available time channel into M individual time slots, during each of which one user is activated to send information to the destination, as displayed in Figure 2.8 (b). For a single-antenna system, the overall input-output relation can be expressed by

$$\begin{bmatrix} y[1] \\ y[2] \\ \vdots \\ y[M] \end{bmatrix} = \sqrt{\rho} \begin{bmatrix} h_{\mathcal{S}_1} & 0 & \cdots & 0 \\ 0 & h_{\mathcal{S}_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{\mathcal{S}_M} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} + \begin{bmatrix} n[1] \\ n[2] \\ \vdots \\ n[M] \end{bmatrix}, \tag{2.35}$$

in which $y[i]$ and $n[i]$ denote the received signal and noise at the destination during the ith time slot, and $h_{\mathcal{S}_i}$ denotes the channel fading coefficient between $\mathcal{S}_i$ and $\mathcal{D}$. Since the channel is orthogonally allocated to each source, the transmission of each source is not interfered with transmissions from the other sources. Assuming the transmission rate of source $\mathcal{S}_i$ is $\bar{R}_i$ bits per codeword, to guarantee correct decoding at the destination, the following rate constraints should be met

$$\bar{R}_i \leq \log\left(1+\rho|h_{\mathcal{S}_i}|^2\right) \quad i \in \{1, 2, \cdots, M\}. \tag{2.36}$$

Since M time slots are used to transmit only one codeword from each source (or each source uses only $\frac{1}{M}$ of the total channel to send information), the average transmission rate $R_i = \frac{1}{M}\bar{R}_i$ BPCU. Assuming $R_i = r\log\rho$ for infinite-SNR, the achievable DMT for each source is easily obtained as

$$d(r) = 1 - Mr, \qquad 0 \leq r \leq \frac{1}{M}. \tag{2.37}$$

For a general case in which each source are equipped with N_t antennas and the destination has N_r receive antennas, denoting $d^*_{N_t,N_r}(r)$ as the optimal DMT achieved by a point-to-point $N_t \times N_r$ MIMO system (the DMT curve is expressed by (2.34)), the achievable DMT for each source is $d^*_{N_t,N_r}(Mr)$.

2.5.2 DMT for multiple-access channels

The optimal DMT of the multi-user system can be obtained by relaxing the orthogonal transmission requirement and letting all the M sources communicate with the destination simultaneously (such a setup is normally termed a *multiple-access channel*), as displayed in Figure 2.8 (c). The input-output relation for a single-antenna system can be expressed by

$$y = \sqrt{\rho}\underbrace{\begin{bmatrix} h_{\mathcal{S}_1} & h_{\mathcal{S}_2} & \cdots & h_{\mathcal{S}_M} \end{bmatrix}}_{\mathbf{h}} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} + n, \tag{2.38}$$

Clearly, the nonorthogonal multiple access transmission requires high complexity at the destination since M independent codewords rather than one need to be decoded at a time. Since the transmission of each source is independent and interferes the other sources, to guarantee correct decoding at the destination, the transmission rate constraints for the multiple-access channel are expressed as [53]

$$R_i \leq \log\left(1+\rho|h_{i,1}|^2\right) \quad i \in \{1,2,\cdots,M\}, \tag{2.39}$$

$$R_{i_1}+R_{i_2} \leq \log\left(1+\rho|h_{i_1,1}|^2+\rho|h_{i_2,1}|^2\right) \quad i_1,i_2 \in \{1,2,\cdots,M\}, \tag{2.40}$$

$$\cdots \tag{2.41}$$

$$\sum_{i=1}^{M} R_i \leq \log\left(1+\rho\sum_{i=1}^{M}|h_{i,1}|^2\right) = \log\left(1+\rho\mathbf{h}\mathbf{h}^H\right) \tag{2.42}$$

An outage event occurs when any of the above inequalities is not met. The achievable DMT for each source can be expressed by [54]

$$d(r) = \min\{1-r, M(1-Mr)\} = \begin{cases} 1-r & 0 \leq r \leq \frac{1}{M+1} \\ M(1-Mr) & \frac{1}{M+1} \leq r \leq \frac{1}{M} \end{cases}. \tag{2.43}$$

The DMT performance (2.43) shows that for a small multiplexing gain $0 \leq r \leq \frac{1}{M+1}$, the system performs the same as an interference-free channel where users do not interfere with each other (the DMT $d(r) = 1-r$ for a point-to-point single-antenna system is achieved). On the other hand, if the multiplexing gain $\frac{1}{M+1} \leq r \leq \frac{1}{M}$, the achievable DMT is as if the users pool their individual antennas together and transmit with a multiplexing gain Mr (since the overall average transmission rate is $\sum_{i=1}^{M} R_i$ BPCU for the pooled-antenna system). An example of the DMT performance for a two-user system is plotted in Figure 2.9. Comparing (2.37) with (2.43), the same maximal multiplexing gains $\frac{1}{M}$ imply that TDMA achieves the optimal maximal multiplexing gain of the multi-user system. However, the fact that the DMT curve of TDMA is below that of the multiple-access channel (except when $r = 0$ and $r = \frac{1}{M}$ where both schemes have the same diversity gains) means the nonorthogonal multiple access uses the channel resource more efficiently.

In addition, for a general symmetric network where each source is equipped with N_t antennas and the destination is equipped with N_r antennas, the achievable DMT for each source is [54]

$$d(r) = \min\{d^*_{N_t,N_r}(r), d^*_{MN_t,N_r}(Mr)\} = \begin{cases} d^*_{N_t,N_r}(r) & r \leq \min\{N_t, \frac{N_r}{M+1}\} \\ d^*_{MN_t,N_r}(Mr) & r \geq \min\{N_t, \frac{N_r}{M+1}\} \end{cases}. \tag{2.44}$$

Similarly, the achievable DMT performance of a multiple-access channel cannot exceed that of a point-to-point $N_t \times N_r$ single-source system (i.e. $d^*_{N_t,N_r}(r)$). If the multiplexing gain $r \leq \min\{N_t, \frac{N_r}{M+1}\}$, the system behaves like a point-to-point system and users do not inter-

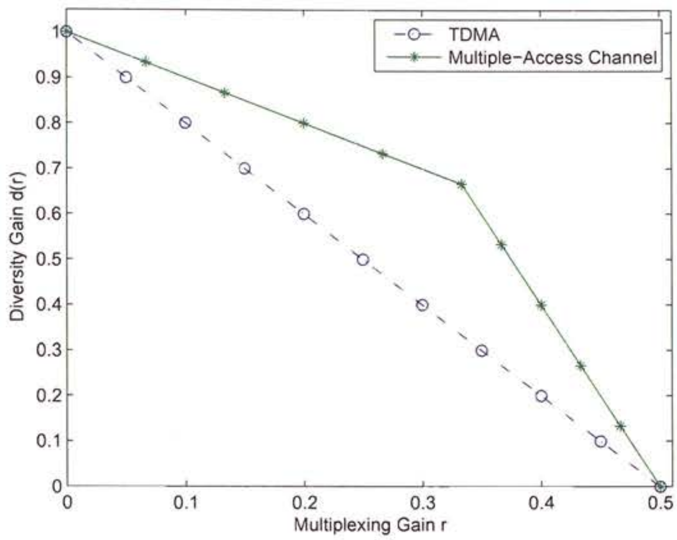

Figure 2.9: *DMT performance of a two-source single-antenna system.*

fere with each other so that the optimal DMT $d^*_{N_t,N_r}(r)$ is obtained. If the multiplexing gain $r \geq \min\{N_t, \frac{N_r}{M+1}\}$, the tradeoff is the same as that of a system where all sources pool their antennas together and transmit with multiplexing gain Mr (the DMT $d^*_{MN_t,N_r}(Mr)$ is attained). Furthermore, if the destination has a sufficiently large number of antennas such that $N_t \leq \frac{N_r}{M+1}$, the single user DMT $d^*_{N_t,N_r}(r)$ can always be achieved. This means the destination can always separate the transmit information from each individual source. Such a system is usually called a space-division-multiple-access (SDMA) system.

2.6 Multi-hop relay systems: Cooperative diversity

We have seen that the use of multiple antennas can provide dramatic advantages over single-antenna systems. However, in practical cellular systems, due to size, cost, or hardware limitations, mobile terminals may not be able to support multiple antennas, which makes it infeasible to extract all the advantages of multiple-antenna systems. To handle this issue, an emerging technique named *cooperative diversity* [29, 30] has attracted much interest in recent years. The basic idea behind cooperative diversity is that single-antenna mobiles in a multi-user scenario pool their antennas to create a virtual antenna array to emulate a multiple-antenna system [55, 56]. Specifically, when one terminal communicates with its intended destination, the other terminals involved in the cooperation framework will offer their antennas to work as *relays* to assist this transmission.

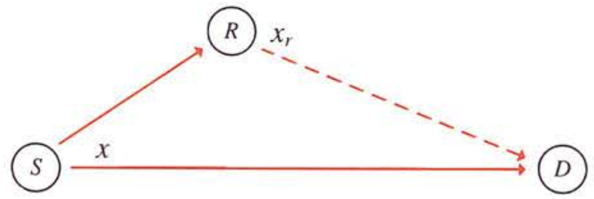

Figure 2.10: *A single-source single-relay cooperative diversity system model. The solid lines denote that the source broadcasts its codeword x to the relay and the destination. The dashed line denotes that the relay forwards x_r, the processed version of the source codeword, to the destination.*

For example, Figure 2.10 displays the simplest cooperative diversity network. In this single-antenna three-node system, a source terminal $\mathcal{S}$ intends to transmit information to its destination $\mathcal{D}$. To attain an extra protection of this transmission, another terminal (denoted as $\mathcal{R}$) which can overhear the source is activated to relay the source information to the destination. In general, the whole transmission process consists of two phases. In a broadcasting phase, the source broadcasts its transmit codeword x to the relay and the destination. And in a forwarding phase, the relay processes its received signal and retransmits it (denoted as x_r) to the destination. The destination combines the signals it received from both phases and performs joint decoding to recover the transmitted source information.[3]

The two most popular forwarding strategies for the relay are amplify-and-forward (AF) relaying and decode-and-forward (DF) relaying [31]. In AF relaying, the relay simply amplifies its received signal and retransmits it to the destination. The relay's transmitted signal x_r is

$$x_r = \kappa y_r = \kappa \left(\sqrt{\rho} h_{\mathcal{S},\mathcal{R}} x + n_r \right), \tag{2.45}$$

in which $y_r = \sqrt{\rho} h_{\mathcal{S},\mathcal{R}} x + n_r$ and n_r denote the received signal and AWGN at the relay in the broadcasting phase, $h_{\mathcal{S},\mathcal{R}}$ denotes the source-relay channel fading coefficient, and κ is the amplifying gain of the relay. The amplifying gain κ can be chosen according to the channel realization, e.g.

$$\kappa = \sqrt{\frac{1}{\rho |h_{\mathcal{S},\mathcal{R}}|^2 + 1}}, \tag{2.46}$$

[3]Conventionally, in multi-hop relaying communications, the signals are transmitted hop-by-hop. In other words, the receivers of any hop can *only* receive signals from the transmitters of the same hop. For the two-hop system displayed in Figure 2.10, it is usually assumed that the destination cannot receive signals transmitted by the source (e.g. [26, 57–59]). However, under the concept of cooperative diversity, the direct source-destination link is considered to provide spatial diversity, which is the focus of this thesis.

so that x_r has a unit average power [31]. Or κ can be simply set as a fixed value [60]. The advantage of the AF setup is that minimal signal processing is required at the relay. However, when the relay amplifies its received signal, it inevitably amplifies the noise and thus corrupts the received signal at the destination.

On the other hand, in DF relaying, the relay completely decodes the data from the source to recover the original source information. The data is then re-encoded and sent to the destination. The simplest DF strategy is to use repetition coding at the relay (i.e. the relay repeats the source codeword) [31]. The relay's transmitted signal is thus

$$x_r = x. \tag{2.47}$$

This arrangement has the advantage that receiver noise at the relay can be removed if the codeword is decoded correctly. Clearly, decoding errors at the relay may cause error propagation at the destination.

Since the source information is conveyed to the destination through two independent paths (i.e. the direct source-destination link and the source-relay-destination link), spatial diversity can be exploited in a single-antenna system to improve the performance over direct source-destination transmission without considering relaying. However, in practical systems, terminals cannot transmit and receive simultaneously in the same frequency band. The so-called *half-duplex limitation* may cause a multiplexing gain reduction in conventional protocol designs when compared with direct source-destination transmission. In the following, we will review the standard transmission protocols and highlight their diversity advantages as well as their resulting spectral *inefficiency*. Advanced transmission protocols aiming at improving the multiplexing performance of the standard protocols will also be discussed.

2.6.1 Single-relay protocols

We begin with the simplest single-relay network displayed in Figure 2.10. Due to the half-duplex limitation, the total number of channel uses has to be divided into two orthogonal portions: one for the relay to listen to the source and one for it to retransmit the source information to the destination. Assuming a TDMA system and the total transmission time for x is T, the standard transmission protocol [31] allocates half of T to the broadcasting and forwarding phases respectively. More specifically, during the first time period of duration $\frac{T}{2}$, the source broadcasts a codeword x to both the relay and the destination. During the second time period of duration $\frac{T}{2}$, the relay transmits x_r while the source is *silent*. The time-division channel allocation is displayed in Figure 2.11 (b).

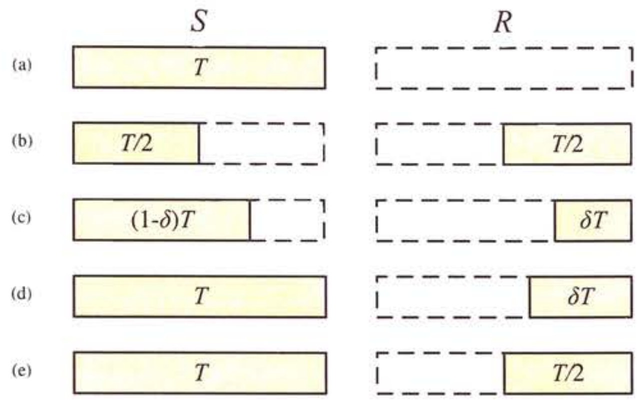

Figure 2.11: *Time-division channel allocations for (a) direct source-destination transmission, (b) standard AF/DF relaying, (c) protocols proposed in [8–10], (d) DDF relaying, and (e) NAF relaying for a single-relay network.*

For AF relaying, the input-output relation of the relay network can be written as

$$\begin{bmatrix} y_d[1] \\ y_d[2] \end{bmatrix} = \sqrt{\rho} \begin{bmatrix} h_{\mathcal{S}} \\ \sqrt{\rho} h_{\mathcal{S},\mathcal{R}} h_{\mathcal{R}} \kappa \end{bmatrix} x + \begin{bmatrix} n_d[1] \\ \sqrt{\rho} h_{\mathcal{S},\mathcal{R}} \kappa n_r + n_d[2] \end{bmatrix}, \tag{2.48}$$

in which $y_d[1]$ and $n_d[1]$ denote the received signal and AWGN at the destination during the first $\frac{T}{2}$ time period, $y_d[2]$ and $n_d[2]$ denote the received signal and AWGN at the destination during the second $\frac{T}{2}$ time period, and h_a ($a \in \{\mathcal{S}, \mathcal{R}\}$) denotes the channel fading coefficient between node a and the destination.

Since only half of the total time is allocated to the source to transmit information to the destination, to guarantee correct decoding at the destination when κ is chosen as that in (2.46), the average transmission rate R needs to satisfy [31]

$$R \leq \frac{1}{2} \log \left(1 + \rho |h_{\mathcal{S}}|^2 + \frac{\rho^2 |h_{\mathcal{S},\mathcal{R}}|^2 |h_{\mathcal{R}}|^2}{1 + \rho |h_{\mathcal{S},\mathcal{R}}|^2 + \rho |h_{\mathcal{R}}|^2} \right). \tag{2.49}$$

Studying the high-SNR outage probability performance, the achievable DMT of the standard AF relaying protocol can be expressed by [31, 61]

$$d(r) = 2\,(1 - 2r)\,, \qquad 0 \leq r \leq \frac{1}{2}. \tag{2.50}$$

The maximal diversity gain 2 can be achieved so that the diversity performance of direct source-destination transmission is improved. This is because the source information is transmitted to the destination through two independent paths. However, only half of the channel bandwidth (i.e. time uses) is allocated to the source to convey information (which induces the scaling factor $\frac{1}{2}$ in inequality (2.49)). The spectral efficiency is significantly reduced when compared with direct source-destination transmission (in which the source uses the whole time T to transmit information, as displayed in Figure 2.11 (a)), especially for the high SNR region. This is confirmed by the fact that the standard AF relaying protocol only achieves maximal multiplexing gain $\frac{1}{2}$. Such a multiplexing gain reduction is termed *multiplexing loss* throughout the thesis.

For the standard DF relaying protocol, assuming successful decoding at the relay, the input-output relation of the relay network is

$$\begin{bmatrix} y_d[1] \\ y_d[2] \end{bmatrix} = \sqrt{\rho} \begin{bmatrix} h_{\mathcal{S}} \\ h_{\mathcal{R}} \end{bmatrix} x + \begin{bmatrix} n_d[1] \\ n_d[2] \end{bmatrix}. \tag{2.51}$$

To guarantee correct decoding at the destination, the average transmission rate should be limited by

$$R \leq \frac{1}{2} \log \left(1 + \rho |h_{\mathcal{S}}|^2 + \rho |h_{\mathcal{R}}|^2\right). \tag{2.52}$$

The achievable DMT thus is

$$d(r) = 2\,(1 - 2r)\,, \qquad 0 \leq r \leq \frac{1}{2}. \tag{2.53}$$

The diversity performance of direct source-destination transmission is improved. However, the good diversity performance is attained under an assumption of perfect decoding at the relay, i.e.

$$R \leq \frac{1}{2} \log \left(1 + \rho |h_{\mathcal{S},\mathcal{R}}|^2\right). \tag{2.54}$$

Inequality (2.54) only leads to the achievable DMT $d(r) = 1 - 2r$. Combining (2.52) and (2.54), the system achievable DMT is actually

$$d(r) = \min\left\{1 - 2r, 2\,(1 - 2r)\right\} = 1 - 2r, \qquad 0 \leq r \leq \frac{1}{2}. \tag{2.55}$$

This means the system diversity performance of the DF relaying protocol is limited by the quality of the source-relay link. The good diversity gain is achieved only when the source-relay link is sufficiently good to hold (2.54), which is however difficult to guarantee in general.

To solve this problem, a simple adaptive protocol named *selection relaying* [31] is considered under general source-relay channel conditions. For this protocol, the relay listens to and tries to decode the source in the first $\frac{T}{2}$ time period. If the decoding is successful, the relay retransmits the source codeword in the second $\frac{T}{2}$ time period. Otherwise, it remains silent (the source also remains silent or it can retransmit the codeword to the destination if the relay sends back an acknowledgement to notify the source). In this way, the system DMT performance is not affected by the quality of the source-relay channel and the DMT (2.53) can be attained under general source-relay channel conditions[4].

Although the standard DF relaying protocol (under the assumption of perfect decoding at the relay or using the adaptive protocol) can obtain higher maximal diversity gain than direct source-destination transmission, the maximal multiplexing gain is only $\frac{1}{2}$. Spectral efficiency is significantly reduced for the high SNR region. Consequently, advanced transmission protocols aiming to recover the multiplexing loss have attracted more and more interest.

One very simple approach is the incremental relaying protocol proposed in [31]. After the source broadcasts its information during the first $\frac{T}{2}$ time period, the destination sends an acknowledgement (1 bit feedback) to confirm whether it has correctly decoded the source transmission. If yes, the source continues transmitting new information to the destination and the relay does nothing. Otherwise, the relay forwards its received signal to the destination to assist in decoding during the second $\frac{T}{2}$ time period. This method means that the relay only transmits when the destination needs assistance so that the overall efficiency of the link is improved.

If feedback from the destination is difficult to be exploited by the source (e.g. due to a large distance between the source and the destination in practical systems), the multiplexing performance of the standard protocol can be improved by allocating a larger portion of the total transmission time to the source. For example, the protocols proposed in [8, 9] assume that the source first broadcasts its codeword during the a time period of duration $(1-\delta)T$ $(0 < \delta < 0.5)$. Then the relay uses a new *independent* Gaussian random codebook to re-encode the received information to a higher rate codeword and retransmits the new codeword to the destination

[4]Another example, which can avoid the achievable DMT degradation due to source-relay transmissions, is a decode-amplify-forward (DAF) protocol proposed in [62]. The DAF protocol can be considered as a combination of AF and DF protocols and the relay can choose using either DF mode (if the decoding is successful) or AF mode (if the decoding is not successful) to retransmit the source information to the destination. The DAF protocol increases the complexity of the relay and does not improve the multiplexing performance of the standard DF relaying protocol. Therefore, we do not consider this approach.

during the remaining δT time period, as displayed in Figure 2.11 (c). Since more resource is allocated to the broadcasting phase (because $(1-\delta)T > \frac{T}{2} > \delta T$), smaller multiplexing loss compared with the standard protocol is induced. However, the information theory based independent coding strategy demands highly complex processing at the relay and in particular at the destination. The other problem is the small fraction of time allocated to the relay's transmission forces the relay to encode the source information into a high rate codeword and thus results in a high probability of erroneous decoding at the destination. This means the larger portion of time assigned to the source (i.e. the better multiplexing performance), the less protection can be provided by the relay. As a result, to maintain a good diversity performance, the maximal multiplexing gain performance actually cannot approach that of direct source-destination transmission because the relay's very high rate codeword would inevitably cause problems of correct decoding at the destination. The complexity problem can be partly solved by the partial repetition-coding protocol proposed in [10] where the relay uses repetition coding but repeats only part of the source's codeword to the destination during the forwarding phase. Similarly, to obtain a good multiplexing performance, a very large portion of the total time should be assigned to the broadcasting phase so that only a very small part of the source codeword can be protected by the relay. The initial purpose of using a relay to provide diversity is nearly lost. Clearly, the orthogonal channel allocation requirement is the crucial limitation of effectively recovering the multiplexing loss.

Nonorthogonal transmission protocols permitting the source and the relay to transmit simultaneously have been investigated by many researchers. For example, for a dynamic DF (DDF) relaying protocol [34], the source transmits a codeword during the whole transmission time T. The relay listens to the source until it gathers enough energy to decode the source codeword. Then the relay re-encodes the source information using a new independent codebook, and forwards it to the destination when the source transmits the remaining part of the codeword to the destination. The time-division channel allocation for the DDF protocol is displayed in Figure 2.11 (d). The DDF protocol obtains good diversity performance and does not induce multiplexing loss (since the source uses the whole channel to convey information to the destination). However, a major drawback is the high complexity of the independent-coding strategy. Further, since it is difficult to know when the relay starts transmitting, synchronization becomes a problem in practical systems.

For AF relaying networks, a nonorthogonal AF (NAF) relaying protocol was proposed and analyzed in [34, 63]. For the NAF protocol, the source transmits using the whole transmission time T. The relay listens to the source during the first $\frac{T}{2}$ time period and retransmits the scaled version of its received signal to the destination during the second $\frac{T}{2}$ time period, as displayed in Figure 2.11 (e). Since half of the source's transmission is protected by the relay, some diversity improvement over direct transmission can be achieved without any multiplexing loss. Reference [64] extends the NAF protocol to multiple-source networks and Reference [65] extends it to multiple-antenna systems. Both references observe the diversity and multiplexing advantages of the nonorthogonal transmission schemes. However, since only half of the source transmission is protected by the relay, the system error performance may be dominated by that of the unprotected signal. For the NAF protocol, the signals transmitted by the source during the first and second $\frac{T}{2}$ time periods belong to a single codeword so that diversity improvement is observed. But if the two signals are independent, the transmission would be problematic. It is observed in [66] that if the source transmits a new codeword to the destination during the second $\frac{T}{2}$ time period, even with the assistance of the relay, the protocol does not provide maximal diversity gain improvement. This means, such a transmission scheme cannot be directly adopted in a repetition-coding DF relaying network.

2.6.2 Multiple-relay protocols

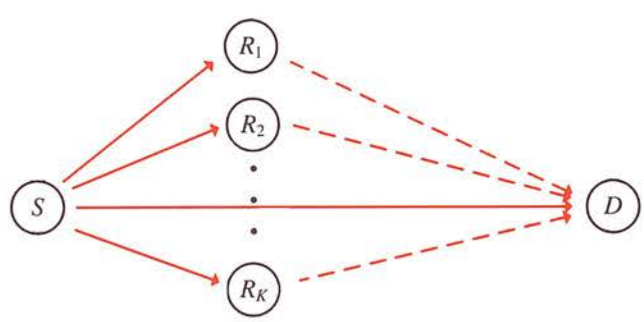

Figure 2.12: *A multiple-relay cooperative diversity system with one source, K relays and one destination.*

The use of multiple relays is considered as a means to further improve diversity performance over single-relay systems. For example, we assume there are K terminals in the network which can overhear the source and serve as potential relays for the source as displayed in Figure 2.12. Due to the half-duplex limitation in relays, the standard multiple-relay transmission protocol [32] allocates time period of duration $\frac{T}{K+1}$ for each terminal's transmission. During the first $\frac{T}{K+1}$ time period, the source broadcasts a codeword to all the relays and the destination. Then the relays forward the source information to the destination successively.[5] The time-division channel allocation of the standard protocol (e.g. $K = 2$) is displayed in Figure 2.13 (b). The achievable DMT for both AF relaying and DF relaying (assuming the source-relay channels are sufficiently good or using adaptive protocol under general source-relay link conditions) is calculated by

$$d(r) = (K+1)\left(1-(K+1)r\right), \qquad 0 \le r \le \frac{1}{K+1}. \tag{2.56}$$

The maximal diversity gain of such a standard orthogonal transmission protocol is $K+1$ instead of 2 for the single-relay protocols. However, the maximal multiplexing gain is only $\frac{1}{K+1}$ of that of direct transmission because the source only uses $\frac{1}{K+1}$ of the total time channel to transmit information.

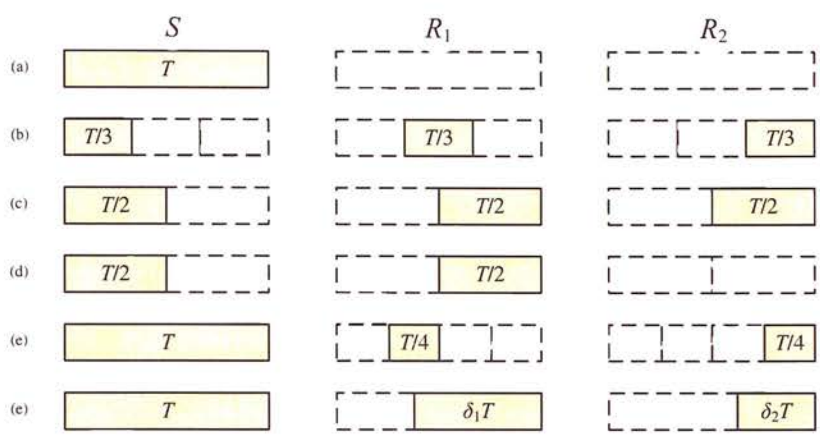

Figure 2.13: *Time-division channel allocations for (a) direct source-destination transmission, (b) standard AF/DF relaying, (c) space-time-coded DF relaying, (d) opportunistic relaying, (e) NAF relaying, and (f) DDF relaying for a two-relay network.*

To avoid allocating the relays orthogonal channels to improve the multiplexing performance, reference [32] proposes a space-time-coded protocol (a practical example, in which two relays are available in the network, is analyzed in [69]). In such an independent-coding based protocol, the time channel is divided into two equal parts. During the first $\frac{T}{2}$ time period, the source broadcasts its codeword. During the second $\frac{T}{2}$ time period, the relays which can correctly decode the source re-encode the source information using a distributed space-time code (the codebooks used by relays are independent from that used by the source) and forward the code-

[5]Here we consider that each relay receives signals only from the source. In fact, when any relay is transmitting, the other relays which have not transmitted yet can also listen to the transmitting relay. An advanced approach thus is that each relay combines and processes all the signals it received from the previous transmitting terminals to form its own transmitted signal (e.g. [67, 68]). Although better error performance than the standard protocol we discussed can be obtained, such a protocol does not improve the infinite-SNR DMT performance.

words to the destination simultaneously. The time-division channel allocation for a two-relay network (assuming both relays can correctly decode the source) is displayed in Figure 2.13 (c). By this means, the maximal diversity gain $K + 1$ can be achieved and the multiplexing performance of the standard multiple-relay protocol is improved since half of the total channel uses is allocated to the source. However, compared with direct source-destination transmission, the multiplexing loss is still notable.

Another method to avoid the orthogonal channel allocation requirement for the multiple relays is to use an opportunistic relaying strategy [70], where only the "best" relay is used to assist the source during the second $\frac{T}{2}$ time period. Specifically, each relay sets up a local timer and uses a ready-to-send (RTS) signal broadcasted by the source as well as a clear-to-send (CTS) signal broadcasted by the destination to measure its instantaneous channel condition. The timer of the relay with the best channel condition (e.g. the relay with the largest $|h_{\mathcal{S},\mathcal{R}_i}|^2|h_{\mathcal{R}_i}|^2$ [70]) expires first. This relay is selected as the forwarding relay and it broadcasts a "flag" signal to notify other relays (the notification can also be performed by the destination in the case where some relays may not be able to overhear other relays). Opportunistic relaying can be considered as a random-access based protocol since during the forwarding phase the relays communicate with the destination without a centralized coordination of the relays' transmission time (this is the case when multiple mobile terminals communicate with the access point in random-access networks such as IEEE 802.11 wireless local-area network (WLAN) [71]). The opportunistic relaying protocol provides the same diversity performance (i.e. the maximal diversity gain $K + 1$ is achieved) but much better multiplexing performance than the standard multiple-relay protocol. However, the transmissions of the source and the selected relay still use orthogonal channels and only half of the total time is assigned to the source. The multiplexing loss still exists.

The NAF relaying and DDF relaying methods discussed in the previous subsection can be extended to the multiple-relay network to fully compensate the multiplexing loss [34]. For the multiple-relay NAF relaying, the total channel is divided into $2K$ parts. The source transmits a codeword using the whole time T. During the $(2i-1)$th $\frac{T}{2K}$ time period ($i = 1, \ldots, K$), relay $\mathcal{R}_i$ is activated to listen to the source. It then amplifies and forwards its received signal during the next time period of duration $\frac{T}{K+1}$ (i.e. the $(2i)$th $\frac{T}{2K}$ time period). For the multiple-relay DDF relaying, the source broadcasts a long codeword which spans the whole time channel T. The relays listen to the source and each other. As long as one relay receives enough energy to decode the source, it re-encodes the source information using an independent codebook and transmits this codeword during the remaining time. The time-division channel allocations for these two protocols in a two-relay network are displayed in Figure 2.13 (d) and (e), respectively. The advantage of these protocols is that the whole channel is used by the source to send information. However, for the multiple-relay NAF relaying, only half of the whole transmission is protected by relays. For the multiple-relay DDF relaying, complexity and synchronization are again major drawbacks.

2.7 Summary

Wireless communications suffer from fading. For fast fading environments, choosing a sufficiently long codeword with transmission rate smaller than the ergodic capacity can provide error free transmission. However, there is no such 100% guarantee of reliable communication for slow fading environments. Diversity techniques are needed to improve the system error performance. Although the use of multiple transmit and/or receive antennas serves as an efficient approach to realize diversity, in practical cellular systems, mobiles may not be able to support this structure. Therefore, cooperative diversity techniques have been considered to use distributed antennas to attain spatial diversity.

Due to the half-duplex limitation at relays, the diversity improvement of the standard cooperative diversity relaying transmission protocols generally comes along with a multiplexing loss. This is because the standard protocols are based on an orthogonal channel allocation requirement and the source uses only no more than half of the total channel to send information. Hence, to recover the multiplexing loss, an explicit approach is to increase the fraction of channel uses that the source's transmission spans. This can be done by sending feedback from the destination to the source. By this means, the source's transmission can use the whole channel as long as the destination can correctly decode it. However, the destination-source feedback may be difficult to exploit in practice.

Without feedback, in a single-relay system, the source can directly use a larger fraction of the total channel to broadcast its information to obtain a better multiplexing performance than the standard protocol. Nevertheless, a smaller fraction of channel left to the relay's transmission may result in a higher probability of decoding errors at the destination. Approaching the multiplexing performance of direct source-destination transmission is almost impossible. Clearly, the orthogonal channel allocation requirement is another crucial limitation of recovering the multiplexing loss.

By permitting the source and the relay to transmit simultaneously, the commonly adopted nonorthogonal transmission protocols use the channel more efficiently. Since the source uses the whole channel to convey information, there is no multiplexing loss induced. However, for the NAF protocol, only part of the source signal is forwarded by relays. The system diversity gain may be limited by the unprotected signals. For the DDF protocol, the high complexity of the independent-coding strategy and the synchronization difficulty of the dynamic transmission fashion are two drawbacks which may hinder its practical implementation.

In this thesis, we also try to use advanced transmission protocols to recover the multiplexing loss for cooperative diversity systems, more specifically, for multiple-relay DF relaying systems. Unlike the spectrally efficient protocols described in this chapter, we try to keep the systems *simple*: there is no feedback from the destination to the source and the relays only utilize a repetition-coding strategy to re-encode the source information. In addition, we try to make our protocols *efficient*: the multiplexing loss induced by the standard protocols can be significantly recovered, even fully compensated under certain conditions. Finally, we try to make our protocols *effective*: the diversity gain of direct source-destination transmission can be substantially improved and the good diversity performance is not limited by the quality of source-relay channels or any un-relayed signals. In order to achieve the three goals, we utilize the multiple relays in a different way from the approaches discussed above. Further, we tightly follow the two principles: to increase the fraction of channel uses that the source's transmission spans and to permit nonorthogonal transmission to efficiently use the channel. Besides single-source networks, more attention will be drawn on multiple-source systems. The details of our protocols will be presented in the next two chapters.

Chapter 3
Recovering Multiplexing Loss

3.1 Introduction

We have introduced that using relays to assist in the communication between a source and its intended destination can provide higher diversity gain to improve communication reliability. However, due to the half-duplex limitation, in the standard transmission protocols, the source inefficiently uses only at most half of the channel bandwidth (i.e. time uses) to transmit information to the destination. This fact leads to a significant reduction of spectral efficiency, especially for the high SNR region. We have also mentioned that although some existing advanced transmission protocols can attain better multiplexing performance compared with the standard protocols, when considering simple repetition-coding DF relay systems, these protocols have their limitations. In this chapter, we will present a novel approach which can effectively recover the multiplexing loss by still using the simple repetition coding strategy without demanding destination-source feedback.

For a single-relay system, a nonorthogonal repetition-coding DF transmission scheme was proposed in [63]. The transmission process is similar to the NAF protocol we introduced in the previous chapter except that the source transmits new independent information during the second half time period when the relay is repeating the source codeword it received during the first half time period. For simplicity, we assume the transmission duration of each codeword is one time slot. The total transmission process thus takes two time slots. More specifically, during the first time slot, the source broadcasts its first codeword to the relay and the destination. During the second time slot, the relay repeats its received codeword and the source transmits a new codeword (contains new information). This protocol does not induce any multiplexing loss since the source uses the whole channel to send information. However, the major drawback is that the relay cannot listen to the second codeword when it is retransmitting the first codeword to the destination. The second codeword is conveyed to the destination through only the direct source-destination link. As a result, the system diversity gain is actually the same as that of direct transmission since the overall error probability is dominated by the probability of incorrect decoding of the second codeword [34, 66].

A simple solution of this problem is to add in a second relay to the network and let this relay listen to the second codeword during the second time slot (when the first relay is forwarding) and retransmit it during the third time slot. In this way, each of the two codewords is protected by one relay and the source uses $\frac{2}{3}$ of the total channel (i.e. time uses) to send information to the destination. A smaller multiplexing loss than the standard DF relaying protocols can be obtained. In addition, during the third time slot, the first relay is free again. If the source transits another new codeword at this time slot, the first relay can be used to assist the source so that the source's transmission spans $\frac{3}{4}$ of the total time channel. The process can be repeated for as many time slots as is desired. In summary, during each time slot, when one relay is forwarding the source codeword it decoded, the other relay is used to listen to the new codeword transmitted from the source. Clearly, the more codewords the source transmits, the source uses the channel bandwidth more efficiently and better multiplexing performance can be achieved. This is the basic idea of our approaches to recover the multiplexing loss for repetition-coded DF relay systems. It is worth noting that our use of the two relays is different from standard approaches. Adding one more relay is conventionally considered as a means to further improve diversity performance but suffer from further reduction of multiplexing performance. However, the purpose of our approach is to use two successively activated relays to reduce the negative impact of the half-duplex limitation at relays on the system multiplexing performance.

In this chapter, we first use a *repetition-coded successive DF relaying* protocol to recover the multiplexing loss for a simple single-source network. Two specific scenarios (i.e. an isolated-relay scenario and a strong-interference scenario) are considered so that the interference between relays does not affect the system performance. Assuming good source-relay channel conditions, an upper bound of the maximal average transmission rate is provided. Through simulations, it can be seen that the upper bound substantially outperforms the maximal average transmission rates of the standard DF relaying protocols. Since the use of the upper bound is not sufficient to prove that the multiplexing loss induced by the standard protocols is actually recovered, we calculate the achievable DMT of the considered protocol to offer a concrete theocratical evidence. Further, although diversity performance of DF relaying protocols is limited by the quality of source-relay links in principle, we propose an adaptive form of the repetition-coded successive DF relaying protocol which permits the relays to choose between transmitting and remaining silent according to whether the source-relay transmissions are successful. It is proved through DMT analysis that the adaptive protocol still provides link reliability and spectral efficiency advantages in general source-relay channel conditions. Then, the repetition-coded successive DF relaying protocol is extended to a two-source network (termed *repetition-coded concurrent DF relaying*). The analysis shows that the same good performance of the single-source protocol also holds for a multiple-source network. The generalized case in which any number of sources exist in the network is summarized finally.

3.2 System model and standard approaches

The general system model studied in this chapter is an uplink transmission in an $(M+3)$-node network with M sources $\mathcal{S}_i$ $(i = 1, \ldots, M)$, two *half-duplex* DF relays $\mathcal{R}_1$ and $\mathcal{R}_2$, and one common destination $\mathcal{D}$, as displayed in Figure 3.1. Each terminal is equipped with a single antenna. Each source $\mathcal{S}_i$ intends to transmit a frame with L codewords (denoted as $x_i[j]$, $j = 1, \ldots, L$) to the destination. The M sources use M *independent* Gaussian random codebooks, which are known by both relays. We assume each codeword $x_i[j]$ has a unit average power and is *independently* chosen from the associated Gaussian random codebook. The transmission rate of each codeword from source $\mathcal{S}_i$ is $\bar{R}_i$ bits per codeword. We assume that sending one codeword from any transmitter to any receiver takes one time slot (i.e. one channel). For fair comparison, we consider the average transmission rate R_i BPCU (i.e. bits per time slot) for different transmission protocols. The relationship between R_i and $\bar{R}_i$ can be expressed as

$$R_i = \frac{L}{T_L}\bar{R}_i, \tag{3.1}$$

where T_L denotes the *total* time used to transmit the L codewords from each source (i.e. the overall time used to finish the transmission of the ML codewords from the M sources). To characterize the system achievable DMT, we assume that the system is *symmetric* [54], where the M sources have identical multiplexing gains r. In other words, for infinite SNR, the average transmission rate R_i of each source changes with respect to SNR as

$$R_i = r \log \rho, \quad i \in \{1, 2, \ldots, M\}. \tag{3.2}$$

A frequency-flat, slow, block Rayleigh fading environment is assumed, where the channel remains static for a coherence interval (no less than T_L) and changes independently in different coherence intervals. We use $h_{\mathcal{S}_i,\mathcal{R}_j}$ $(i \in \{1, \ldots, M\}, j \in \{1, 2\})$ to denote the channel fading coefficient between the source $\mathcal{S}_i$ and the relay $\mathcal{R}_j$ and use h_a $(a \in \{\mathcal{S}_1, \ldots, \mathcal{S}_M, \mathcal{R}_1, \mathcal{R}_2\})$ to

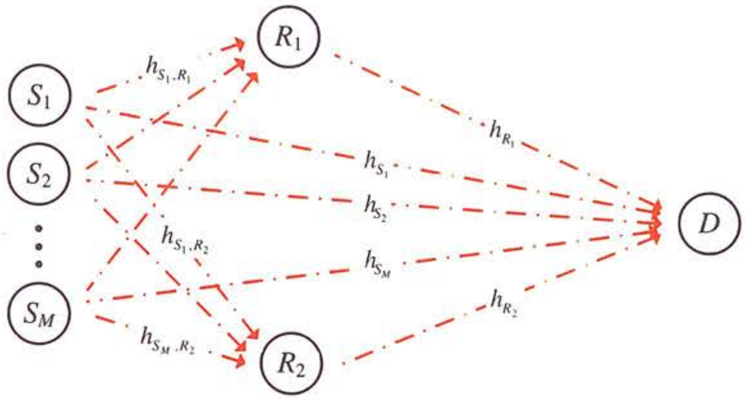

Figure 3.1: *The M-source ($\mathcal{S}_1 - \mathcal{S}_M$), two-relay ($\mathcal{R}_1, \mathcal{R}_2$), one-destination ($\mathcal{D}$) system model.*

denote the channel fading coefficient between node a and the destination. We assume perfect channel knowledge only at the receiver of each link. No cooperation among the sources and perfect synchronization are also assumed. Moreover, each terminal transmits with *equal* power.

We assume the ML codewords are conveyed to the destination through TDMA. For this single-antenna M-source network, the input-output relation of different protocols can be generalized as the following equation

$$\mathbf{y} = \sqrt{\rho}\mathbf{H}\mathbf{x} + \mathbf{n}, \tag{3.3}$$

where $\mathbf{y} = [y[1], y[2], \ldots, y[T_L]]^T$ is the $T_L \times 1$ receive signal vector, $y[k]$ ($k \in \{1, \ldots, T_L\}$) denotes the received signal at the destination during the kth time slot, $\mathbf{H}$ denotes the $T_L \times ML$ channel transfer matrix, $\mathbf{x} = [x_1[1], x_2[1], \ldots, x_M[1], x_1[2], x_2[2], \ldots, x_{M-1}[L], x_M[L]]^T$ is the $ML \times 1$ transmit vector, and $\mathbf{n}$ denotes the $T_L \times 1$ unit power complex circular AWGN vector at the destination. Note that in equation (3.3), the dimensions of the channel matrix, the input signal, output signal, and the noise vectors are expanded in the time domain.

For *direct source-destination transmission* without the help of any relay, the M sources take turns transmitting their codewords to the destination (i.e. TDMA). A total of $T_L = ML$ time slots are used to finish the transmission of the ML codewords and each source uses $\frac{1}{M}$ of the total time to transmit information to the destination (the time-division channel allocation is

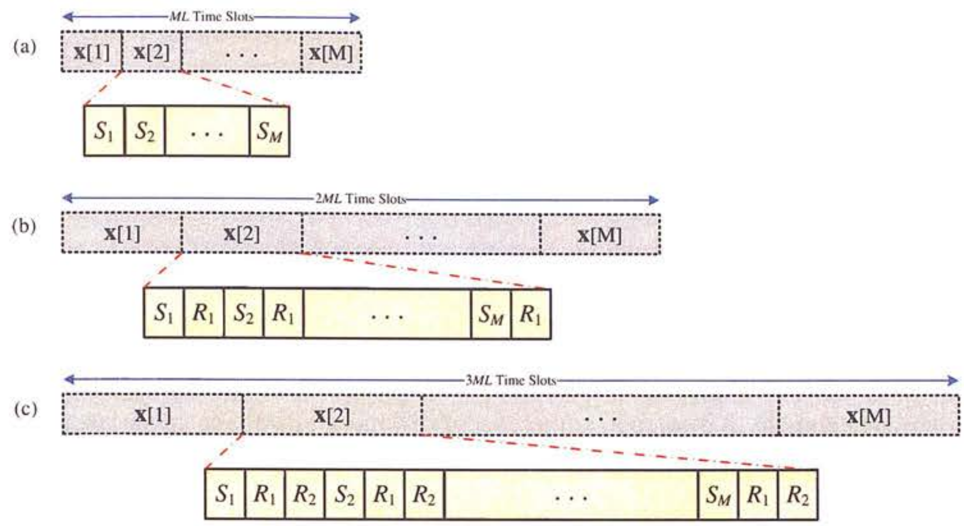

Figure 3.2: *Time-division channel allocations for (a) direct source-destination transmission, (b) one-relay standard DF relaying protocol, and (c) two-relay standard DF relaying protocol for an M-source network. $\mathbf{x}[i]$ denotes that all the sources take turns transmitting their ith codewords. The terminal displayed in each time slot (each yellow box) denotes the transmitter during that time slot.*

displayed in Figure 3.2 (a)). The channel matrix $\mathbf{H}$ is expressed by

$$\mathbf{H} = \begin{bmatrix} \bar{\mathbf{H}} & \mathbf{O} & \cdots & \mathbf{O} \\ \mathbf{O} & \bar{\mathbf{H}} & \cdots & \mathbf{O} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{O} & \mathbf{O} & \cdots & \bar{\mathbf{H}} \end{bmatrix}, \tag{3.4}$$

where $\mathbf{O}$ denotes an all-zero matrix, and the sub-matrix

$$\bar{\mathbf{H}} = \begin{bmatrix} h_{\mathcal{S}_1} & 0 & \cdots & 0 \\ 0 & h_{\mathcal{S}_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{\mathcal{S}_M} \end{bmatrix}. \tag{3.5}$$

To guarantee that the destination can correctly decode the sources, the average transmission rates need to satisfy the following inequalities

$$R_i = \frac{1}{M}\bar{R}_i \leq \frac{1}{M}\log\left(1+\rho|h_{\mathcal{S}_i}|^2\right), \quad i \in \{1, 2, \ldots, M\}. \tag{3.6}$$

The achievable DMT for each source is calculated by

$$d(r) = 1 - Mr, \qquad 0 \le r \le \frac{1}{M}. \tag{3.7}$$

For infinite SNR, TDMA direct source-destination transmission obtains maximal diversity gain 1 and maximal multiplexing gain $\frac{1}{M}$ (i.e. the optimal multiplexing gain of the system).

If only one of the two half-duplex relays (e.g. relay $\mathcal{R}_1$) is used to assist the sources, the repetition-coded standard DF relaying protocol [31] demands two time slots for each codeword's transmission: one for the source to broadcast the codeword to the relay and the destination and one for the relay to repeat the codeword to the destination while the source remains silent. Therefore, a total of $T_L = 2ML$ time slots are used to transmit the ML codewords (Figure 3.2 (b) displays the time-division channel allocation). Assuming the source-relay links are sufficiently good such that the source codewords are correctly decoded by the relay, the equivalent channel matrix can be expressed as (3.4) except that the sub-matrix $\bar{\mathbf{H}}$ is

$$\bar{\mathbf{H}} = \begin{bmatrix} h_{\mathcal{S}_1} & 0 & \cdots & 0 \\ h_{\mathcal{R}_1} & 0 & \cdots & 0 \\ 0 & h_{\mathcal{S}_2} & \cdots & 0 \\ 0 & h_{\mathcal{R}_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{\mathcal{S}_M} \\ 0 & 0 & \cdots & h_{\mathcal{R}_1} \end{bmatrix}. \tag{3.8}$$

The transmission rates for reliable communication are limited by

$$R_i = \frac{1}{2M}\bar{R}_i \le \frac{1}{2M}\log\left(1 + \rho|h_{\mathcal{S}_i}|^2 + \rho|h_{\mathcal{R}_1}|^2\right), \quad i \in \{1, 2, \ldots, M\}. \tag{3.9}$$

We call this protocol *one-relay standard DF relaying*. The achievable DMT for each source can be written as

$$d(r) = 2\left(1 - 2Mr\right), \qquad 0 \le r \le \frac{1}{2M}. \tag{3.10}$$

As mentioned in Chapter 2, although it brings diversity improvement compared with direct transmission, the one-relay standard DF relaying protocol reduces transmission data rate at high SNR because it can only achieve maximal multiplexing $\frac{1}{2M}$ for each source.

Conventionally, adding more relays to assist in the sources' transmissions is considered as a means to further improve the system diversity performance. The standard approach to make use of both relays [32] (we call it *two-relay standard DF relaying* throughout the thesis) allocates three time slots to $\mathcal{S}_i$, $\mathcal{R}_1$, and $\mathcal{R}_2$ respectively to complete a single codeword's transmission. Thus $T_L = 3ML$ time slots have to be used as displayed in Figure 3.2 (c). On assuming perfect decoding at both relays, the equivalent channel matrix is expressed as (3.4) with the sub-matrix

$$\bar{\mathbf{H}} = \begin{bmatrix} h_{\mathcal{S}_1} & 0 & \cdots & 0 \\ h_{\mathcal{R}_1} & 0 & \cdots & 0 \\ h_{\mathcal{R}_2} & 0 & \cdots & 0 \\ 0 & h_{\mathcal{S}_2} & \cdots & 0 \\ 0 & h_{\mathcal{R}_1} & \cdots & 0 \\ 0 & h_{\mathcal{R}_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{\mathcal{S}_M} \\ 0 & 0 & \cdots & h_{\mathcal{R}_1} \\ 0 & 0 & \cdots & h_{\mathcal{R}_2} \end{bmatrix}. \tag{3.11}$$

The average transmission rates, which allow successful decoding at the destination, can be expressed by

$$R_i = \frac{1}{3M}\bar{R}_i \le \frac{1}{3M}\log\left(1 + \rho|h_{\mathcal{S}_i}|^2 + \rho|h_{\mathcal{R}_1}|^2 + \rho|h_{\mathcal{R}_2}|^2\right), \quad i \in \{1, 2, \ldots, M\}. \tag{3.12}$$

The achievable DMT for each source is

$$d(r) = 3\left(1 - 3Mr\right), \qquad 0 \le r \le \frac{1}{3M}. \tag{3.13}$$

Clearly, the maximal diversity gain of the one-relay standard DF relaying protocol is improved to 3. This is because each codeword is protected by both relays now. However, the maximal multiplexing gain $\frac{1}{3M}$ implies that the cost of such a diversity improvement is a further sacrifice of transmission rate.

For the one-relay standard DF relaying protocol, the transmission rate reduction comes from the inherent half-duplex limitation at practical terminals and thus cannot be avoided by applying the simple repetition coding strategy at the relay. However, for the two-relay network, from a multiplexing viewpoint, we argue that the two relays are not efficiently used in the two-relay standard DF relaying protocol. In what follows, we present a novel approach which considers the use of two relays as a means to increase not the diversity performance but the multiplexing performance of the one-relay standard DF relaying protocol.

3.3 Single-source systems

We start from the simplest network in which $M = 1$.[1] The time-division channel allocations for direct source-destination transmission, the single-relay and two-relay standard DF relaying protocols are displayed in Figure 3.3 (a) - (c).

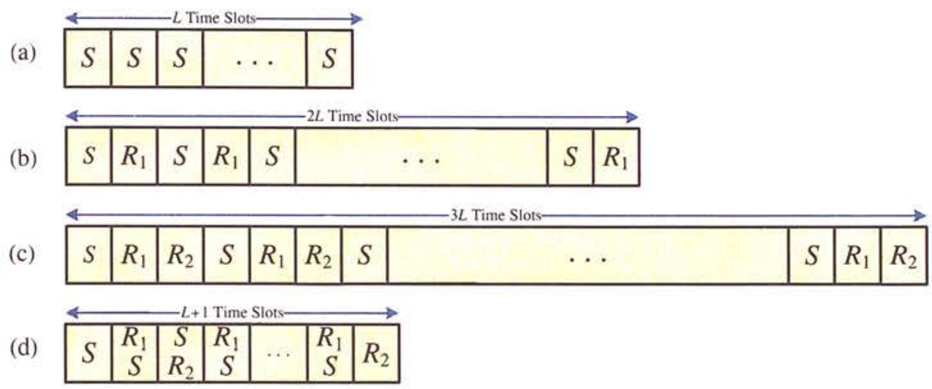

Figure 3.3: *Time-division channel allocations for (a) direct source-destination transmission, (b) one-relay standard DF relaying protocol, (c) two-relay standard DF relaying protocol, and (d) repetition-coded successive DF relaying protocol (when L is even) for a single-source network. The terminals displayed in each time slot denote the transmitters during that time slot.*

3.3.1 Protocol design

In order to recover the multiplexing loss induced by the standard protocols, we relax the orthogonal transmission requirement. Concurrent transmission [26] among the network nodes is permitted so that the source and one relay can communicate with the destination simultaneously[2]. Unlike the two-relay standard DF relaying protocol, which demands each relay to forward all the L source codewords to the destination, each relay needs to individually decode, re-encode and retransmit only half of them. Specifically, the source transmits its L codewords

[1] For simplicity, we drop the subscripts 1 from $\mathcal{S}_1$, $\mathcal{R}_1$, and x_1^j.

[2] The protocol was proposed by Dr. Yijia Fan in [1] for the case of one source. Similar approaches considering AF relay networks were also independently proposed and studied in references [72, 73].

to the destination continuously during the first L time slots. From the second time slot, the two relays take turns assisting the source (i.e. for odd j, $x[j]$ is received and forwarded by $\mathcal{R}_1$, and for even j, $x[j]$ is received and forwarded by $\mathcal{R}_2$). By this means, each of the L codewords is transmitted to the destination through two independent paths and only $T_L = L + 1$ time slots are used to complete the transmission. The detailed transmission process of the frame can be described as follows [2]:

Time slot 1: $\mathcal{S}$ broadcasts the first codeword $x[1]$ to both $\mathcal{R}_1$ and $\mathcal{D}$; $\mathcal{R}_2$ remains silent.

Time slot 2: $\mathcal{R}_1$ forwards $x[1]$ to $\mathcal{D}$. $\mathcal{S}$ transmits the second codeword $x[2]$. $\mathcal{R}_2$ listens to $\mathcal{S}$ while being interfered by $x[1]$ from $\mathcal{R}_1$. $\mathcal{D}$ receives $x[1]$ from $\mathcal{R}_1$ and $x[2]$ from $\mathcal{S}$.

Time slot 3: $\mathcal{R}_2$ forwards $x[2]$ to $\mathcal{D}$. $\mathcal{S}$ transmits the third codeword $x[3]$. $\mathcal{R}_1$ listens to $\mathcal{S}$ while being interfered by $x[2]$ from $\mathcal{R}_2$. $\mathcal{D}$ receives $x[2]$ from $\mathcal{R}_2$ and $x[3]$ from $\mathcal{S}$.

This process repeats until the (L)th time slot.

Time slot ($L + 1$): $\mathcal{R}_\alpha$ ($\alpha = 1$ if L is odd, and $\alpha = 2$ if L is even) forwards $x[L]$, the last codeword, to $\mathcal{D}$.

After all the L codewords are received via both direct and relay links, the destination performs joint decoding to recover the information transmitted by the source (e.g. a V-BLAST decoding scheme is analyzed in reference [2]). We refer to this transmission protocol as *repetition-coded successive DF relaying* throughout the thesis and its time-division channel allocation and transmission schedule (e.g. L is even) are displayed in Figure 3.3 (d) and Figure 3.4, respectively.

3.3.2 Interference cancellation

The major issue with the proposed repetition-coded successive DF relaying protocol is the interference generated between the two relays when one relay is listening to the source while the other relay is forwarding the source codeword to the destination. This situation mimics a two-user Gaussian interference channel [38] with $\mathcal{S}$ and $\mathcal{R}_1$ ($\mathcal{S}$ and $\mathcal{R}_2$) acting as two senders which intend to communicate with two receivers $\mathcal{R}_2$ and $\mathcal{D}$ ($\mathcal{R}_1$ and $\mathcal{D}$) respectively, in even (odd) time slots. The optimal solution for this problem is still open and we consider a simple SIC-based decoding criterion for the relays to suppress the interference: If the interference between relays is stronger than the desired signal, the relay decodes the interference signal and

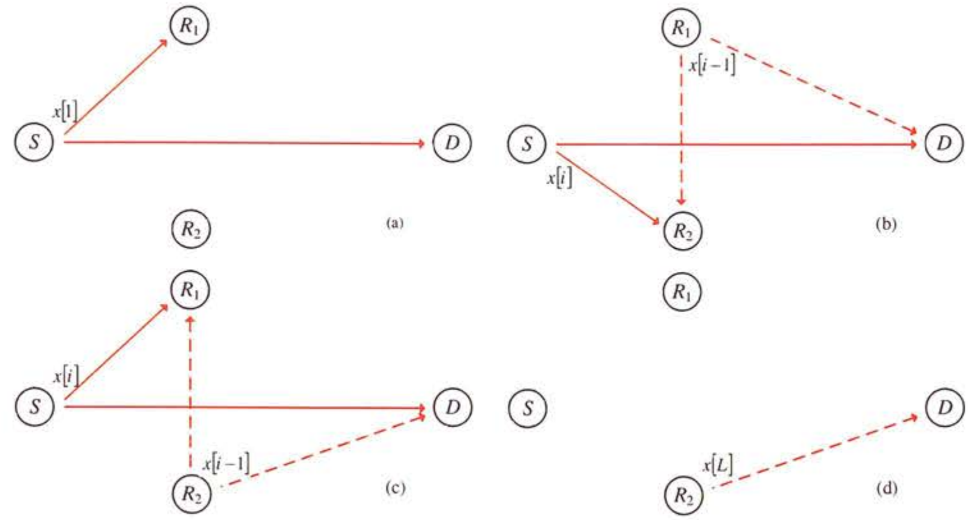

Figure 3.4: *Transmission schedule for the repetition-coded successive DF relaying protocol (L is even) in (a) time slot 1, (b) even time slot i ($1 < i \leq L$), (c) odd time slot i ($1 < i \leq L$), and (d) time slot $L + 1$. Solid lines and dashed lines denote the transmissions of the source and the relays respectively.*

subtracts it from the received signal before decoding the desired codeword. Otherwise, the relay decodes the desired codeword directly while treating the interference as Gaussian noise.

It is not known a priori how the inter-relay interference affects the system capacity and error performance under general relay-relay channel conditions [72]. In this thesis, we focus on two specific scenarios so that the influence can be eliminated. Specifically, the first scenario is an *isolated-relay scenario* [73], in which the quality of the inter-relay link is much worse than those of the links between the source and the relays. The second scenario is a *strong-interference scenario* [38], in which the channel between the two relays is sufficiently stronger than the source-relay links. For both scenarios, when using the above decoding criterion at the relays, the system capacity and error performance would not be affected by the inter-relay interference. It is worth noting that such two extreme scenarios are not uncommon in practice. For example, by using relay selection schemes, the isolated-relay scenario can be realized by choosing two relays which are located sufficiently far away from each other and the strong-interference scenario can be realized by choosing two relays which are sufficiently close to each other.

3.3.3 Perfect source-relay links

Similar to the standard DF relaying protocols, the quality of the source-relay links limits the network capacity and error performance. In the following, we first assume that the source-relay links are sufficiently strong. Under this condition, we will show how much performance improvement over the standard protocols the repetition-coded successive DF relaying protocol can bring. In the next subsection, with the use of a simple adaptive transmission protocol, it will be shown that the good diversity and multiplexing performances are in fact not affected by the source-relay channel conditions.

If both source-relay links are strong enough to hold the following inequalities:

$$\bar{R} \le \log\left(1 + \rho|h_{\mathcal{S},\mathcal{R}_1}|^2\right), \tag{3.14}$$

$$\bar{R} \le \log\left(1 + \rho|h_{\mathcal{S},\mathcal{R}_2}|^2\right), \tag{3.15}$$

the relays can always correctly decode the source codewords. The proposed repetition-coded successive DF relaying protocol mimics an L-user multiple-access SIMO channel (except that the signal dimensions are expended in the time domain rather than the space domain). The associated input-output channel relation for the relay network can be written as

$$\begin{bmatrix} y[1] \\ y[2] \\ y[3] \\ \vdots \\ y[L+1] \end{bmatrix} = \sqrt{\rho} \underbrace{\begin{bmatrix} h_{\mathcal{S}} & 0 & 0 & \cdots & 0 & 0 \\ h_{\mathcal{R}_1} & h_{\mathcal{S}} & 0 & \cdots & 0 & 0 \\ 0 & h_{\mathcal{R}_2} & h_{\mathcal{S}} & \cdots & 0 & 0 \\ 0 & 0 & h_{\mathcal{R}_1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & h_{\mathcal{R}_\alpha} & h_{\mathcal{S}} \\ 0 & 0 & 0 & \cdots & 0 & h_{\mathcal{R}_\beta} \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} x[1] \\ x[2] \\ x[3] \\ \vdots \\ x[L] \end{bmatrix} + \mathbf{n}, \tag{3.16}$$

in which $\mathcal{R}_\alpha$ ($\alpha = mod\{L, 2\} + 1$) denotes the relay used to retransmit $x[L-1]$ at the Lth time slot, and $\mathcal{R}_\beta$ ($\beta = mod\{L+1, 2\} + 1$) denotes the relay used to retransmit $x[L]$ at the $(L+1)$th time slot. Following the capacity calculation for multiple-access MIMO channels in [53], to guarantee the destination to correctly decode every source codeword, there are $(2^L - 1)$ constraints for the source transmission rate for a given realization of the channel. These rate constraints can be expressed as

$$\bar{R} \leq \log\left(\det\left(\mathbf{I} + \rho \mathbf{h}_{j_1} \mathbf{h}_{j_1}^H\right)\right), \tag{3.17}$$

$$2\bar{R} \leq \log\left(\det\left(\mathbf{I} + \rho \mathbf{h}_{j_1} \mathbf{h}_{j_1}^H + \rho \mathbf{h}_{j_2} \mathbf{h}_{j_2}^H\right)\right), \tag{3.18}$$

$$3\bar{R} \leq \log\left(\det\left(\mathbf{I} + \rho \mathbf{h}_{j_1} \mathbf{h}_{j_1}^H + \rho \mathbf{h}_{j_2} \mathbf{h}_{j_2}^H + \rho \mathbf{h}_{j_3} \mathbf{h}_{j_3}^H\right)\right), \tag{3.19}$$

$$\cdots$$

$$L\bar{R} \leq \log\left(\det\left(\mathbf{I} + \rho \mathbf{H}\mathbf{H}^H\right)\right) \tag{3.20}$$

where $j_i \in \{1, \ldots, L\}$, and $\mathbf{h}_{j_i}$ denotes the j_ith column of $\mathbf{H}$. It is extremely complicated to give an exact description for the average transmission rate R when $L > 2$. Therefore we first concentrate only on the rate constraints (3.17) and (3.20) to provide an upper bound of the maximally reliable average transmission rate. Through simulations we will show this upper bound significantly outperforms the maximal average transmission rates of the standard DF relaying protocols and is comparable to that of direct transmission when L is large. Afterwards, we will use DMT analysis to prove theoretically that the repetition-coded successive DF relaying protocol can indeed recover the multiplexing loss induced by the standard protocols while still providing a diversity improvement over direct transmission.

Since the repetition-coded successive DF relaying protocol uses $(L+1)$ time slots to transmit L codewords, the average transmission rate R BPCU can be calculated by

$$R = \frac{L}{L+1}\bar{R}. \tag{3.21}$$

Substituting (3.21) into the rate constraints (3.17) and (3.20) and using $R_{\max}$ to denote the maximal average transmission rate which can guarantee reliable communication, $R_{\max}$ is upper bounded by

$$R_{\max} \leq \frac{L}{L+1} \min\left\{\log\left(\mathbf{I} + \rho \mathbf{h}_1 \mathbf{h}_1^H\right), \log\left(\mathbf{I} + \rho \mathbf{h}_2 \mathbf{h}_2^H\right), \frac{1}{L}\log\left(\mathbf{I} + \rho \mathbf{H}\mathbf{H}^H\right)\right\}. \tag{3.22}$$

Assuming perfect decoding at the relays, Figure 3.5 plots the 10% outage capacity comparison of different protocols through Monte Carlo simulations. The outage capacity for the repetition-coded successive DF relaying protocol is calculated by (3.22) so that the associated curves displayed in Figure 3.5 denote upper bounds. Clearly, the standard protocols significantly lose spectral efficiency compared with direct source-destination transmission for the high SNR region. However, the capacity upper bound of the repetition-coded successive DF relaying protocol is much higher than the capacity of the standard protocols. This is roughly because the proposed protocol allocates $\frac{L}{L+1}$ of the total channel uses to the source to send information to the destination, while only $\frac{1}{2}$ and $\frac{1}{3}$ are used in the one-relay and two-relay standard DF relaying protocols respectively. Further, it can be seen that when L increases, the outage capacity performance of the repetition-coded successive DF relaying protocol improves. For large L, the outage capacity upper bound is comparable to the outage capacity of direct source-destination transmission.

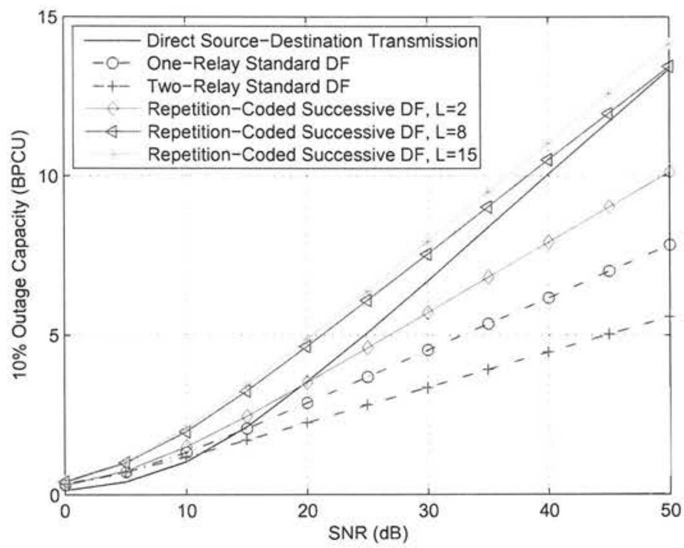

Figure 3.5: 10% *outage capacity comparison. It is assumed that decoding at the relays is always successful. The curves for the repetition-coded successive DF relaying protocol are upper bounds calculated by (3.22).*

As discussed previously, calculating the exact expression of the maximal average transmission rate is very involved so that we can only compute an upper bound in closed form, i.e. (3.22). Since it is not known how tight the upper bound is, the good outage capacity performance plotted in Figure 3.5 does not serve as a sufficient evidence that can prove whether the repetition-coded successive DF relaying protocol actually compensates the spectral efficiency loss of the standard protocols. In the following, we use the DMT analysis to give a more concrete answer to this question.

Considering the rate constraints (3.17)-(3.20), an outage event occurs when any of the constraints is not met. For each constraint there is a probability of not meeting it. The system outage probability is the highest one among all these probabilities. Therefore, there are $(2^L - 1)$ diversity-multiplexing tradeoffs corresponding to those conditions and the lowest curve within the range of multiplexing gain is the achievable tradeoff for the system. By substituting $R = r \log \rho$ and (3.21) into (3.17)-(3.20), we can calculate the achievable DMT. We summarize the result as the following theorem (the proof is offered in Appendix A.1.1).

Theorem 1 *For an isolated-relay scenario or a strong-interference scenario, on assuming that the relays correctly decode the source, the achievable DMT of the repetition-coded successive DF relaying protocol (i.e. the system model in (3.16)) is*

$$d(r) = 2\left(1 - \frac{L+1}{L}r\right), \qquad 0 \leq r \leq \frac{L}{L+1}. \tag{3.23}$$

Theorem 1 implies that if the relays correctly decode the source, the repetition-coded successive DF relaying protocol achieves maximal diversity gain 2 and maximal multiplexing gain $\frac{L}{L+1}$ for infinite SNR. The DMT performance strictly outperforms that of the one-relay standard DF relaying protocol (with maximal diversity gain 2 and maximal multiplexing gain $\frac{1}{2}$) when $L > 1$. Compared with the two-relay standard DF relaying protocol (with maximal diversity gain 3 and maximal multiplexing gain $\frac{1}{3}$), the repetition-coded successive DF relaying protocol achieves a smaller maximal diversity gain but a much better maximal multiplexing gain. Such a fact indicates that adding one more relay can be considered as a means to recover efficiently the multiplexing loss induced by the repetition-coded one-relay standard DF relaying protocol rather than just to improve the diversity performance with the sacrifice of more multiplexing gain. Furthermore, if the frame length L is chosen as a large value, the maximal multiplexing gain can approach 1, which is achieved by direct source-destination transmission. The multiplexing loss is thus *fully* recovered.

Figure 3.6 displays the DMT performance of the repetition-coded successive DF relaying protocol for some values of L versus SNR. Clearly, the proposed protocol achieves a better DMT performance than the one-relay standard DF relaying protocol even when L is chosen as a small value 2. Compared with the two-relay standard DF relaying protocol, as long as the multiplexing gain $r \geq \frac{1}{6}$, the repetition-coded successive DF relaying protocol (with $L = 2$) obtains better diversity performance. With increasing L, our proposed protocol becomes more advantageous. Generally, large L induces large decoding delay because the destination has to wait for

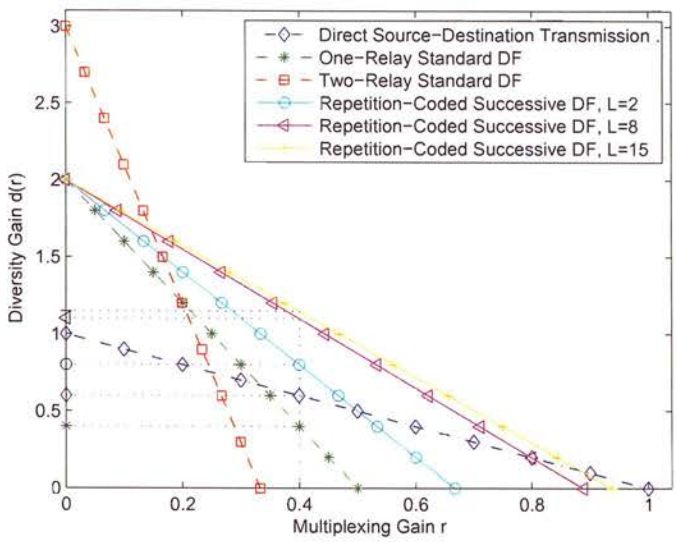

Figure 3.6: *DMT comparison of different protocols under perfect source-relay channel conditions. The dotted lines mark the achievable diversity gains of different protocols when the multiplexing gain* $r = 0.4$.

$L + 1$ time slots before it performs joint decoding. However, comparing the DMT curves of the cases when $L = 2$, $L = 8$ and $L = 15$, it can be seen that when L increases, the performance improvement becomes less significant. This observation indicates that to achieve a good DMT performance does not necessarily require a sufficiently large value of frame length. When L is chosen as 15, the maximal multiplexing gain is very close to 1, that of direct source-destination transmission.

Assuming $L = 2$, we compare the outage probability performance for different protocols. In our simulations, when $r > 0$, we assume the average transmission rate scales with SNR like $R = r \log(1 + \rho)$, which is defined in [74, 75] as the finite-SNR DMT. For the high-SNR region, the expression approaches $R = r \log \rho$ defined in (3.2). The transmission rates versus SNR for $r = 0$ (R is fixed at 2 BPCU) and $r = 0.4$ are plotted in Figure 3.7. The associated outage probability comparisons of different protocols are plotted in Figure 3.8 and Figure 3.9, respectively. It can be seen from the figures that, when the source transmission rate is fixed (i.e. $r = 0$), the repetition-coded successive DF relaying protocol obtains the same diversity gain as the one-relay standard DF relaying protocol. The diversity performance of direct source-destination transmission is significantly improved. With multiplexing gain $r = 0$, the two-relay standard DF relaying protocol achieves the highest diversity gain because both relays are used to protect each codeword. However, when we chose multiplexing gain $r = 0.4$, which is larger than the maximal achievable multiplexing gain of the two-relay standard DF relaying protocol, the outage probability does not decrease with increasing SNR (because the transmission rate increases too fast and the protocol no longer provides positive diversity gain). For the one-relay standard DF relaying protocol, when $r = 0.4$, the diversity performance drops below that of direct source-destination transmission so that the outage probability of the standard protocol decreases more slowly than that of direct transmission. Due to the multiplexing gain advantage, the repetition-coded successive DF relaying protocol still outperforms direct transmission in terms of diversity gain when $r = 0.4$. These observations confirm the results displayed in Figure 3.6. Further, following the analysis in Theorem 1, larger source frame length L leads to better DMT performance. If L is chosen as a large value, the diversity performance of the repetition-coded successive DF relaying protocol will be better than that of direct transmission within the range of almost all possible multiplexing gains.

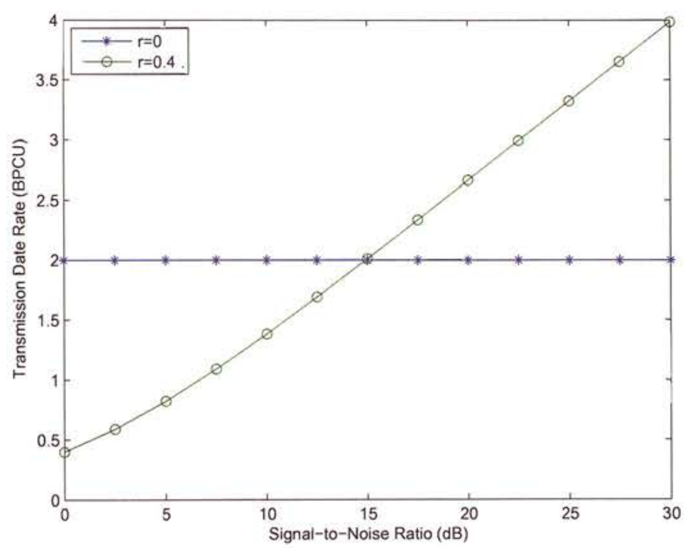

Figure 3.7: *Transmission rates versus SNR for* $r = 0$ *(*$R = 2$ *BPCU) and* $r = 0.4$.

Similar to the standard DF relaying protocols, the diversity performance of the proposed protocol is also limited by the quality of the source-relay links. The DMT performance (3.23) is attained only under the assumption that the source-relay links are sufficiently good. However, such a situation is difficult to be guaranteed in general. Throughout the thesis, we term the achievable DMT with perfect source-relay transmissions as *full DMT*. In the following, we will present a simple adaptive protocol which can achieve the full DMT (3.23) under general

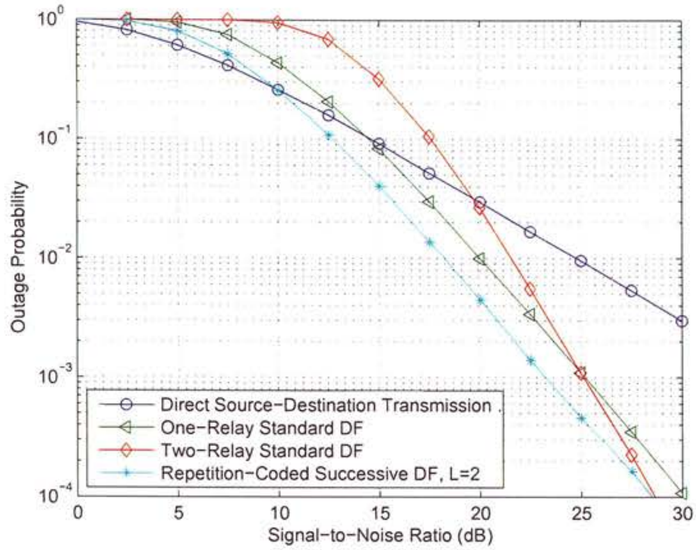

Figure 3.8: *Outage probability comparison of the repetition-coded successive DF relaying protocol ($L = 2$) and the standard protocols assuming perfect decoding at relays when $r = 0$ (the average source transmission rate is fixed at 2 BPCU).*

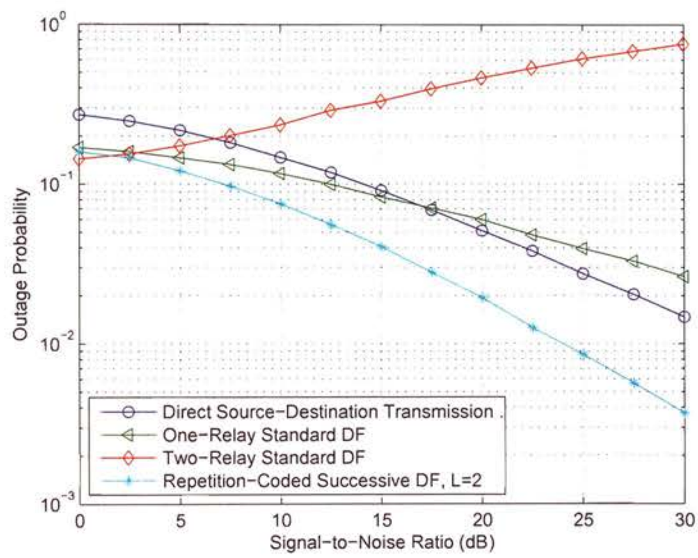

Figure 3.9: *Outage probability comparison of the repetition-coded successive DF relaying protocol ($L = 2$) and the standard protocols assuming perfect decoding at relays when $r = 0.4$.*

source-relay channel conditions.

3.3.4 General source-relay links

To allow for the case when perfect decoding at the relays is not guaranteed (i.e. (3.14) or (3.15) is not met), an adaptive protocol similar to the selection relaying protocol [31] is now considered. Specifically, the source continues transmitting one codeword during each of the first L time slots. Each relay listens to and tries to decode the source. If the decoding is successful, the relay retransmits the codeword to the destination. Otherwise, it remains *silent*. It is assumed that the source is not aware of whether the relays are used to assist it so that the whole transmission process always takes $T_L = L + 1$ time slots (i.e. the average transmission rate $R = \frac{L}{L+1}\bar{R}$). Therefore, if neither relay is activated, the adaptive protocol acts as direct source-destination transmission except that $(L+1)$ time slots are used to finish the transmission (during the last time slot, the destination does not receive information from any transmitter). If both relays are activated, the equivalent input-output relation of the channel is expressed by (3.16). If only one relay is activated, e.g. $\mathcal{S}$ is only assisted by $\mathcal{R}_1$, the equivalent channel matrix can be written as (e.g. L is even)

$$\mathbf{H} = \begin{bmatrix} h_{\mathcal{S}} & 0 & 0 & \cdots & 0 & 0 \\ h_{\mathcal{R}_1} & h_{\mathcal{S}} & 0 & \cdots & 0 & 0 \\ 0 & 0 & h_{\mathcal{S}} & \cdots & 0 & 0 \\ 0 & 0 & h_{\mathcal{R}_1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & h_{\mathcal{R}_1} & h_{\mathcal{S}} \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}. \tag{3.24}$$

The rate constraints (3.17)-(3.20) can still be applied to this case except that the matrix $\mathbf{H}$ is defined in (3.24). Using Bayes' rule, the overall system outage probability of the adaptive repetition-coded successive DF relaying protocol can be expressed by

$$\begin{aligned} P_{\text{out}} &= P_{\mathcal{R}_1} P_{\mathcal{R}_2} P_{\mathcal{SD}} + (1 - P_{\mathcal{R}_1}) P_{\mathcal{R}_2} P_{\mathcal{SR}_1\mathcal{D}} \\ &\quad + P_{\mathcal{R}_1}(1 - P_{\mathcal{R}_2}) P_{\mathcal{SR}_2\mathcal{D}} + (1 - P_{\mathcal{R}_1})(1 - P_{\mathcal{R}_2}) P_{\mathcal{SR}_1\mathcal{R}_2\mathcal{D}}, \end{aligned} \tag{3.25}$$

where $P_{\mathcal{R}_i}$ denotes the outage probability at relay $\mathcal{R}_i$ when $\mathcal{R}_i$ decodes the source, $P_{\mathcal{SD}}$ denotes the outage probability at the destination when no relay is used to assist the source, $P_{\mathcal{SR}_i\mathcal{D}}$ denotes the outage probability at the destination when only $\mathcal{R}_i$ can successfully decode the source and is used, and $P_{\mathcal{S}\mathcal{R}_1\mathcal{R}_2\mathcal{D}}$ denotes the outage probability at the destination when the source codewords can be correctly decoded at both relays. By studying the overall outage probability P_{out}, the achievable DMT can be summarized in the following theorem (the proof is provided in Appendix A.2).

Theorem 2 *For an isolated-relay scenario or a strong-interference scenario, under general source-relay channel conditions, the adaptive repetition-coded successive DF relaying protocol achieves the full DMT (3.23), i.e.*

$$d(r) = 2\left(1 - \frac{L+1}{L}r\right), \qquad 0 \leq r \leq \frac{L}{L+1}. \tag{3.26}$$

Theorem 2 implies that the extreme assumption of sufficiently good source-relay links discussed in the last subsection can actually be relaxed without affecting the good DMT performance. The repetition-coded successive DF relaying protocol can thus be considered in practical systems to improve transmission reliability over direct source-destination transmission. The protocol uses the same simple repetition-coding strategy at relays to enhance diversity as the standard DF relaying protocols but no longer significantly loses spectral efficiency. Therefore, the relays are more beneficially used.

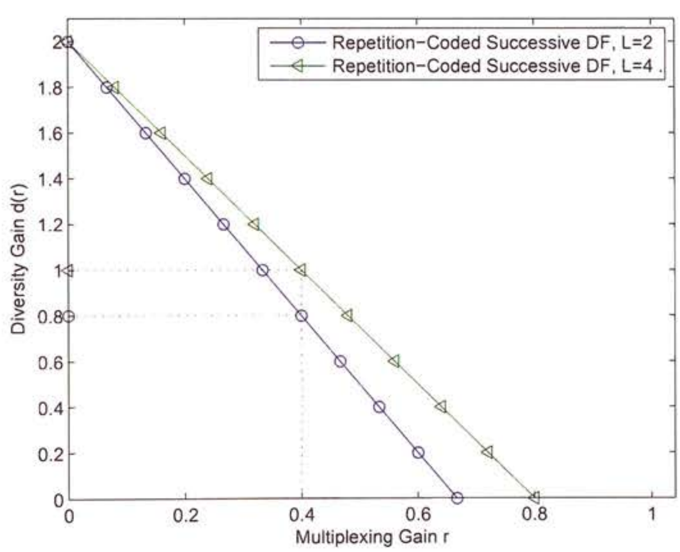

Figure 3.10: *DMT performance of the repetition-coded successive DF relaying protocol when $L = 2$ and $L = 4$. The dotted lines mark the achievable diversity gains when the multiplexing gain $r = 0.4$.*

Assuming $L = 2$ and $L = 4$, Figure 3.10 displays the DMT performance of the repetition-coded successive DF relaying protocol. Here we do not specify whether the quality of source-relay channels is sufficiently good or not because the use of the adaptive protocol under general source-relay channel conditions can obtain the same DMT as under perfect source-relay channel conditions. To see this more explicitly, we also plot the outage probability comparison in Figure 3.11 (when $r = 0$) and Figure 3.12 (when $r = 0.4$). Clearly, when the average multiplexing gain $r = 0$ (the average source transmission rate R is fixed at 2 BPCU), the achieved diversity gains when $L = 2$ and $L = 4$ are the same (i.e. the maximal diversity gain 2). The outage probability curves obtained by using the adaptive protocol under general source-relay channel conditions have the same high-SNR slopes as those obtained assuming perfect decoding at the relays. When $r = 0.4$, the repetition-coded successive DF relaying protocol with $L = 4$ has higher diversity gain than the case where $L = 2$. This observation reaffirms the DMT performance displayed in Figure 3.10. Again, for each individual frame length, the diversity gains attained in both perfect and general source-relay channel conditions remain the same.

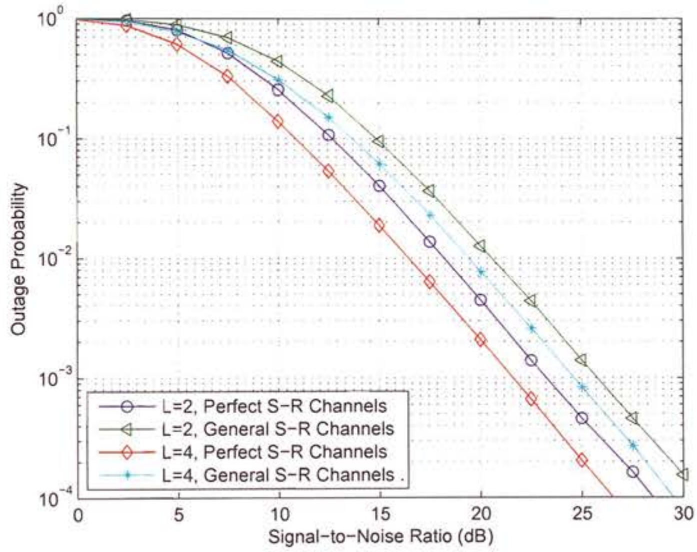

Figure 3.11: *Outage probability performance of the repetition-coded successive DF relaying protocol when $r = 0$ ($R = 2$ BPCU).*

In the following, we extend the idea of using two successively activated half-duplex relays to assist the communication between a single source and its intended destination to multiple-source networks. We begin with the simplest case in which two sources communicate with

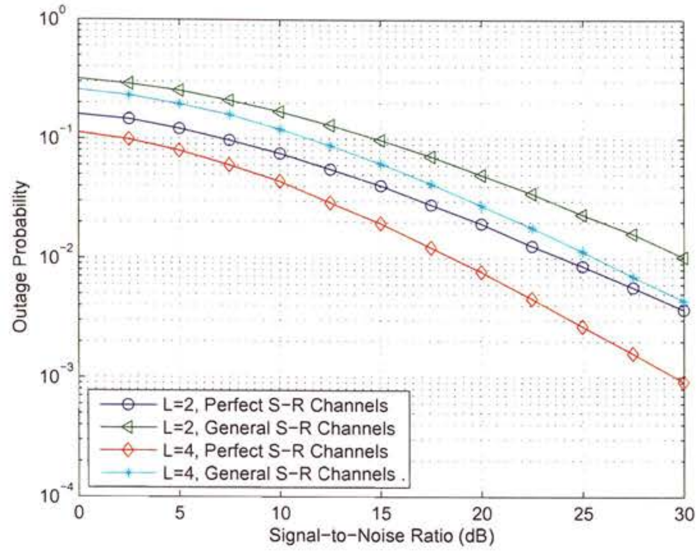

Figure 3.12: *Outage probability performance of the repetition-coded successive DF relaying protocol when $r = 0.4$.*

a common destination with the help of two relays. We also use DMT analysis to show that, for the multi-user network, the multiplexing loss induced by the standard protocols can be effectively recovered and higher diversity performance than TDMA direct transmission can still be obtained by our proposed protocol. Afterwards, the analysis will be generalized to M-source networks.

3.4 Two-source systems

For a two-source network, the time-division channel allocations for TDMA direct source-destination transmission, the single-relay standard DF relaying, and the two-relay standard DF relaying protocols are displayed in Figure 3.13 (a) - (c) respectively. Extending the repetition-coded successive DF relaying protocol proposed for a single-source network to such a multiple-source network is straightforward. The transmission process will be clearly described in what follows.

3.4.1 Protocol design

Unlike the repetition-coded successive DF relaying protocol in which a source is assisted by both relays, we require $\mathcal{R}_1$ and $\mathcal{R}_2$ to listen to $\mathcal{S}_1$ and $\mathcal{S}_2$ respectively. By requiring the relays to take turns forwarding their source codewords to the destination, the $2L$ codewords from the two sources are finished transmitting using $T_L = 2L + 1$ time slots. The transmission process can be described as follows:

Time slot 1: $\mathcal{S}_1$ broadcasts its first codeword $x_1[1]$ to both $\mathcal{R}_1$ and $\mathcal{D}$; $\mathcal{S}_2$ and $\mathcal{R}_2$ remain silent.

Time slot 2: $\mathcal{R}_1$ forwards $x_1[1]$ to $\mathcal{D}$. $\mathcal{S}_2$ transmits its first codeword $x_2[1]$. $\mathcal{R}_2$ listens to $\mathcal{S}_2$ while being interfered by $x_1[1]$ from $\mathcal{R}_1$. $\mathcal{D}$ receives $x_1[1]$ from $\mathcal{R}_1$ and $x_2[1]$ from $\mathcal{S}_2$.

Time slot 3: $\mathcal{R}_2$ forwards $x_2[1]$ to $\mathcal{D}$. $\mathcal{S}_1$ transmits its second codeword $x_1[2]$. $\mathcal{R}_1$ listens to $\mathcal{S}_1$ while being interfered by $x_2[1]$ from $\mathcal{R}_2$. $\mathcal{D}$ receives $x_2[1]$ from $\mathcal{R}_2$ and $x_1[2]$ from $\mathcal{S}_1$.

This process repeats until the $(2L)$th time slot.

Time slot $(2L + 1)$: $\mathcal{R}_2$ decodes, re-encodes and retransmits $x_2[L]$, the last codeword from $\mathcal{S}_2$, to $\mathcal{D}$.

After all the $2L$ codewords are received, the destination performs joint decoding to recover each codeword $x_i[j]$. The time-division channel allocation and transmission schedule are respectively displayed in Figure 3.13 (d) and Figure 3.14. We refer to this protocol as *repetition-coded concurrent DF relaying* throughout the thesis.

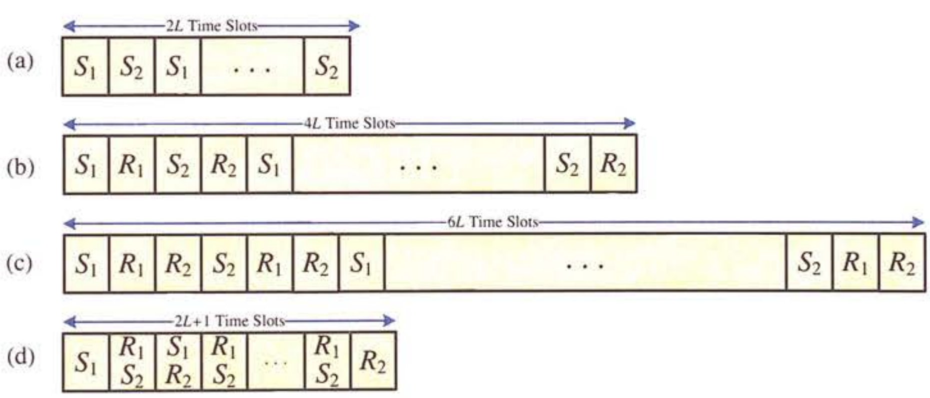

Figure 3.13: *Time-division channel allocations for (a) TDMA direct source-destination transmission, (b) one-relay standard DF relaying protocol, (c) two-relay standard DF relaying protocol, and (d) repetition-coded concurrent DF relaying protocol for the two-source network.*

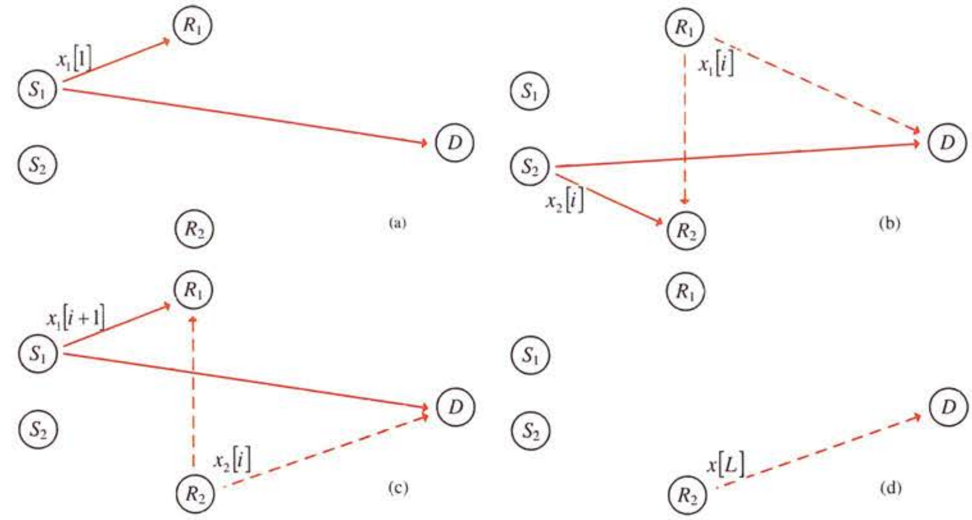

Figure 3.14: *Transmission schedule for the repetition-coded concurrent DF relaying protocol in (a) time slot* 1*, (b) time slot* $2i$ $(i = 1, \ldots, L)$*, (c) slot* $2i+1$ $(i = 1, \ldots, L-1)$*, and (d) time slot* $2L + 1$*.*

Similar to the repetition-coded successive DF relaying protocol, we assume that the quality of the inter-relay channel is either sufficiently weak (i.e. the isolated-relay scenario) or sufficiently strong (i.e. the strong-interference scenario) so that the inter-relay interference does not affect the system capacity and error performance.

3.4.2 Perfect source-relay links

Firstly, we assume the source-relay links are sufficiently good, i.e.

$$\bar{R}_1 \leq \log\left(1 + \rho |h_{\mathcal{S}_1, \mathcal{R}_1}|^2\right) \tag{3.27}$$

$$\bar{R}_2 \leq \log\left(1 + \rho |h_{\mathcal{S}_2, \mathcal{R}_2}|^2\right). \tag{3.28}$$

The relays can always perfectly decode their source codewords. The repetition-coded concurrent DF relaying protocol mimics a $2L$-user multiple-access SIMO channel with input-output channel relation,

$$\begin{bmatrix} y[1] \\ y[2] \\ y[3] \\ \vdots \\ y[2L] \\ y[2L+1] \end{bmatrix} = \sqrt{\rho} \underbrace{\begin{bmatrix} h_{\mathcal{S}_1} & 0 & 0 & \cdots & 0 & 0 \\ h_{\mathcal{R}_1} & h_{\mathcal{S}_2} & 0 & \cdots & 0 & 0 \\ 0 & h_{\mathcal{R}_2} & h_{\mathcal{S}_1} & \cdots & 0 & 0 \\ 0 & 0 & h_{\mathcal{R}_1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & h_{\mathcal{R}_1} & h_{\mathcal{S}_2} \\ 0 & 0 & 0 & \cdots & 0 & h_{\mathcal{R}_2} \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} x_1[1] \\ x_2[1] \\ x_1[2] \\ \vdots \\ x_1[L] \\ x_2[L] \end{bmatrix} + \mathbf{n}. \qquad (3.29)$$

Thus there are $(2^{2L} - 1)$ constraints for the source transmission rates for a given realization of the channel, which can be expressed as

$$\bar{R}_1 \leq \log\left(\det\left(\mathbf{I} + \rho\mathbf{h}_{2j-1}\mathbf{h}_{2j-1}^H\right)\right), \quad j = 1, \ldots, L \qquad (3.30)$$

$$\bar{R}_2 \leq \log\left(\det\left(\mathbf{I} + \rho\mathbf{h}_{2j}\mathbf{h}_{2j}^H\right)\right), \quad j = 1, \ldots, L \qquad (3.31)$$

$$2\bar{R}_1 \leq \log\left(\det\left(\mathbf{I} + \rho\mathbf{h}_{2i-1}\mathbf{h}_{2i-1}^H + \rho\mathbf{h}_{2j-1}\mathbf{h}_{2j-1}^H\right)\right), \quad i, j = 1, \ldots, L, i \neq j \qquad (3.32)$$

$$2\bar{R}_2 \leq \log\left(\det\left(\mathbf{I} + \rho\mathbf{h}_{2i}\mathbf{h}_{2i}^H + \rho\mathbf{h}_{2j}\mathbf{h}_{2j}^H\right)\right), \quad i, j = 1, \ldots, L, i \neq j \qquad (3.33)$$

$$\bar{R}_1 + \bar{R}_2 \leq \log\left(\det\left(\mathbf{I} + \rho\mathbf{h}_{2i-1}\mathbf{h}_{2i-1}^H + \rho\mathbf{h}_{2j}\mathbf{h}_{2j}^H\right)\right), \quad i, j = 1, \ldots, L \qquad (3.34)$$

$$\cdots$$

$$L\bar{R}_1 \leq \log\left(\det\left(\mathbf{I} + \rho\mathbf{H}_1\mathbf{H}_1^H\right)\right), \qquad (3.35)$$

$$L\bar{R}_2 \leq \log\left(\det\left(\mathbf{I} + \rho\mathbf{H}_2\mathbf{H}_2^H\right)\right), \qquad (3.36)$$

$$L\bar{R}_1 + L\bar{R}_2 \leq \log\left(\det\left(\mathbf{I} + \rho\mathbf{H}\mathbf{H}^H\right)\right), \qquad (3.37)$$

where

$$\mathbf{H}_1 = \begin{bmatrix} h_{\mathcal{S}_1} & 0 & 0 & \cdots & 0 \\ h_{\mathcal{R}_1} & 0 & 0 & \cdots & 0 \\ 0 & h_{\mathcal{S}_1} & 0 & \cdots & 0 \\ 0 & h_{\mathcal{R}_1} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & h_{\mathcal{S}_1} \\ 0 & 0 & 0 & \cdots & h_{\mathcal{R}_1} \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{3.38}$$

and

$$\mathbf{H}_2 = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ h_{\mathcal{S}_2} & 0 & 0 & \cdots & 0 \\ h_{\mathcal{R}_2} & 0 & 0 & \cdots & 0 \\ 0 & h_{\mathcal{S}_2} & 0 & \cdots & 0 \\ 0 & h_{\mathcal{R}_2} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & h_{\mathcal{S}_2} \\ 0 & 0 & 0 & \cdots & h_{\mathcal{R}_2} \end{bmatrix}. \tag{3.39}$$

Since a total of $T_L = 2L + 1$ time slots are used to transmit L codewords from each source, we have

$$R_i = \frac{L}{2L+1}\bar{R}_i, \quad i \in \{1, 2\}. \tag{3.40}$$

By substituting $R_i = r \log \rho$ and (3.40) into (3.30)-(3.37) and looking for the lowest DMT curve induced by all the $(2^{2L} - 1)$ rate constraints, the system achievable DMT performance is described in the following corollary to Theorem 1 (the proof is provided in Appendix A.1.2).

Corollary 1 *For the symmetric five-node network and an isolated-relay scenario (or a strong-interference scenario), on assuming that the relays correctly decode the sources, the achievable DMT for each source of the repetition-coded concurrent DF relaying protocol (i.e. the system model in (3.29)) is expressed by*

$$d(r) = 2\left(1 - \frac{2L+1}{L}r\right), \qquad 0 \le r \le \frac{L}{2L+1}. \tag{3.41}$$

Corollary 1 shows that under the assumption of perfect source-relay channel conditions, the repetition-coded concurrent DF relaying protocol achieves maximal multiplexing gain $\frac{L}{2L+1}$ and maximal diversity gain 2. Compared with the maximal multiplexing gain $\frac{1}{4}$ obtained by the one-relay standard DF relaying protocol (with achievable DMT $d(r) = 2(1 - 4r)$) and $\frac{1}{6}$ obtained by the two-relay standard DF relaying protocol (with achievable DMT $d(r) = 3(1-6r)$) for the two-source network, the proposed scheme significantly improves the multiplexing performance. When L is chosen as a large value, the maximal multiplexing gain can approach $\frac{1}{2}$, which matches the result for TDMA direct source-destination transmission. The multiplexing loss induced by the standard protocols is fully recovered. In addition, since even the use of full-duplex relays cannot further improve the maximal multiplexing gain performance over direct transmission for infinite-SNR [76] and TDMA obtains the optimal maximal multiplexing gain for the multiple-source system, the proposed protocol in fact achieves the optimal maximal multiplexing gain for the two-source network for large L. Although the achievable diversity gain of the repetition-coded concurrent DF relaying protocol outperforms the two-relay standard DF relaying protocol for only the large r region (specifically, for $\frac{L}{14L-2} \leq r \leq \frac{L}{2L+1}$), when compared with TDMA direct transmission, our protocol also increases diversity gain significantly. Figure 3.15 displays the DMT comparison. The advantages of the repetition-coded concurrent DF relaying protocol regarding both diversity gain improvement over TDMA direct source-destination transmission and multiplexing gain improvement over the repetition-coded standard DF relaying protocols can be clearly seen. By choosing a relatively large frame length $L = 15$, the proposed protocol outperforms TDMA direct transmission for almost all possible multiplexing gains.

Similar to the repetition-coded successive DF relaying protocol we discussed in the last section, we can also use a simple adaptive protocol under general source-relay link conditions to obtain the full DMT. The adaptive repetition-coded concurrent DF relaying protocol is presented in the following subsection.

3.4.3 General source-relay links

If the quality of source-relay links is not good enough to guarantee (3.27) and (3.28) hold, we require each relay to forward its source codewords to the destination *only if* it can correctly decode them. Otherwise, the relay remains silent for the whole transmission process. Again, we assume the sources are not aware of whether their relays can correctly decode them so that the transmission of the $2L$ codewords from the two sources always takes $T_L = 2L + 1$ time

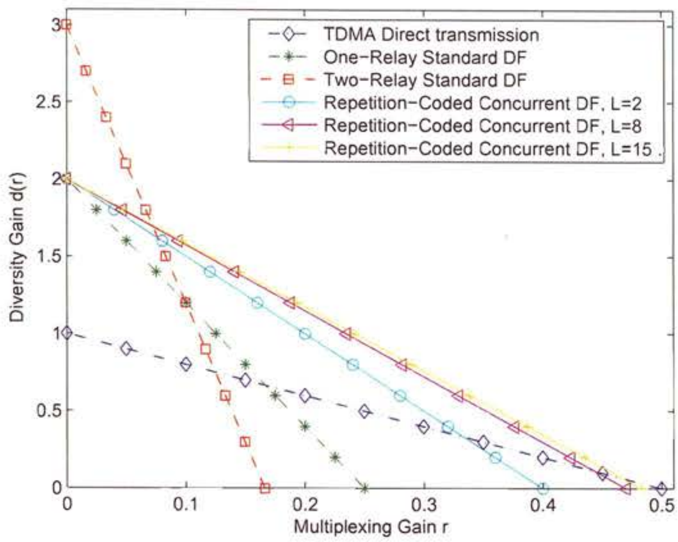

Figure 3.15: *DMT comparison of different protocols for the two-source network. It is assumed that the relays can always correctly decode the sources.*

slots.

There are four situations to be considered. Firstly, both channels are good enough so that both (3.27) and (3.28) are met. Both relays are used to assist the sources and the associated equivalent channel matrix is expressed by $\mathbf{H}$ in (3.29). Secondly, the condition (3.27) is met but (3.28) is not. In this case, only relay $\mathcal{R}_1$ is activated to assist its source $\mathcal{S}_1$. The equivalent channel matrix can be expressed as $\mathbf{H}$ in (3.29) by replacing $h_{\mathcal{R}_2}$ with 0. The third case is that only inequality (3.28) is satisfied so that only $\mathcal{S}_2$ is helped by its relay $\mathcal{R}_2$. The equivalent channel matrix can be expressed as $\mathbf{H}$ in (3.29) by replacing $h_{\mathcal{R}_1}$ with 0. Finally, when decoding at both relays fails (i.e. neither (3.27) nor (3.28) is met), the equivalent channel matrix is expressed as $\mathbf{H}$ in (3.29) by replacing both $h_{\mathcal{R}_1}$ and $h_{\mathcal{R}_2}$ with 0.

Denoting $P_{\mathcal{S}\mathcal{R}_1\mathcal{R}_2\mathcal{D}}$, $P_{\mathcal{S}\mathcal{R}_1\mathcal{D}}$, $P_{\mathcal{S}\mathcal{R}_2\mathcal{D}}$, and $P_{\mathcal{D}}$ as the outage probabilities at the destination for the four situations respectively, the overall outage probability can be calculated by

$$\begin{aligned} P_{\text{out}} &= P_{\mathcal{R}_1} P_{\mathcal{R}_2} P_{\mathcal{S}\mathcal{D}} + (1 - P_{\mathcal{R}_1}) P_{\mathcal{R}_2} P_{\mathcal{S}\mathcal{R}_1\mathcal{D}} \\ &\quad + P_{\mathcal{R}_1}(1 - P_{\mathcal{R}_2}) P_{\mathcal{S}\mathcal{R}_2\mathcal{D}} + (1 - P_{\mathcal{R}_1})(1 - P_{\mathcal{R}_2}) P_{\mathcal{S}\mathcal{R}_1\mathcal{R}_2\mathcal{D}} \end{aligned} \tag{3.42}$$

where $P_{\mathcal{R}_1}$ and $P_{\mathcal{R}_2}$ denote the outage probabilities at $\mathcal{R}_1$ and $\mathcal{R}_2$ respectively (i.e. $P_{\mathcal{R}_1}$ and $P_{\mathcal{R}_2}$ are calculated by (3.27) and (3.28) respectively). By studying the overall outage probability, the achievable DMT performance can be summarized as the following corollary to Theorem 2.

Corollary 2 *For the symmetric five-node network and an isolated-relay scenario (or a strong-interference scenario), under general source-relay channel conditions, the adaptive repetition-coded concurrent DF relaying protocol achieves the full DMT (3.41), i.e.*

$$d(r) = 2\left(1 - \frac{2L+1}{L}r\right), \qquad 0 \leq r \leq \frac{L}{2L+1}. \tag{3.43}$$

Corollary 2 shows that higher maximal multiplexing gain over the standard protocols and higher maximal diversity gain over TDMA direct transmission without the help of relays can indeed be achieved by the adaptive protocol under general source-relay channel conditions. Assuming $L = 2$, the outage probability performance of the repetition-coded concurrent DF relaying protocol (when $r = 0$ and $r = 0.2$) is plotted in Figure 3.16. Clearly, with the same multiplexing gain r, the diversity performance under both good and general source-relay channel conditions are the same. Corollary 2 is confirmed.

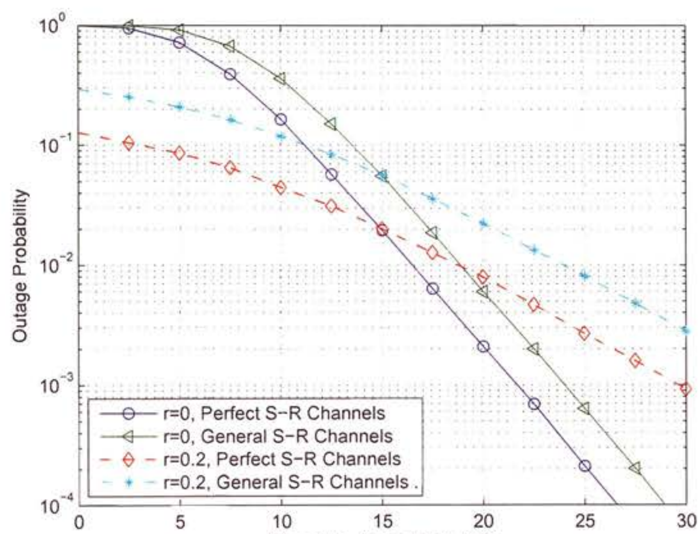

Figure 3.16: *Outage probability performance of the repetition-coded concurrent DF relaying protocols with $L = 2$. $R_1 = R_2 = 1$ BPCU when $r = 0$.*

3.5 M-source systems

In this section, we extend the protocols we discussed in the last two sections to the generalized $(M+3)$-node network, in which M sources communicate with one common destination with the help of two relays, as displayed in Figure 3.17. The basic idea is that the M sources communicate with the common destination using TDMA and the two relays take turns helping each source until the transmission of the L codewords from each source is finished. As mentioned in Section 3.2, to complete the transmission of the ML codewords, TDMA direct transmission uses ML time slots, the one-relay standard DF relaying protocol uses $2ML$ time slots, and the two-relay standard DF relaying protocol uses $3ML$ time slots. However, our approach demands only one more time slot than TDMA direct transmission (i.e. $T_L = ML + 1$). The specific transmission process can be described as follows (e.g. M is even):

Time slot 1: $\mathcal{S}_1$ broadcasts $x_1[1]$ to $\mathcal{R}_1$ and $\mathcal{D}$.

Time slot 2: $\mathcal{R}_1$ forwards $x_1[1]$ if it correctly decodes $x_1[1]$. $\mathcal{S}_2$ broadcasts $x_2[1]$ to $\mathcal{R}_2$ and $\mathcal{D}$.

Time slot 3: $\mathcal{R}_2$ forwards $x_2[1]$ if it correctly decodes $x_2[1]$. $\mathcal{S}_3$ broadcasts $x_3[1]$ to $\mathcal{R}_1$ and $\mathcal{D}$.

The transmission proceeds similarly until the Mth time slot.

Time slot $(M+1)$: $\mathcal{R}_2$ forwards $x_M[1]$ if it correctly decodes $x_M[1]$. $\mathcal{S}_1$ broadcasts $x_1[2]$ to $\mathcal{R}_1$ and $\mathcal{D}$.

Time slot $(M+2)$: $\mathcal{R}_1$ forwards $x_1[2]$ if it correctly decodes $x_1[2]$. $\mathcal{S}_2$ broadcasts $x_2[2]$ to $\mathcal{R}_2$ and $\mathcal{D}$.

The progress repeats until the (ML)th time slot.

Time slot $(ML+1)$: $\mathcal{R}_2$ forwards $x_M[L]$, the last codeword from $\mathcal{S}_M$, to $\mathcal{D}$, if it correctly decodes $x_M[L]$.

Thus, if the decoding at both relays is successful, the proposed protocol mimics an ML-user

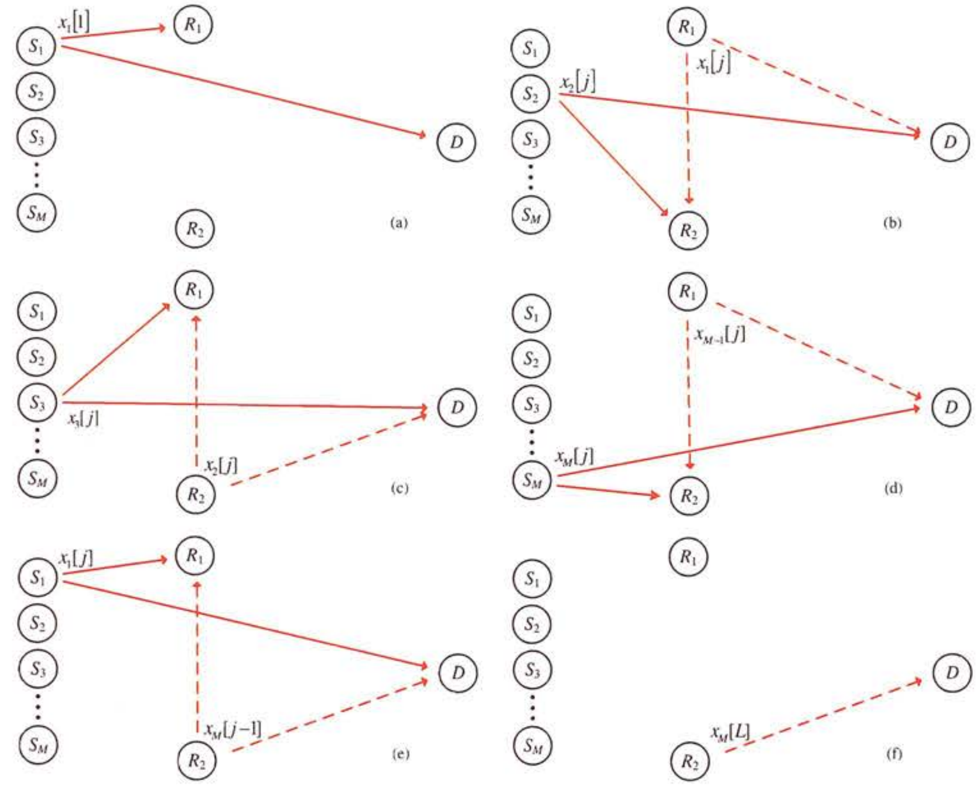

Figure 3.17: *Transmission schedule for the M-source protocol in (a) time slot 1, (b) time slot $(j-1)M+2$, $j \in \{1,\ldots,L\}$, (c) time slot $(j-1)M+3$, $j \in \{1,\ldots,L\}$, (d) time slot jM, , $j \in \{1,\ldots,L\}$, (e) time slot $(j-1)M+1$, , $j \in \{2,\ldots,L\}$, and (f) time slot $LM+1$.*

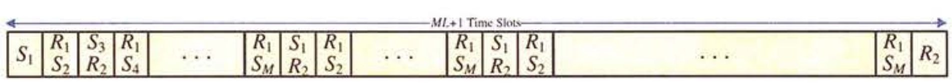

Figure 3.18: *Time-division channel allocation of the M-source protocol.*

multiple-access SIMO channel with equivalent channel matrix (e.g. M is even)

$$\mathbf{H} = \begin{bmatrix} h_{\mathcal{S}_1} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ h_{\mathcal{R}_1} & h_{\mathcal{S}_2} & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & h_{\mathcal{R}_2} & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & h_{\mathcal{S}_M} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & h_{\mathcal{R}_2} & h_{\mathcal{S}_1} & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & h_{\mathcal{R}_1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & h_{\mathcal{R}_1} & h_{\mathcal{S}_M} \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & h_{\mathcal{R}_2} \end{bmatrix}. \tag{3.44}$$

The transmission schedule and time-division channel allocation are illustrated in Figure 3.17 and Figure 3.18, respectively. The system achievable DMT for each source is summarized as the following corollary to Theorem 1 and Theorem 2.

Corollary 3 *For the symmetric M-source network and an isolated-relay scenario (or a strong-interference scenario), using two relays take turns assisting the sources obtains the DMT for each source*

$$d(r) = 2\left(1 - \frac{ML+1}{L}r\right), \qquad 0 \leq r \leq \frac{L}{ML+1}. \tag{3.45}$$

Note that we do not specify whether the source-relay channels are sufficiently good or not because the use of the adaptive protocol under general source-relay channel conditions achieves the same DMT performance as the case where the relays are assumed to always correctly decode the sources. Compared with the standard DF relaying protocols for the M-source network, our protocol can effectively improve the maximal multiplexing gain from $\frac{1}{2M}$ (for the one-relay standard DF relaying) or $\frac{1}{3M}$ (for the two-relay standard DF relaying) to $\frac{L}{ML+1}$. If ML is a large number, the maximal multiplexing gain approaches $\frac{1}{M}$ (the maximal multiplexing gain for TDMA direct source-destination transmission). This means the multiplexing loss is fully recovered and the requirement of L being very large is relaxed. Obviously, when $M = 1$, the protocol is the repetition-coded successive DF relaying protocol discussed in Section 3.3. And when $M = 2$, the protocol is the repetition-coded concurrent DF relaying protocol discussed in Section 3.4.

3.6 Summary

The use of relays to assist in direct source-destination communications has been intensively studied as a means to increase the system diversity performance. However, the diversity improvement normally causes a significant multiplexing performance reduction for the high-SNR region. In this chapter, our attention has been on using a novel protocol to recover the multiplexing loss induced by the standard DF relaying protocols. The first key feature of our protocol is that we relax the orthogonal transmission requirement so that the sources use the channel bandwidth more efficiently than the standard protocols. More specifically, for an M-source network, $\frac{L}{ML+1}$ of the total time channel is allocated to each source to convey information to the destination, while for the standard protocols, only $\frac{1}{2M}$ or $\frac{1}{3M}$ is used. Our protocol ob-

Transmission Protocols	Achievable DMT	d_{max}	r_{max}
TDMA Direct Transmission	$1-Mr$	1	$\frac{1}{M}$
One-relay Standard DF Relaying	$2(1-2Mr)$	2	$\frac{1}{2M}$
Two-relay Standard DF Relaying	$3(1-3Mr)$	3	$\frac{1}{3M}$
Proposed Protocol	$2\left(1-\frac{ML+1}{L}r\right)$	2	$\frac{L}{ML+1}$

Table 3.1: *Summary of Chapter 3: DMT comparison for different transmission protocols in an M-source network.*

tains significantly better multiplexing performance than the standard protocols. Conventionally, nonorthogonal transmission is not considered in repetition-coded DF relay networks because the system diversity gain is limited by the non-relayed codewords. However, by the use of two successively activated relays, each codeword can be conveyed to the destination through two independent paths so that higher diversity performance than direct source-destination transmission is achieved. This is another key feature of our protocol. Using $d_{\max}$ and $r_{\max}$ to denote respectively the maximal achievable diversity gain and multiplexing gain, Table 3.1 summarizes the key results of this chapter.

Finally, it is worth noting that when the source frame length L is sufficiently large, the maximal multiplexing gain of the proposed protocol approaches the result for direct source-destination transmission. This means, our protocol can attain the *optimal* multiplexing performance of the considered system.

Chapter 4
Improving Diversity Gain

4.1 Introduction

In the last chapter, we used two successively activated half-duplex relays to assist in the communications between M sources and a common destination. For an isolated-relay scenario and a strong-interference scenario, both the multiplexing performance of the standard DF relaying protocols and the diversity performance of direct source-destination transmission are improved with the use of a simple repetition-coding strategy at relays. When the source frame length L is large, the optimal multiplexing gain of the considered system [76] can actually be achieved so that the multiplexing loss due to the half-duplex limitation at relays is no longer an issue. Our attention is thus drawn back to the system *diversity* performance. One may ask whether even higher diversity gain can be obtained by still using repetition coding at relays without sacrificing the system multiplexing performance? In this chapter, we use advanced protocols to give affirmative answers to this question, especially for multiple-source networks.

Throughout this chapter, we mainly concentrate on *symmetric two-source* networks[1]. Similar to the last chapter, it is assumed that each of the two sources intends to transmit a frame with L codewords to a common destination. Assuming each codeword is transmitted with transmission rate $\bar{R}_i$ bits per codeword, the average transmission rate R_i BPCU from source $\mathcal{S}_i$ can be expressed by

$$R_i = \frac{L}{T_L}\bar{R}_i, \quad i \in \{1, 2\}, \tag{4.1}$$

where T_L denotes the overall time used to finish the transmission of the L codewords from each source. Since the aim of the new protocols we will present is to further improve the diversity performance over the repetition-coded concurrent DF relaying protocol without significantly losing multiplexing performance, the repetition-coded concurrent DF relaying protocol we discussed in the last chapter serves as a performance benchmark of all the protocols. Thereby, for the slow, frequency-flat, block Rayleigh fading environment, we rewrite its achievable DMT

[1]For single-source networks, applicable protocols will also be discussed.

(on assuming $R_i = r\log\rho$)

$$d(r) = 2\left(1 - \frac{2L+1}{L}r\right), \qquad 0 \le r \le \frac{L}{2L+1}. \tag{4.2}$$

The performance improvements of the new protocols are shown by comparing their achievable DMTs with (4.2).

In what follows, we first argue that, for the strong-interference scenario, the advantage of the *multi-relay* structure of the network is not fully exploited by the repetition-coded concurrent DF relaying protocol. In the previous chapter, we noticed that the achievable diversity gain of the repetition-coded concurrent DF relaying protocol is better than that of the two-relay standard DF relaying protocol only for the large multiplexing gain region. This is because the relays are required to individually serve the two sources and each codeword is protected by only one of the two successively activated relays. Is it possible that we can use both relays to assist each source? Or more explicitly, is it possible that each codeword can be retransmitted to the destination by both relays? We recall that, for the strong-interference scenario, the repetition-coded concurrent DF relaying protocol treats the inter-relay interference as useful information at only the relays but not at the destination (i.e. each relay discards the interference signal after using it to facilitate decoding). However, each interference signal is actually a codeword transmitted by one of the two sources so that it may also be useful to the destination. To make use of the inter-relay interference, we present a *superposition-coded concurrent DF relaying* protocol for the strong-interference scenario, in which each relay uses a superposition-coding strategy to retransmit both the interference and its desired signals to the destination. With only one extra transmission time slot, each codeword is forwarded (protected) by both relays. The achievable diversity gain of the repetition-coded concurrent DF relaying protocol is improved with a small multiplexing loss. When the signal frame length L is large, the multiplexing loss induced by the extra transmission time is negligible.

On the other hand, for the isolated-relay scenario where the inter-relay interference cannot be used, how can we improve the diversity performance? To answer this question, we use a *multiple-access concurrent DF relaying* protocol to take advantage of the *multi-source* structure of the network, which is also not exploited by the repetition-coded concurrent DF relaying protocol. In this new protocol, we permit the two sources to transmit simultaneously to use the channel more efficiently than requiring the two sources to communicate with the destination using TDMA [13]. For the considered five-node network, we also present a possible repetition-coded DF relaying protocol based on an orthogonal transmission requirement (termed *multiple-access standard DF relaying*) in which sources and relays communicate with destination in orthogonal channels (time slots). It is shown that if the source-relay transmissions are always successful, the multiple-access standard DF relaying protocol does not induce any multiplexing loss. However, for general source-relay channel conditions, spectral efficiency is significantly lost. Such an issue does not appear in the multiple-access concurrent DF relaying protocol, which has no significant multiplexing loss compared with multiple-access direct transmission when the frame length L is sufficiently large. In addition, for large L, the diversity performance of the multiple-access concurrent DF relaying protocol is better than that of the repetition-coded concurrent DF relaying protocol within the region of almost all possible multiplexing gains.

Finally, we focus our attention on the impact of the number of antennas at the destination. In this thesis, we concentrate on uplink transmissions for practical cellular systems so that mobile terminals (i.e. sources and relays) cannot afford a multiple-antenna setup due to hardware or cost limitations, but the base station (i.e. the destination) can. Therefore, we study the DMT performance of the repetition-coded concurrent DF relaying protocol (for the isolated-relay scenario) and the superposition-coded concurrent DF relaying protocol (for the strong-interference scenario) when the destination is equipped with multiple antennas. Assuming perfect decoding at the relays, the system diversity gain is improved with increasing the number of destination antennas. However, for general source-relay channel conditions, it may be difficult to achieve the full DMTs. This is because the system diversity gain is limited by the low diversity gain provided by the source-relay links when only two relays are considered. To handle this issue, adaptive protocols with a relay selection scheme, which selects two relays from many potential relays to assist the sources, are analyzed. Through DMT analysis, it is shown that if the number of potential relays is larger than some specific threshold (related to the number of destination antennas), the system will perform the same as the case in which the source-relay transmissions are always successful. In this way, the diversity performance of the repetition-coded concurrent DF relaying protocol proposed for a single-antenna system and analyzed in the last chapter can be dramatically improved by using multiple-antennas at the destination under general source-relay channel conditions.

4.2 Strong-interference scenario

In this section, we consider the *strong-interference scenario* and exploit the advantage of the multiple-relay structure of the system.

4.2.1 Protocol design

As mentioned in the last chapter, for the strong-interference scenario, during each time slot, the repetition-coded concurrent DF relaying protocol requires one of the two relays to subtract the inter-relay interference and retransmit only its desired source codeword to the destination. In fact, the interference signal is a transmitted codeword from the other source and thus can also be treated as useful information for the destination. To this end, now we permit each relay to use superposition coding [13, 38] to transmit the *sum* of the interference codeword and the desired codeword (each of the two codewords is allocated with part of the transmit power of the relay). For the finite-SNR region, how to allocate transmit power to the two codewords according to different channel conditions or statistics may have an influence on the system capacity and error performance. However, in the infinite-SNR region, power allocation has no consequence for DMT performance. For simplicity, we assume the power allocation at both relays are the same and each relay allocates $\gamma_d^2\rho$ $(0 < \gamma_d < 1)$ to its desired codeword and $\gamma_i^2\rho$ $(\gamma_i = \sqrt{1-\gamma_d^2})$ to the interference codeword. By this means, the two relays are more efficiently used and each source can be considered as being assisted by both relays. To guarantee that every codeword can be conveyed to the destination via three independent paths, one extra time slot is introduced and thus $T_L = 2L + 2$ time slots are used to finish the transmission of the $2L$ codewords from the two sources (i.e. the average transmission rate of each source $R_i = \frac{L}{2L+2}\bar{R}_i$ BPCU). The detailed transmission process is described as follows:

Time slot 1: $\mathcal{S}_1$ broadcasts $x_1[1]$ to both $\mathcal{R}_1$ and $\mathcal{D}$; $\mathcal{S}_2$ and $\mathcal{R}_2$ remain silent.

Time slot 2: $\mathcal{R}_1$ forwards $x_1[1]$ to $\mathcal{R}_2$ and $\mathcal{D}$. $\mathcal{S}_2$ transmits $x_2[1]$. $\mathcal{R}_2$ listens to $\mathcal{S}_2$ while being interfered by $x_1[1]$ from $\mathcal{R}_1$. $\mathcal{D}$ receives $x_1[1]$ from $\mathcal{R}_1$ and $x_2[1]$ from $\mathcal{S}_2$.

Time slot 3: $\mathcal{R}_2$ forwards $(\gamma_d x_2[1] + \gamma_i x_1[1])$ to $\mathcal{R}_1$ and $\mathcal{D}$. $\mathcal{S}_1$ transmits $x_1[2]$. $\mathcal{R}_1$ listens to $\mathcal{S}_1$ while being interfered by $(\gamma_d x_2[1] + \gamma_i x_1[1])$ from $\mathcal{R}_2$. $\mathcal{D}$ receives $(\gamma_d x_2[1] + \gamma_i x_1[1])$ from $\mathcal{R}_2$ and $x_1[2]$ from $\mathcal{S}_1$.

Time slot 4: $\mathcal{R}_1$ forwards $(\gamma_d x_1[2] + \gamma_i x_2[1])$ to $\mathcal{R}_2$ and $\mathcal{D}$. $\mathcal{S}_2$ transmits $x_2[2]$. $\mathcal{R}_2$ listens to $\mathcal{S}_2$

while being interfered by $(\gamma_d x_1[2] + \gamma_i x_2[1])$ from $\mathcal{R}_1$. $\mathcal{D}$ receives $(\gamma_d x_1[2] + \gamma_i x_2[1])$ from $\mathcal{R}_1$ and $x_2[2]$ from $\mathcal{S}_2$.

This process repeats until the $(2L)$th time slot.

Time slot $(2L + 1)$: $\mathcal{R}_2$ retransmits $(\gamma_d x_2[L] + \gamma_i x_1[L])$ to $\mathcal{R}_1$ and $\mathcal{D}$.

Time slot $(2L + 2)$: $\mathcal{R}_1$ decodes, re-encodes and retransmits $x_2[L]$ to $\mathcal{D}$.

Unlike the repetition-coded concurrent DF relaying protocol, from the third time slot to the $(2L + 1)$th time slot, the signal that one relay receives from the other relay is the sum of two codewords. For instance, during the time slot 3, the interference signal received by $\mathcal{R}_1$ is $(\gamma_d x_2[1] + \gamma_i x_1[1])$. Clearly, $x_1[1]$ is $\mathcal{R}_1$'s own transmitted signal during the previous time slot (i.e. time slot 2). We require $\mathcal{R}_1$ to subtract $x_1[1]$ from its received signal before decoding. Since $\mathcal{R}_1$ has full knowledge of $x_1[1]$, cancellation can be performed so that the interference at $\mathcal{R}_1$ is now actually only $x_2[1]$, the transmitted codeword from $\mathcal{S}_2$. The process repeats for each time slot and the situation is thus the same as that for the repetition-coded concurrent DF relaying protocol. After all the $2L$ codewords are received, the destination performs joint decoding. We refer to this protocol as *superposition-coded concurrent DF relaying*. Its time division channel allocation and transmission schedule are illustrated in Figure 4.1.

In the following, we first present the DMT performance of the superposition-coded concurrent DF relaying protocol under the assumption of perfect source-relay transmissions. For general source-relay channel conditions, a simple adaptive protocol is used such that the full DMT can still be attained.

4.2.2 Perfect source-relay links

If both source-relay links are sufficiently good such that the following inequalities are met

$$\bar{R}_1 \leq \log(1 + \rho |h_{\mathcal{S}_1,\mathcal{R}_1}|^2), \tag{4.3}$$

$$\bar{R}_2 \leq \log(1 + \rho |h_{\mathcal{S}_2,\mathcal{R}_2}|^2), \tag{4.4}$$

all source codewords are correctly decoded by the relays. The superposition-coded concurrent DF relaying protocol mimics a $2L$-user multiple-access SIMO channel with input-output

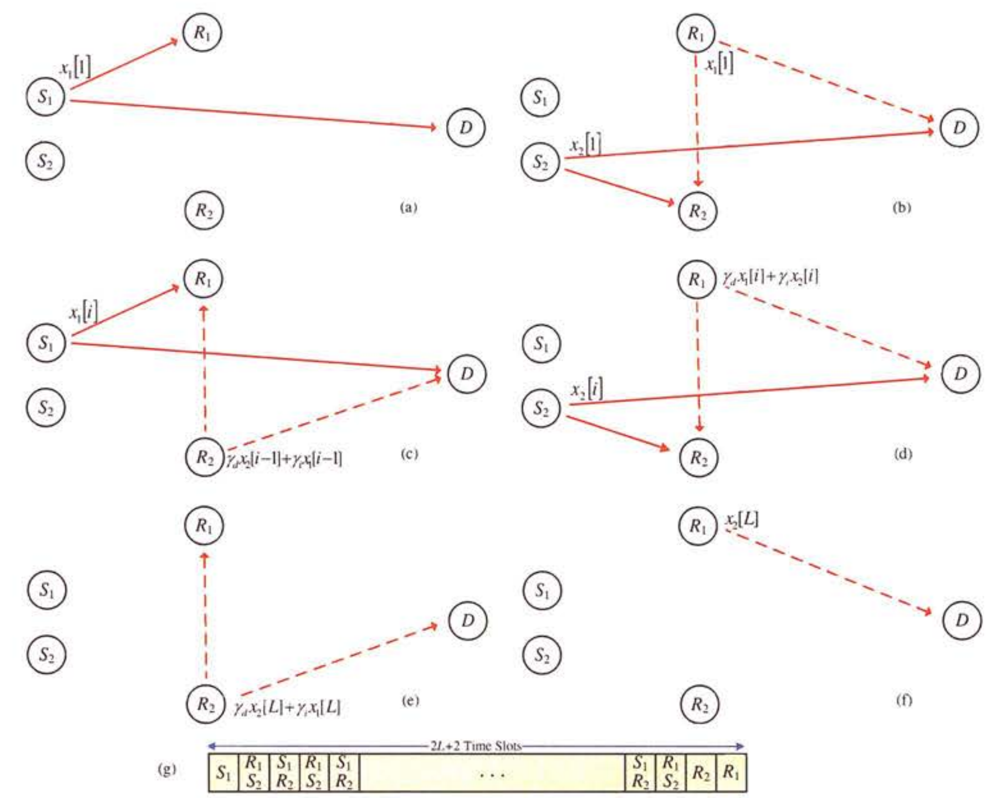

Figure 4.1: *Transmission schedule for the superposition-coded concurrent DF relaying protocol in (a) time slot* 1*, (b) time slot* 2*, (c) time slot* $2i-1$*,* $i \in \{2,\ldots,L\}$*, (d) time slot* $2i$*,* $i \in \{2,\ldots,L\}$*, (e) time slot* $2L+1$*, (f) time slot* $2L+2$*, and (g) the time division channel allocation.*

relation:

$$\begin{bmatrix} y[1] \\ y[2] \\ y[3] \\ \vdots \\ y[2L+1] \\ y[2L+2] \end{bmatrix} = \sqrt{\rho} \underbrace{\begin{bmatrix} h_{\mathcal{S}_1} & 0 & 0 & \cdots & 0 & 0 \\ h_{\mathcal{R}_1} & h_{\mathcal{S}_2} & 0 & \cdots & 0 & 0 \\ \gamma_i h_{\mathcal{R}_2} & \gamma_d h_{\mathcal{R}_2} & h_{\mathcal{S}_1} & \cdots & 0 & 0 \\ 0 & \gamma_i h_{\mathcal{R}_1} & \gamma_d h_{\mathcal{R}_1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \gamma_d h_{\mathcal{R}_1} & h_{\mathcal{S}_2} \\ 0 & 0 & 0 & \cdots & \gamma_i h_{\mathcal{R}_2} & \gamma_d h_{\mathcal{R}_2} \\ 0 & 0 & 0 & \cdots & 0 & h_{\mathcal{R}_1} \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} x_1[1] \\ x_2[1] \\ x_1[2] \\ \vdots \\ x_1[L] \\ x_2[L] \end{bmatrix} + \mathbf{n} \tag{4.5}$$

where $\mathbf{n}$ is the $(2L+2) \times 1$ AWGN vector at the destination. To guarantee that the destination can correctly decode all the $2L$ codewords, the source transmission rates need to satisfy the

$(2^{2L}-1)$ constraints (3.30) - (3.37) except that the channel matrix $\mathbf{H}$ is defined in (4.5). With $\bar{R}_i = \frac{T_L}{L}R_i = \frac{T_L}{L}r\log\rho$ for infinite SNR, we simplify the rate constraints (3.30) - (3.37) to the following inequality

$$\frac{T_L}{L}|\mathcal{L}|r\log\rho \leq \log\left(\det\left(\mathbf{I}+\rho\sum_{j\in\mathcal{L}}\mathbf{h}_j\mathbf{h}_j^H\right)\right), \quad \forall\mathcal{L}\subseteq\{1,\cdots,2L\}, \tag{4.6}$$

where $|\mathcal{L}|$ denotes the cardinality of set $\mathcal{L}$.

As we discussed in Section 3.3.3, an outage event occurs when any of the $(2^{2L}-1)$ constraints is not met. The achievable diversity gain of the system is the lowest one calculated by all the outage events. Regarding the achievable system DMT, we have the following theorem (the proof is provided in Appendix A.3.1).

Theorem 3 *For the symmetric five-node network and a strong-interference scenario, on assuming that the relays correctly decode the sources, the achievable DMT for each source of the superposition-coded concurrent DF relaying protocol (i.e. the system model in (4.5)) is expressed by*

$$d(r) = 3\left(1-\frac{2L+2}{L}r\right), \qquad 0\leq r\leq\frac{L}{2L+2}. \tag{4.7}$$

Equation (4.7) indicates that maximal diversity gain 3 and maximal multiplexing gain $\frac{L}{2L+2}$ can be achieved conditioned on (4.3) and (4.4). Compared with (4.2), it can be seen that the diversity performance of the repetition-coded concurrent DF relaying protocol is further improved by making use of the inter-relay interference. Therefore, unlike the repetition-coded concurrent DF relaying protocol whose diversity gain is only larger than that of the two-relay standard DF relaying protocol for the high r region, the superposition-coded concurrent DF relaying protocol strictly outperforms the two-relay standard DF relaying protocol in terms of DMT. Although there exists a slight difference for the maximal multiplexing gains $\frac{L}{2L+1}-\frac{L}{2L+2}=\frac{L}{(2L+1)(2L+2)}$ between the repetition-coded and superposition-coded concurrent DF relaying protocols (due to the extra transmission time slot introduced in the latter protocol), when the frame length L is sufficiently large this difference is negligible. The maximal multiplexing gains for both protocols approach $\frac{1}{2}$. No multiplexing loss is induced and the optimal multiplexing performance of the system can be achieved.

It is worth noting that, when working in a strong-interference scenario, the superposition-coded concurrent DF relaying protocol does not require a much complex encoding strategy at the relays than for the repetition-coded concurrent DF relaying protocol. In other words, after decoding and re-encoding both the desired and interference codewords, the superposition-coded concurrent DF relaying protocol divides each relay's transmit power into two parts, allocates them to the two codewords, and transmits the sum of the two codewords to the other relay and the destination. The repetition-coded concurrent DF relaying protocol is lightly simpler as it only transmits one codeword with full power. The diversity improvement of the superposition-coded concurrent DF relaying protocol comes from taking advantage of the multiple-relay structure and efficiently using both relays rather than only one to assist each source.

An example of the DMT comparison of different protocols is displayed in Figure 4.2. It can be seen that the DMT performance of the superposition-coded concurrent DF relaying protocol strictly outperforms that of the two-relay standard DF relaying protocol. When L is chosen as a large value, e.g. $L = 15$, the superposition-coded concurrent DF relaying protocol does not induce significant multiplexing loss compared with TDMA direct transmission and its diversity performance is better than that of the repetition-coded concurrent DF relaying protocol within the range of almost all possible multiplexing gains.

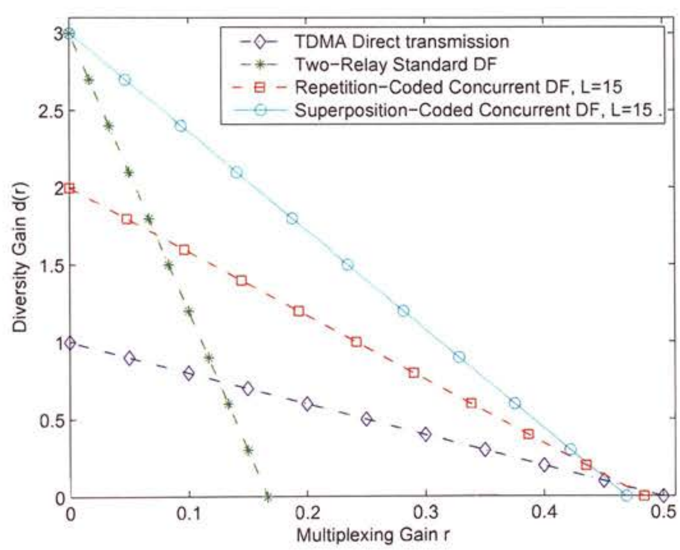

Figure 4.2: *DMT performance of different protocols for the strong-interference scenario under perfect source-relay channel conditions. Each terminal is equipped with a single-antenna.*

4.2.3 General source-relay links

If the source-relay links are not always good enough to guarantee both conditions (4.3) and (4.4) hold, following the analysis in Theorem 2, it is not difficult to prove that the full DMT cannot be achieved by requiring relay $\mathcal{R}_1$ to listen only to source $\mathcal{S}_1$ and relay $\mathcal{R}_2$ to listen only to source $\mathcal{S}_2$. This is because the adaptive protocol with such a requirement only provides maximal diversity 2, which is clearly lower than the full DMT (4.7). However, we have two sources and two relays in the network and the transmission of each source can in fact be overheard by both relays. Besides the $\mathcal{S}_1$-$\mathcal{R}_1$ and $\mathcal{S}_2$-$\mathcal{R}_2$ links we considered for the repetition-coded concurrent DF relaying protocol, the $\mathcal{S}_1$-$\mathcal{R}_2$ link and the $\mathcal{S}_2$-$\mathcal{R}_1$ link can provide extra protection for each codeword (i.e. extra diversity gain for the source-relay links). In particular, if either (4.3) or (4.4) is not met (i.e. $\mathcal{R}_1$ cannot decode $\mathcal{S}_1$ and/or $\mathcal{R}_2$ cannot decode $\mathcal{S}_2$) but the following two inequalities are satisfied

$$\bar{R}_1 \leq \log(1 + \rho |h_{\mathcal{S}_1,\mathcal{R}_2}|^2), \tag{4.8}$$

$$\bar{R}_2 \leq \log(1 + \rho |h_{\mathcal{S}_2,\mathcal{R}_1}|^2), \tag{4.9}$$

the two relays can still be used to assist the sources because $\mathcal{R}_1$ and $\mathcal{R}_2$ can correctly decode $\mathcal{S}_2$ and $\mathcal{S}_1$ respectively. Thereby, for the adaptive superposition-coded concurrent DF relaying protocol, we assume the relays can be configured to assist the sources *if and only if* one relay can decode one source and the other relay can decode the other source. Otherwise, *both* relays remain silent. The overall system outage probability can thus be expressed by

$$P_{\text{out}} = P_{\mathcal{R}} P_{\mathcal{SD}} + (1 - P_{\mathcal{R}}) P_{\mathcal{SRD}}, \tag{4.10}$$

where $P_{\mathcal{R}}$ denotes the probability that relays are *not* used, $P_{\mathcal{SD}}$ denotes the associated outage probability at the destination, and $P_{\mathcal{SRD}}$ denotes the outage probability at the destination given that the source codewords are also retransmitted by the relays. By studying the high-SNR expression of P_{out}, we have the following theorem regarding the achievable DMT performance (the proof is provided in Appendix A.4.1).

Theorem 4 *For the symmetric five-node network and a strong-interference scenario, under general source-relay channel conditions, the adaptive superposition-coded concurrent DF relaying protocol achieves the full DMT*

$$d(r) = 3\left(1 - \frac{2L+2}{L} r\right), \qquad 0 \leq r \leq \frac{L}{2L+2}. \tag{4.11}$$

The DMT performance implies that the adaptive superposition-coded concurrent DF relaying protocol is another protocol capable of efficiently exploiting the multiple-source multiple-relay structure of the considered system.

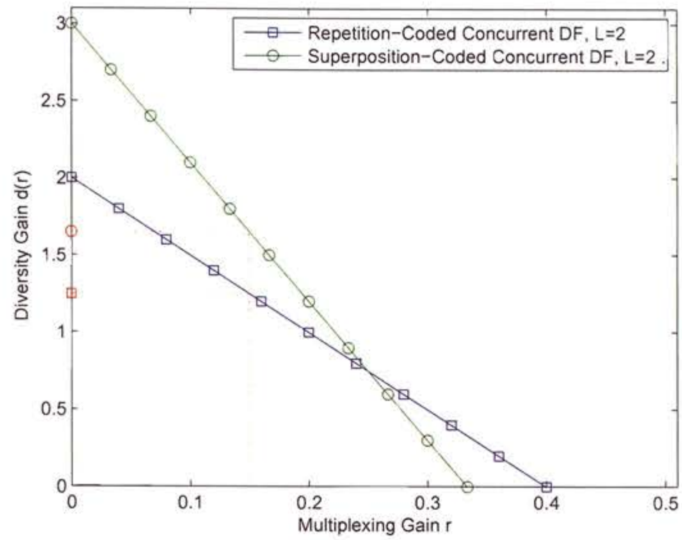

Figure 4.3: *DMT comparison of the repetition-coded and superposition-coded concurrent DF relaying protocols when the frame length $L = 2$. The dashed lines mark the achievable diversity gains of the two protocols when the multiplexing gain $r = 0.15$.*

When the frame length $L = 2$, the DMT comparison of the repetition-coded and superposition-coded concurrent DF relaying protocols is displayed in Figure 4.3. Setting $\gamma_i = \gamma_d = \sqrt{\frac{1}{2}}$, we plot the outage probability performance of the two protocols when the multiplexing gains $r = 0$ and $r = 0.15$ in Figure 4.4 and Figure 4.5 respectively. In both figures, it can be seen that the superposition-coded concurrent DF relaying protocol has deeper slopes of high-SNR outage probability curves than the repetition-coded concurrent DF relaying protocol. This observation clearly shows that the diversity performance of making use of the inter-relay interference is better than simply requiring each relay to retransmit only its desired codeword after subtracting the interference. The superposition-coded concurrent DF relaying protocol under the assumption of perfect source-relay transmissions achieves better outage probability performance than the adaptive protocol because of a power gain (due to the fact that the relays are always used to retransmit the source codewords to the destination). However, the slops of the high-SNR outage probability curves for both cases are the same, which implies that the diversity performance under perfect source-relay channel conditions can be attained under general source-relay channel conditions. The result of Theorem 4 is confirmed. When $L > 2$, the performance will be better than those shown in the figures and the diversity gain of the superposition-coded concurrent DF relaying protocol will outperform that of the repetition-coded concurrent DF relaying protocol for a wider region of r.

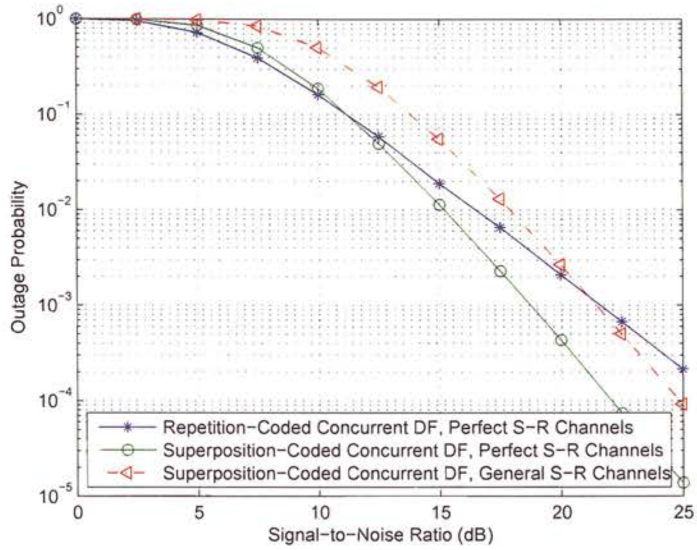

Figure 4.4: *Outage probability comparison of the repetition-coded and superposition-coded concurrent DF relaying protocols ($L = 2$) for the strong-interference scenario when $r = 0$ (the average transmission rate is fixed at 1 BPCU).*

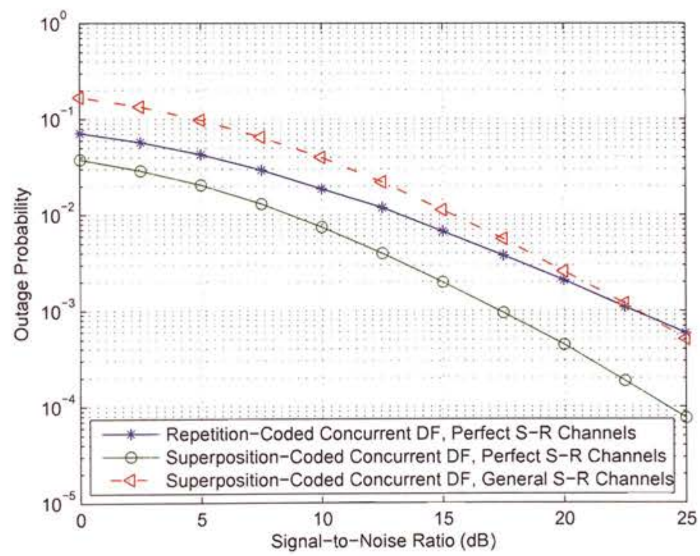

Figure 4.5: *Outage probability comparison of the repetition-coded and superposition-coded concurrent DF relaying protocols ($L = 2$) for the strong-interference scenario when $r = 0.15$.*

It is worth noting that in this thesis the superposition coding is an information-theoretical consideration and it denotes the sum of two i.i.d. Gaussian random codewords. In practice, transmitting a combination of two messages in the superposition-coded concurrent DF relaying protocol (and also the multiple-access concurrent DF relaying protocol which will be discussed in the next section) can be realized by simply requiring each relay to retransmit the sum of the modulated symbols of the two messages, which is similar to *superposition modulation* as discussed in [77]. Another simple method for the relays to retransmit simultaneously both messages is similar to *code superposition* [78] such that the transmit signal of each relay is the XORed version of the two messages. It has been shown in [78] that code superposition brings error performance improvement over superposition modulation. In addition, if one of the two messages is correctly decoded at the destination, it is as if the relay transmits the other message with full power. Therefore, we conjecture that its performance is upper bounded by that of the system model in (4.5) when $\gamma_i = \gamma_d = 1$. Since the values of γ_i and γ_d have no impact on the infinite-SNR DMT performance, it is conjectured that such an approach also attains the full DMT (4.7). In recent years, a similar coding strategy has been commonly considered in the context of *network coding* [79]. Therefore, it would be interesting to consider combining results from network coding with our protocols as a future work.

4.2.4 Single-source network

Straightforwardly, the use of superposition coding at relays to take advantage of the strong-interference scenario can also be applied to improve diversity performance over the repetition-coded successive DF relaying protocol (the achievable DMT is expressed in (3.23)) discussed in Section 3.3, which considers a single-source two-relay one-destination network. Specifically, during each time slot, one relay retransmits the sum of the interference codeword it received from the other relay and its desired source codeword it received from the source. A total of $T_L = L + 2$ time slots are used to finish the transmission of a frame of L codewords so that every codeword is protected by both relays. We refer to this protocol as *superposition-coded successive DF relaying*. Assuming perfect decoding at the relays, following the proof of Theorem 3, it can be proved that the achievable DMT is calculated by

$$d(r) = 3\left(1 - \frac{L+2}{L}r\right), \qquad 0 \leq r \leq \frac{L}{L+2}. \tag{4.12}$$

Maximal diversity gain 3 and maximal multiplexing gain $\frac{L}{L+2}$ can thus be attained. For small value of the frame length L, the superposition-coded successive DF relaying protocol has a multiplexing loss $\frac{L}{L+1} - \frac{L}{L+2} = \frac{L}{(L+1)(L+2)}$ compared with the repetition-coded successive DF relaying protocol. With very large L, the multiplexing loss is negligible and (4.12) strictly outperforms (3.23). The diversity performance is improved without losing multiplexing performance.

However, under general source-relay channel conditions, it is difficult to achieve the full DMT (4.12) by using only two relays. This is because the two links between the source and the two relays can only provide maximal diversity gain 1 for each codeword. Such a diversity gain is smaller than the maximal diversity gain difference (i.e. 2) between the case that relays are used to assist the source and the case that relays are not used. The system achievable DMT is limited to be smaller than the full DMT. Thus we may need other approaches to obtain higher diversity (for each codeword) between the source and the relays. Here we temporarily skip the exact solution of this problem. In Section 4.4.3, when we study how the full DMT is attained in general source-relay channel conditions when the destination is equipped with multiple antennas, we will return to this question. The adaptive protocol presented in that section can be applied in this case.

4.3 Isolated-relay scenario

In this section, we move on to discuss how to improve the diversity performance of the repetition-coded concurrent DF relaying protocol for the isolated-relay scenario in which no inter-relay interference exists.

4.3.1 Protocol design

For the *isolated-relay scenario*, the interference between relays is sufficiently weak compared with the source-relay transmissions. Thus the superposition-coding strategy cannot be used at the relays. In order to improve the diversity performance of the repetition-coded concurrent DF relaying protocol, we permit the two sources to transmit simultaneously. In fact, such a transmission process is similar to the single-source repetition-coded successive DF relaying protocol discussed in the last chapter except that the two relays take turns assisting *two* simultaneously transmitted sources rather than a single one. After decoding and re-encoding (using the same codebooks as the sources) *both* sources' codewords, each relay splits its transmit power and transmits the *sum* of the two codewords. Since the power allocation to each codeword does not affect the system infinite-SNR DMT performance, for simplicity, we assume that both relays use the same two power allocation factors γ_i and γ_d ($0 < \gamma_i < 1$, $\gamma_d = \sqrt{1-\gamma_i^2}$). We refer to this protocol as *multiple-access concurrent DF relaying*. A total of $T_L = L + 1$ time slots are used for completing the transmission. The detailed transmission process is described as follows:

Time slot 1: $\mathcal{S}_1$ broadcasts $x_1[1]$ and $\mathcal{S}_2$ broadcasts $x_2[1]$ to both $\mathcal{R}_1$ and $\mathcal{D}$; $\mathcal{R}_2$ remains silent.

Time slot 2: $\mathcal{R}_1$ forwards $(\gamma_i x_1[1] + \gamma_d x_2[1])$ to $\mathcal{D}$. $\mathcal{S}_1$ broadcasts $x_1[2]$ and $\mathcal{S}_2$ broadcasts $x_2[2]$. $\mathcal{R}_2$ listens to $\mathcal{S}_1$ and $\mathcal{S}_2$. $\mathcal{D}$ receives $(\gamma_i x_1[1] + \gamma_d x_2[1])$ from $\mathcal{R}_1$, $x_1[2]$ from $\mathcal{S}_1$, and $x_2[2]$ from $\mathcal{S}_2$.

Time slot 3: $\mathcal{R}_2$ forwards $(\gamma_i x_2[2] + \gamma_d x_1[2])$ to $\mathcal{D}$. $\mathcal{S}_1$ broadcasts $x_1[3]$ and $\mathcal{S}_2$ broadcasts $x_2[3]$. $\mathcal{R}_1$ listens to $\mathcal{S}_1$ and $\mathcal{S}_2$. $\mathcal{D}$ receives $(\gamma_i x_2[2] + \gamma_d x_1[2])$ from $\mathcal{R}_2$, $x_1[3]$ from $\mathcal{S}_1$, and $x_2[3]$ from $\mathcal{S}_2$.

This process repeats until the (L)th time slot.

Time slot $(L+1)$: $\mathcal{R}_\alpha$ ($\alpha = 1$ if L is odd, and $\alpha = 2$ if L is even) retransmits $(\gamma_i x_\alpha[L] + \gamma_d x_{\bar{\alpha}}[L])$ ($\bar{\alpha} = 2$ if $\alpha = 1$, and $\bar{\alpha} = 1$ if $\alpha = 2$) to $\mathcal{D}$.

The destination combines the signals it received from all the $(L+1)$ time slots and jointly decodes the transmitted source codewords. The time-division channel allocation and transmission schedule (when L is even) are displayed in Figure 4.6.

4.3.2 Perfect source-relay links

When the sources transmit simultaneously, the two sources and each relay form a two-user multiple-access channel. Since for infinite SNR $R_i = \frac{L}{L+1}\bar{R}_i = r\log\rho$, assuming all the source-relay channels are sufficiently good such that the following inequalities are satisfied:

$$\frac{L+1}{L} r \log\rho \leq \log(1 + \rho|h_{\mathcal{S}_1,\mathcal{R}_j}|^2), \tag{4.13}$$

$$\frac{L+1}{L} r \log\rho \leq \log(1 + \rho|h_{\mathcal{S}_2,\mathcal{R}_j}|^2), \tag{4.14}$$

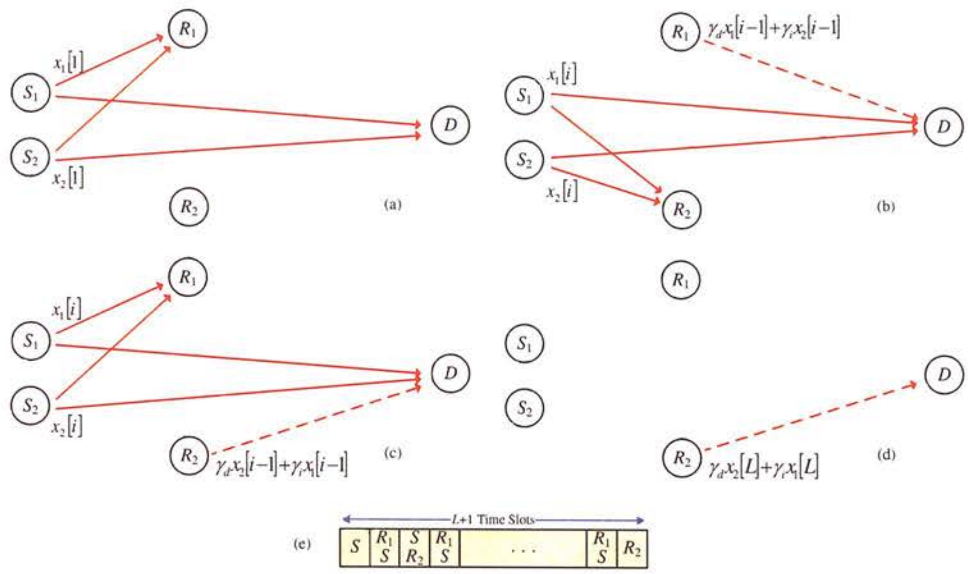

Figure 4.6: *Transmission schedule for the multiple-access concurrent DF relaying protocol (L is even) in (a) time slot 1, (b) even time slot i ($2 \leq i \leq L$), (c) odd time slot i ($2 < i \leq L$), (d) time slot $L+1$, and (e) the time-division channel allocation, where S means the two sources $\mathcal{S}_1$ and $\mathcal{S}_2$ transmit simultaneously.*

$$2\frac{L+1}{L} r \log \rho \leq \log(1 + \rho|h_{\mathcal{S}_1,\mathcal{R}_j}|^2 + \rho|h_{\mathcal{S}_2,\mathcal{R}_j}|^2), \tag{4.15}$$

in which $j \in \{1, 2\}$, each relay can always correctly decode both sources. The multiple-access concurrent DF relaying protocol mimics a $2L$-user multiple-access SIMO channel

$$\begin{bmatrix} y[1] \\ y[2] \\ y[3] \\ \vdots \\ y[L] \\ y[L+1] \end{bmatrix} = \sqrt{\rho} \underbrace{\begin{bmatrix} h_{\mathcal{S}_1} & h_{\mathcal{S}_2} & 0 & 0 & \cdots & 0 & 0 \\ \gamma_i h_{\mathcal{R}_1} & \gamma_d h_{\mathcal{R}_1} & h_{\mathcal{S}_1} & h_{\mathcal{S}_2} & \cdots & 0 & 0 \\ 0 & 0 & \gamma_d h_{\mathcal{R}_2} & \gamma_i h_{\mathcal{R}_2} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & h_{\mathcal{S}_1} & h_{\mathcal{S}_2} \\ 0 & 0 & 0 & 0 & \cdots & \gamma_a h_{\mathcal{S}_\alpha} & \gamma_b h_{\mathcal{R}_\alpha} \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} x_1[1] \\ x_2[1] \\ x_1[2] \\ \vdots \\ x_1[L] \\ x_2[L] \end{bmatrix} + \mathbf{n} \tag{4.16}$$

where $\alpha = 1$, $\gamma_a = \gamma_i$ and $\gamma_b = \gamma_d$ if L is odd, and $\alpha = 2$, $\gamma_a = \gamma_d$ and $\gamma_b = \gamma_i$ if L is even. Substituting $T_L = L + 1$ and the channel matrix $\mathbf{H}$ defined in (4.16) into the rate constraints (4.6), we have the following theorem regarding the achievable DMT (the proof is provided in Appendix A.5).

Theorem 5 *For the symmetric five-node network and an isolated-relay scenario, on assum-*

ing that the relays correctly decode the sources, the achievable DMT for each source of the multiple-access concurrent DF relaying protocol (i.e. the system model in (4.16)) is lower bonded *by*

$$d(r) = \begin{cases} 2(1 - \frac{L+1}{L}r) & 0 \leq r \leq \frac{3L}{10L+10} \\ \frac{7}{2}(1 - \frac{2L+2}{L}r) & \frac{3L}{10L+10} \leq r \leq \frac{L}{2L+2} \end{cases}. \tag{4.17}$$

We can only have a lower bound (4.17) of the achievable DMT because the exact expression of the determinant of the matrix $(\mathbf{I} + \rho\mathbf{H}\mathbf{H}^H)$ is difficult to obtain. From the proof part in Appendix A.5, it can be seen that the bound is tight for $0 \leq r \leq \frac{3L}{10L+10}$ (i.e. the first equation in (4.17) is tight). Moreover, since for large L the maximal multiplexing gain $\frac{L}{2L+2}$ approaches $\frac{1}{2}$, which matches the result for the multiple-access direct source-destination transmission (i.e. a single-antenna two-user multiple-access channel, which achieves the DMT $d(r) = \min\{1 - r, 2(1-2r)\}$), the point $(r, d(r)) = (\frac{1}{2}, 0)$ is also tight for large L.

Unlike the superposition-coded concurrent DF relaying protocol, which takes advantage of the multi-relay structure, the multiple-access concurrent DF relaying protocol takes advantage of the *multi-source* structure of the considered network. For large L, (4.17) approaches $\min\{2\,(1-r)\,, \frac{7}{2}\,(1-2r)\}$ and (4.2) approaches $2\,(1-2r)$. Hence, although the DMT (4.17) serves only as a lower bound, it is sufficient to show that for large L the diversity performance of the multiple-access concurrent DF relaying protocol outperforms that of the repetition-coded concurrent DF relaying protocol within the range of almost all possible multiplexing gains (the maximal diversity gains of the two protocols are the same).

Clearly, the multiple-access concurrent DF relaying protocol requires higher complexity at relays when compared with the repetition-coded concurrent DF relaying protocol because two codewords instead of only one need to be decoded by relays during each time slot. Since each relay re-encodes each source codeword using the same codebook as the source, the multiple-access concurrent DF relaying protocol is still a repetition-coding based protocol. Therefore, diversity improvement comes not from applying complex coding strategies at the relays but from more efficiently using the channel.

Interestingly, for such a five-node network, if the relays can always correctly decode the sources (i.e. the inequalities (4.13)-(4.15) are met), we notice that the use of a standard approach (orthogonal channels (time slots) are assigned to the transmissions of sources and relays) does not induce any multiplexing loss. More specifically, two time slots are used to transmit one codeword from each source. During the first time slot, both sources broadcast their codewords to the relays and the destination. During the second time slot, $\mathcal{R}_1$ forwards the codeword transmitted by $\mathcal{S}_1$ and $\mathcal{R}_2$ forwards the codeword transmitted by $\mathcal{S}_2$ to the destination. Thus $T_L = 2L$ time slots are used to transmit the $2L$ codewords from the two sources (i.e. $R_i = \frac{1}{2}\bar{R}_i$). The transmission process can be described as follows:

Time slot 1: $\mathcal{S}_1$ broadcasts $x_1[1]$ and $\mathcal{S}_2$ broadcasts $x_2[1]$ to $\mathcal{R}_1$, $\mathcal{R}_2$ and $\mathcal{D}$.

Time slot 2: $\mathcal{R}_1$ forwards $x_1[1]$ to $\mathcal{D}$. $\mathcal{R}_2$ forwards $x_2[1]$ to $\mathcal{D}$. Both $\mathcal{S}_1$ and $\mathcal{S}_2$ remain silent.

Time slot 3: $\mathcal{S}_1$ broadcasts $x_1[2]$ and $\mathcal{S}_2$ broadcasts $x_2[2]$ to $\mathcal{R}_1$, $\mathcal{R}_2$ and $\mathcal{D}$.

Time slot 4: $\mathcal{R}_1$ forwards $x_1[2]$ to $\mathcal{D}$. $\mathcal{R}_2$ forwards $x_2[2]$ to $\mathcal{D}$. Both $\mathcal{S}_1$ and $\mathcal{S}_2$ remain silent.

This process repeats until the $(2L)$th time slot.

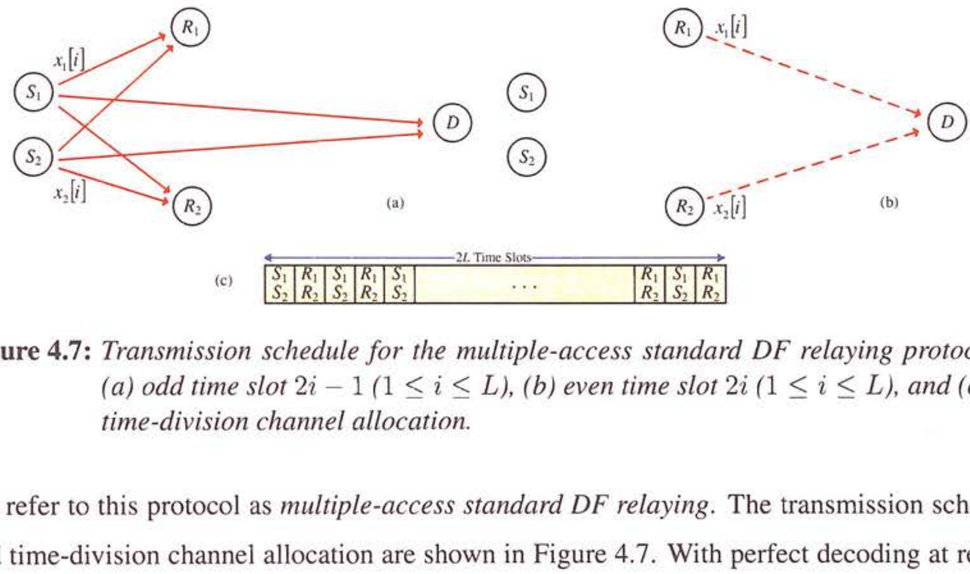

Figure 4.7: *Transmission schedule for the multiple-access standard DF relaying protocol in (a) odd time slot $2i-1$ ($1 \leq i \leq L$), (b) even time slot $2i$ ($1 \leq i \leq L$), and (c) the time-division channel allocation.*

We refer to this protocol as *multiple-access standard DF relaying*. The transmission schedule and time-division channel allocation are shown in Figure 4.7. With perfect decoding at relays, the input-output relation can be expressed by

$$\begin{bmatrix} y[1] \\ y[2] \\ y[3] \\ y[4] \\ \vdots \\ y[2L-1] \\ y[2L] \end{bmatrix} = \sqrt{\rho} \underbrace{\begin{bmatrix} h_{\mathcal{S}_1} & h_{\mathcal{S}_2} & 0 & 0 & \cdots & 0 & 0 \\ h_{\mathcal{R}_1} & h_{\mathcal{R}_2} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & h_{\mathcal{S}_1} & h_{\mathcal{S}_2} & \cdots & 0 & 0 \\ 0 & 0 & h_{\mathcal{R}_1} & h_{\mathcal{R}_2} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & h_{\mathcal{S}_1} & h_{\mathcal{S}_2} \\ 0 & 0 & 0 & 0 & \cdots & h_{\mathcal{R}_1} & h_{\mathcal{R}_2} \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} x_1[1] \\ x_2[1] \\ x_1[2] \\ x_2[2] \\ \vdots \\ x_1[L] \\ x_2[L] \end{bmatrix} + \mathbf{n} \quad (4.18)$$

Substituting $T_L = 2L$ and the channel matrix $\mathbf{H}$ defined in (4.18) into the rate constraints (4.6), it can be proved that the achievable DMT for each source is[2]

$$d(r) = 2(1-2r), \qquad 0 \le r \le \frac{1}{2}. \quad (4.19)$$

The diversity performance of the multiple-access direct transmission is improved and no multiplexing loss is induced (i.e. the maximal multiplexing gain remains $\frac{1}{2}$). However, under general source-relay channel conditions, good multiplexing performance is difficult to obtain. This is mainly because the system multiplexing performance is limited by the low multiplexing performance provided by the source-relay links (which achieves DMT $d(r) = \min\{1-2r, 2(1-4r)\}$ and maximal multiplexing gain only $\frac{1}{4}$). For example, assuming an adaptive protocol in which the relays are used only if both relays can correctly decode the sources, we have the following corollary to Theorem 4 regarding the achievable DMT (the proof is provided in Appendix A.4.2).

Corollary 4 *For the symmetric five-node network and under general source-relay channel conditions, the achievable DMT for each source of the adaptive multiple-access standard DF relaying protocol is*

$$d(r) = \begin{cases} 2(1-2r) & 0 \le r \le \frac{1}{6} \\ 4(1-4r) & \frac{1}{6} \le r \le \frac{1}{4} \end{cases} \quad (4.20)$$

[2] In fact, assuming perfect decoding at the relays, the multiple-access standard DF relaying protocol mimics a multiple-access channel with two single-antenna sources and one two-antenna destination. Substituting $r = \frac{1}{2}r'$ into the achievable DMT for such a multiple-access channel $d(r') = 2(1-r')$ [54] also leads to (4.19).

The maximal achievable multiplexing gain $\frac{1}{4}$ implies that if perfect source-relay transmissions are not guaranteed, the multiple-access standard DF relaying protocol always induces significant multiplexing loss compared with multiple-access direct source-destination transmission.

In the following, we will show that this limitation can be relaxed for the proposed multiple-access concurrent DF relaying protocol. Under general source-relay channel conditions, the adaptive protocol can still recover the multiplexing loss induced by the multiple-access standard DF relaying protocol and improve diversity gain over multiple-access direct transmission and the repetition-coded concurrent DF relaying protocol.

4.3.3 General source-relay links

For the adaptive multiple-access concurrent DF relaying protocol under general source-relay channel conditions, the two relays listen to and try to decode the two sources. We assume the relays are used to assist the sources *if and only if* the sources are correctly decoded at both relays (i.e. the inequalities (4.13)-(4.15) are satisfied); otherwise, both relays remain silent for the whole $(L+1)$ time slots. Studying the overall outage probability (which has the identical form as (4.10)), we generalize the achievable DMT result to the following corollary to Theorem 4 (the proof is provided in Appendix A.4.3).

Corollary 5 *For the symmetric five-node network and an isolated-relay scenario, under general source-relay channel conditions, the achievable DMT for each source of the adaptive multiple-access concurrent DF relaying protocol is* lower bounded *by (4.17), i.e.,*

$$d(r) = \begin{cases} 2(1-\frac{L+1}{L}r) & 0 \le r \le \frac{3L}{10L+10} \\ \frac{7}{2}(1-\frac{2L+2}{L}r) & \frac{3L}{10L+10} \le r \le \frac{L}{2L+2} \end{cases}, \tag{4.21}$$

and is upper bounded *by*

$$d(r) = \begin{cases} 2(1-\frac{L+1}{L}r) & 0 \le r \le \frac{L}{3L+3} \\ 4(1-\frac{2L+2}{L}r) & \frac{L}{3L+3} \le r \le \frac{L}{2L+2} \end{cases}. \tag{4.22}$$

An example of the DMT bounds ($L = 15$) is displayed in Figure 4.8. Clearly, the point $(r, d(r)) = (\frac{L}{2L+2}, 0)$ is tight for the adaptive protocol. When L is sufficiently large, the multiple-access concurrent DF relaying protocol strictly outperforms the repetition-coded con-

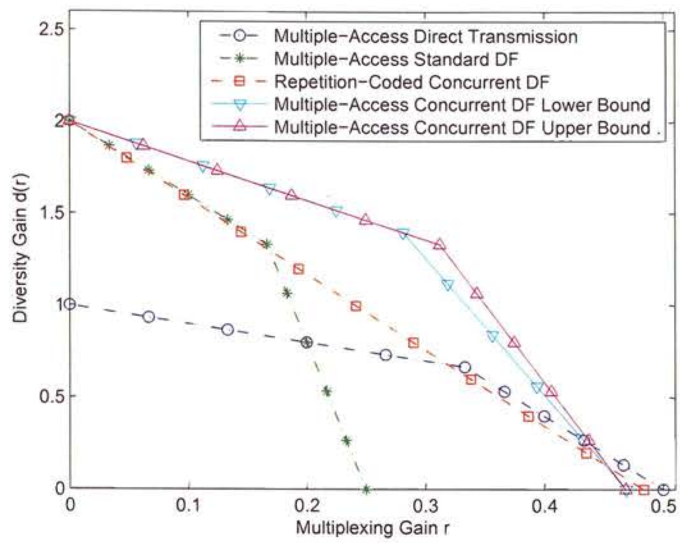

Figure 4.8: *DMT performance of different protocols for the isolated-relay scenario under general source-relay channel conditions. For the repetition-coded and multiple-access concurrent DF relaying protocols, the frame length* $L = 15$.

current DF relaying protocol and effectively recovers the multiplexing loss induced by the adaptive multiple-access standard DF relaying protocol.

Assuming $\gamma_i = \gamma_d = \sqrt{\frac{1}{2}}$, the outage probability performance (when $r = 0.13$ and $r = 0.23$) of the multiple-access standard DF relaying protocol, the repetition-coded and multiple-access concurrent DF relaying protocols with $L = 2$ (the associated DMT comparison is displayed in Figure 4.9) is plotted in Figure 4.10. It can be seen that for a relatively small multiplexing gain (e.g. $r = 0.13$), the multiple-access standard DF relaying protocol can achieve higher diversity gain than the repetition-coded concurrent DF relaying protocol. This is because for small r, the multiple-access standard DF relaying protocol attains the diversity gain $d(r) = 2(1-2r)$, which is larger than that of the repetition-coded concurrent DF relaying protocol $d(r) = 2(1 - \frac{2L+1}{L}r)$ (for large L, such a diversity difference is negligible, which can be seen from Figure 4.8). However, when r approaches $\frac{1}{4}$ (e.g. $r = 0.23$), the diversity performance of the multiple-access standard DF relaying protocol decreases significantly because of its multiplexing limitation. Clearly, the proposed multiple-access concurrent DF relaying protocol obtains better diversity performance than the repetition-coded concurrent DF relaying protocol for both multiplexing gains $r = 0.13$ and $r = 0.23$.

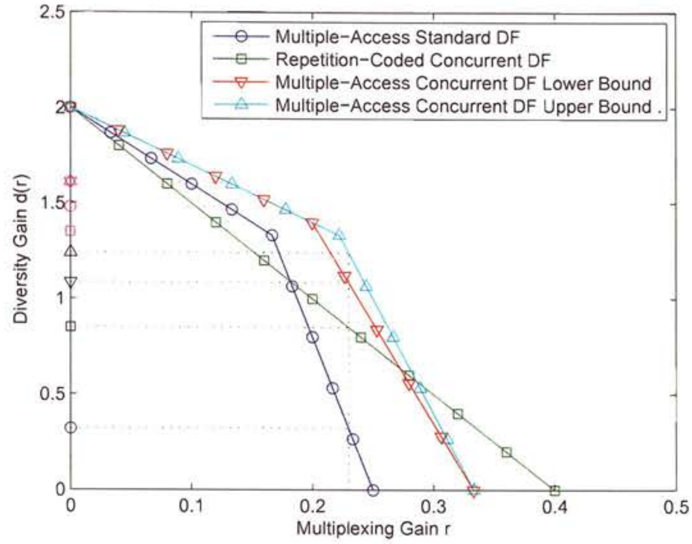

Figure 4.9: *DMT comparison of different protocols ($L = 2$) for the isolated-relay scenario under general source-relay channel conditions. The dotted lines mark the achievable diversity gains of the three protocols when $r = 0.13$ and $r = 0.23$.*

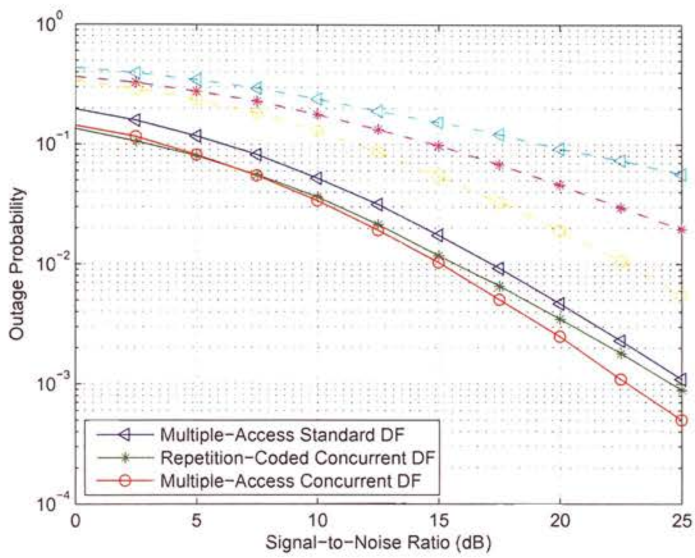

Figure 4.10: *Outage probability comparison of different protocols ($L = 2$) for the isolated-relay scenario under general source-relay channel conditions. Solid lines and dashed lines denote the cases in which $r = 0.13$ and $r = 0.23$, respectively.*

After seeing how to take advantages of the multi-relay or multi-source structure of the network to further improve the system diversity gain, in the following section, we will show that the same task can also be fulfilled by using multiple antennas at the destination.

4.4 Multiple-antenna scenarios

So far, all the nodes in the considered network are assumed to be single-antenna terminals. In fact, as mentioned in Chapter 2, the use of multiple antennas at terminals is an effective way to improve system performance. In this thesis, the multi-hop relaying cooperative diversity concept is considered for an uplink transmission in cellular systems, in which multiple mobile terminals (operating as sources and relays) transmit signals to the base station (i.e. the destination). In general practical systems, base stations are able to support a multiple-antenna setup but mobile terminals cannot due to hardware and cost limitations. Thus, in what follows, we study the achievable DMT performance of the repetition-coded and superposition-coded concurrent DF relaying protocols when the *destination* is equipped with N antennas ($N \geq 1$). We first present the full DMT performance of the two protocols. And then adaptive protocols under general source-relay channel conditions, which select two relays from K potential relays ($K > 1$) to assist the sources, are followed. Note that when we consider the repetition-coded concurrent DF relaying protocol, we assume the K potential relays are isolated from each other. On the other hand, for the superposition-coded concurrent DF relaying protocol, it is assumed that the interference between each relay pair is sufficiently stronger than the source-relay links.

4.4.1 Perfect source-relay links

When N antennas are used at the destination, with perfect decoding at the relays (i.e. inequalities (4.3) and (4.4) are satisfied), the input-output relation of the repetition-coded concurrent DF relaying protocol can be expressed by (each codeword $x_i[j]$ is transmitted with average transmission rate $R_i = \frac{L}{2L+1}\bar{R}_i$ BPCU)

$$\begin{bmatrix} \mathbf{y}[1] \\ \mathbf{y}[2] \\ \mathbf{y}[3] \\ \vdots \\ \mathbf{y}[2L+1] \end{bmatrix} = \sqrt{\rho} \underbrace{\begin{bmatrix} \mathbf{h}_{\mathcal{S}_1} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{h}_{\mathcal{R}_1} & \mathbf{h}_{\mathcal{S}_2} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{h}_{\mathcal{R}_2} & \mathbf{h}_{\mathcal{S}_1} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{h}_{\mathcal{R}_1} & \mathbf{h}_{\mathcal{S}_2} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h}_{\mathcal{R}_2} \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} x_1[1] \\ x_2[1] \\ x_1[2] \\ \vdots \\ x_1[L] \\ x_2[L] \end{bmatrix} + \mathbf{n} \qquad (4.23)$$

where $\mathbf{0}$ denotes an $N \times 1$ all zero vector, $\mathbf{h}_a$ is the $N \times 1$ channel fading vector between node a and the destination, and $\mathbf{y}[i]$ is the $N \times 1$ received signal vector at the destination during the

ith time slot. For the superposition-coded concurrent DF relaying protocol, the input-output relation is expressed as (each codeword $x_i[j]$ is transmitted with average transmission rate $R_i = \frac{L}{2L+2}\bar{R}_i$ BPCU)

$$\begin{bmatrix} \mathbf{y}[1] \\ \mathbf{y}[2] \\ \mathbf{y}[3] \\ \vdots \\ \mathbf{y}[2L+1] \\ \mathbf{y}[2L+2] \end{bmatrix} = \sqrt{\rho} \underbrace{\begin{bmatrix} \mathbf{h}_{\mathcal{S}_1} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{h}_{\mathcal{R}_1} & \mathbf{h}_{\mathcal{S}_2} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \gamma_i \mathbf{h}_{\mathcal{R}_2} & \gamma_d \mathbf{h}_{\mathcal{R}_2} & \mathbf{h}_{\mathcal{S}_1} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \gamma_i \mathbf{h}_{\mathcal{R}_1} & \gamma_d \mathbf{h}_{\mathcal{R}_1} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \gamma_d \mathbf{h}_{\mathcal{R}_1} & \mathbf{h}_{\mathcal{S}_2} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \gamma_i \mathbf{h}_{\mathcal{R}_2} & \gamma_d \mathbf{h}_{\mathcal{R}_2} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h}_{\mathcal{R}_1} \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} x_1[1] \\ x_2[1] \\ x_1[2] \\ \vdots \\ x_1[L] \\ x_2[L] \end{bmatrix} + \mathbf{n}. \tag{4.24}$$

Applying the rate constraints (4.6) by replacing the channel matrix $\mathbf{H}$ with those defined in (4.23) and (4.24) respectively, we can have the achievable DMT performance of the two protocols using a similar analysis approach to Theorem 3 (the proof is provided in Appendix A.1.3 and Appendix A.3.2).

Corollary 6 *In a symmetric scenario in which the destination is equipped with N antennas, on assuming that the relays correctly decode the sources, the achievable DMT for each source of the repetition-coded concurrent DF relaying protocol (for the isolated-relay scenario) is expressed by*

$$d(r) = 2N\left(1 - \frac{2L+1}{L}r\right), \qquad 0 \le r \le \frac{L}{2L+1}. \tag{4.25}$$

For the superposition-coded concurrent DF relaying protocol (for the strong-interference scenario), the achievable DMT for each source is expressed by

$$d(r) = 3N\left(1 - \frac{2L+2}{L}r\right), \qquad 0 \le r \le \frac{L}{2L+2}. \tag{4.26}$$

Corollary 6 shows that, assuming perfect source-relay channel conditions, the achievable DMTs of the protocols applied to single-antenna systems are substantially improved by equipping multiple antennas at the destination. In particular, for either protocol, the achievable maximal diversity gain increases linearly with the number of destination antennas and the achievable maximal multiplexing gain remains the same as that in the single-antenna case. Some examples of the DMT performance are displayed in Figure 4.11. It can be seen that the maximal diversity gain of the repetition-coded concurrent DF relaying protocol for a single-antenna system we discussed in the last chapter is dramatically increased from 2 to 8 when the destination is equipped with 4 antennas. In addition, making use of the inter-relay interference in a strong-interference scenario (i.e. the superposition-coded concurrent DF relaying protocol with 4 destination antennas) can further improve the maximal diversity gain to 12. The advantage of using multiple antennas at the destination is obvious.

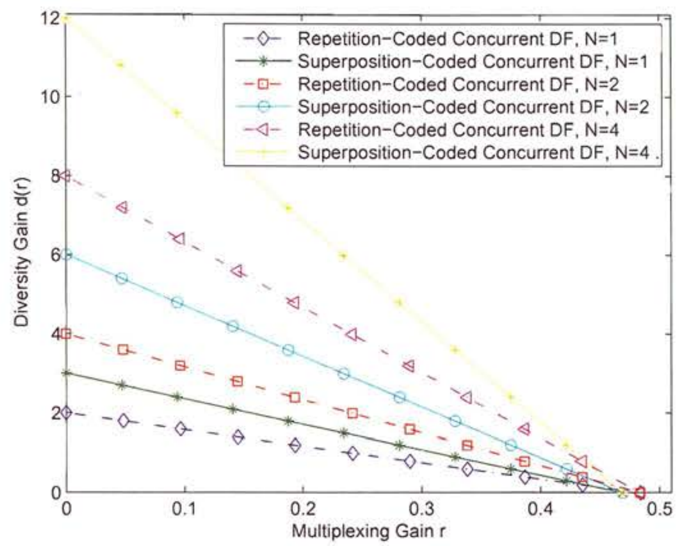

Figure 4.11: *Full DMT performance of the repetition-coded and superposition-coded concurrent DF relaying protocols with N antennas at the destination. The source frame length $L = 15$.*

Nevertheless, the question we encountered in Section 4.2.4 comes up again. When the diversity improvement of the system under perfect source-relay transmissions over direct source-destination transmission is larger than the diversity gain provided by the source-relay channels, how can we obtain the full DMT under general source-relay channel conditions? Clearly, a straightforward answer is to find a way to improve the achievable diversity gain of the source-relay channels. In the following, we use a relay selection scheme in a multi-relay scenario to fulfil the task.

4.4.2 General source-relay links

Besides using multiple antennas at the sources and/or relays, which is normally difficult for mobile users to afford, an efficient way to improve the achievable diversity gain of the source-relay channels is to increase the number of relays. We assume there exist K ($K \geq 2$) single-antenna terminals (mobiles) in the network that can overhear the sources and serve as potential relays for them. If perfect decoding at relays cannot be guaranteed, we consider relay-selection based adaptive forms of our transmission protocols. Specifically, if a relay pair ($\mathcal{R}_\alpha$, $\mathcal{R}_\beta$), in which $\alpha, \beta \in \{1, \ldots, K\}$ and $\alpha \neq \beta$, within the K potential relays can be found such that $\mathcal{R}_\alpha$ can correctly decode $\mathcal{S}_1$ and $\mathcal{R}_\beta$ can correctly decode $\mathcal{S}_2$, the two relays are used to assist the sources. Otherwise, all the relays remain silent and the sources communicate with the destination without the help of relays.

Choosing relays with the highest SNR source-relay links would give robustness to Doppler effects on the network links in practice. However, for simplicity, if there exist more than one pair of such relays, we assume one pair is *randomly* chosen. Further, to minimize the system complexity, we do not consider any specific selection criterion regarding the quality of relay-destination links (e.g. choosing the relays which have the best relay-destination links) or using more than one relay to simultaneously forward each source's codeword to achieve even higher diversity. It is assumed that the sources have no information about whether their codewords can be retransmitted by relays so that the transmission of the two frames from the two sources always takes $(2L+1)$ time slots for the repetition-coded concurrent DF relaying protocol and $(2L+2)$ time slots for the superposition-coded concurrent DF relaying protocol.

Following the analysis in Theorem 4, the achievable DMTs of the adaptive protocols can be summarized as the following corollary (The proof is provided in Appendix A.4.4).

Corollary 7 *For a symmetric two-source network, under general source-relay channel conditions, the achievable DMT for each source of the adaptive repetition-coded concurrent DF relaying protocol with the use of the relay selection scheme in a K-relay scenario (which are isolated from each other) can be expressed by:*

$$d(r) = \min\{N+K, 2N\}\left(1 - \frac{2L+1}{L}r\right), \qquad 0 \leq r \leq \frac{L}{2L+1}. \tag{4.27}$$

The achievable DMT for each source of the adaptive superposition-coded concurrent DF re-

laying protocol (for a strong-interference scenario) can be expressed by

$$d(r) = \min\{N + K, 3N\}\left(1 - \frac{2L+2}{L}r\right), \qquad 0 \le r \le \frac{L}{2L+2}. \tag{4.28}$$

Corollary 7 implies that for the isolated-relay scenario and general source-relay channel conditions, the adaptive repetition-coded concurrent DF relaying protocol always attains the full DMT (4.25) if $K \geq N$. For the strong-interference scenario, the adaptive superposition-coded concurrent DF relaying protocol always obtains the full DMT (4.26) if $K \geq 2N$. These observations indicate that if the number of potential relays is larger than some threshold (related to the number of destination antennas), the system DMT performance is the same as the case in which perfect source-relay transmissions are assumed. Moreover, it can be seen that $\min\{N + K, 3N\} \geq \min\{N + K, 2N\}$. This means, if L is large and $K > N$, the superposition-coded concurrent DF relaying protocol always has better DMT performance than the repetition-coded concurrent DF relaying protocol.

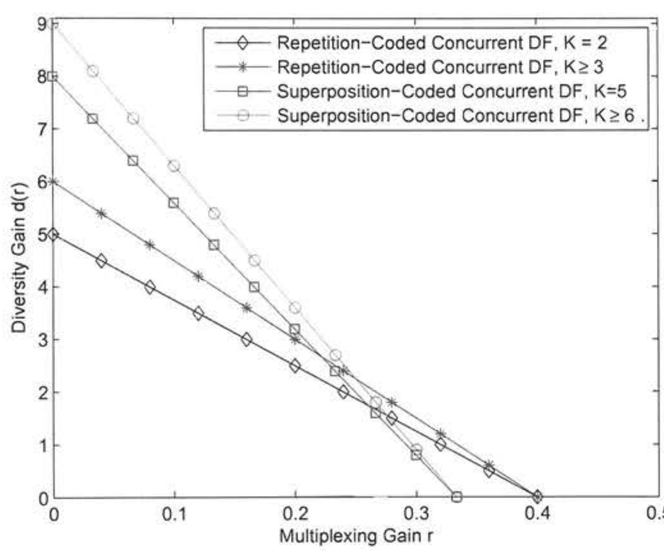

Figure 4.12: *DMT performance of the adaptive repetition-coded and superposition-coded concurrent DF relaying protocols under general source-relay channel conditions. The destination is equipped with $N = 3$ antennas. The source frame length $L = 2$.*

Assuming $L = 2$ and $N = 3$, Figure 4.12 displays the achievable DMT performance of the adaptive protocols with different number of potential relays. Setting $r = 0$ (the average transmission rate is fixed at 2 BPCU) and $\gamma_i = \gamma_d = \sqrt{\frac{1}{2}}$, the outage probability comparison is

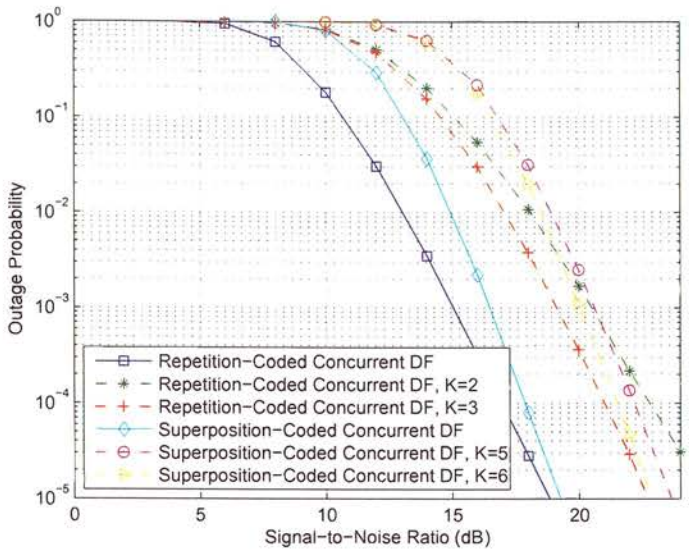

Figure 4.13: *Outage probability comparison of the adaptive repetition-coded and superposition-coded concurrent DF relaying protocols ($L = 2$, $N = 3$). Solid lines denote the cases in which perfect source-relay transmissions are assumed, while the dashed lines denote the performance for the adaptive protocols under general source-relay channel conditions.*

plotted in Figure 4.13. It can be seen that, if the number of potential relays is equivalent to the number of destination antennas (i.e. $K = N$), the diversity performance of the adaptive repetition-coded concurrent DF relaying is the same as that when perfect source-relay transmissions are assumed. However, if $K < N$, the good diversity performance is not achievable. For the adaptive superposition-coded concurrent DF relaying, the diversity performance of the case under perfect source-relay channel conditions can be realized only when $K \geq 2N$.

4.4.3 Single-source networks

Following the above analysis, it is straightforward to prove that when equipping N antennas at the destination in a single-source network and assuming perfect decoding at relays, the use of the repetition-coded successive DF relaying protocol for the isolated-relay scenario obtains the DMT

$$d(r) = 2N\left(1 - \frac{L+1}{L}r\right), \qquad 0 \leq r \leq \frac{L}{L+1}. \tag{4.29}$$

The use of the superposition-coded successive DF relaying protocol for the strong-interference scenario obtains the DMT

$$d(r) = 3N\left(1 - \frac{L+2}{L}r\right), \qquad 0 \le r \le \frac{L}{L+2}. \tag{4.30}$$

For general source-relay channel conditions, we assume there exist K ($K \geq 2$) potential relays in the network. If a relay pair can be chosen from the K potential relays such that the two relays can correctly decode the source, the two relays are used to assist the source to proceed the repetition-coded or superposition-coded successive DF relaying. Otherwise, if there is no such a relay pair, all relays remain silent and the source communicates with the destination without the help of any relay. It is not difficult to prove that the adaptive protocols attain the DMT

$$d(r) = \min\{N+K-1, 2N\}\left(1 - \frac{L+1}{L}r\right), \qquad 0 \le r \le \frac{L}{L+1}, \tag{4.31}$$

for the adaptive repetition-coded successive DF relaying protocol and

$$d(r) = \min\{N+K-1, 3N\}\left(1 - \frac{L+2}{L}r\right), \qquad 0 \le r \le \frac{L}{L+2}, \tag{4.32}$$

for the superposition-coded successive DF relaying protocol. If K and L are large, the two protocols can attain high diversity gain without significantly losing multiplexing performance compared with direct source-destination transmission.

Recall that in Section 4.2.4, when the destination is equipped with only one antenna (i.e. $N = 1$), we mentioned that simply using two relays (i.e. $K = 2$) cannot achieve the full DMT for the superposition-coded successive DF relaying protocol. Equation (4.32) provides an explanation of this issue, i.e. the maximal achievable diversity gain is only $\min\{N + K - 1, 3N\} = 2$. Further, it can be seen from (4.32) that if there exist at least 3 mobile terminals in the network which can overhear the source and serve as potential relays, the use of the adaptive protocol can obtain the full DMT for the strong-interference scenario. This solves the problem we met in Section 4.2.4.

4.5 Summary

In the last chapter, we proposed a repetition-coding based cooperative diversity transmission protocol, in which the conventional orthogonal transmission requirement is relaxed and two successively activated half-duplex relays are used to assist in direct source-destination transmissions. This approach was shown to recover the multiplexing loss induced by the standard repetition-coded DF relaying protocols. However, although this approach can provide better communication reliability over direct transmissions, we still argue that its diversity performance can be further improved without using complex coding strategies at relays or losing multiplexing performance, especially for multiple-source networks. This has been precisely the aim of this chapter. Specifically, we have mainly considered a two-source system. For the strong-interference scenario, we have taken advantage of the multi-relay structure and made further use of the interference between relays. Unlike the previously discussed repetition-coded concurrent DF relaying protocol which neglects the useful inter-relay interference, we have required each relay to retransmit the sum of the interference codeword and its desired source codeword so that each codeword can be protected by both relays. For the isolated-relay scenario, we have taken advantage of the multi-source structure. Instead of using TDMA in the repetition-coded concurrent DF relaying protocol, we have permitted both sources to transmit simultaneously so that the channel can be more efficiently used. Through DMT analysis, it has been shown that for a large frame length L, the diversity performance of the repetition-coded concurrent DF relaying protocol can be further increased within the range of almost all possible multiplexing gains.

Finally, when considering practical uplink cooperative diversity transmissions, we have shown that diversity performance of the repetition-coded concurrent DF relaying protocol can be improved by equipping multiple antennas at the destination (base station). For the strong-interference scenario, we have required each relay to retransmit the sum of its desired codeword and the interference codeword, while for the isolated-relay scenario, each relay only forwards one source codeword. With the use of a relay selection scheme, which chooses two relays from a relatively large number of potential relays, the diversity performance of the single-antenna protocols can be dramatically improved. Since all the protocols we have presented in this chapter are also repetition-coding based (i.e. each relay re-encodes the source information using the same codeword as the source), improving communication reliability without losing spectral efficiency has been accomplished in a very simple way, which has highlighted the important ad-

Transmission Protocols	**Achievable DMT**	d_{max}	r_{max}
Repetition-coded Concurrent DF relaying (1 destination antenna)	$2\left(1-\frac{2L+1}{L}r\right)$	2	$\frac{L}{2L+1}$
Superposition-coded Concurrent DF Relaying (1 destination antenna)	$3\left(1-\frac{2L+2}{L}r\right)$	3	$\frac{L}{2L+2}$
Multiple-access Concurrent DF Relaying (Lower Bound)	$\min\left\{2\left(1-\frac{L+1}{L}r\right),\frac{7}{2}\left(1-\frac{2L+2}{L}r\right)\right\}$	2	$\frac{L}{2L+2}$
Multiple-access Concurrent DF Relaying (Upper Bound)	$\min\left\{2\left(1-\frac{L+1}{L}r\right),4\left(1-\frac{2L+2}{L}r\right)\right\}$		
Repetition-coded Concurrent DF Relaying (N destination antennas)	$2N\left(1-\frac{2L+1}{L}r\right)$	$2N$	$\frac{L}{2L+1}$
Superposition-coded Concurrent DF Relaying (N destination antennas)	$3N\left(1-\frac{2L+2}{L}r\right)$	$3N$	$\frac{L}{2L+2}$

Table 4.1: *Summary of Chapter 4: DMT comparison for different transmission protocols in a two-source network.*

vantages of the proposed protocols. In Table 4.1, we summarize the key results of this chapter (for a two-source network).

When the destination is equipped with N receive antennas, adopting the repetition-coded and superposition-coded protocols in single-source networks has also been analyzed in this chapter. Extending them to general M-source networks is straightforward. For the multiple-access concurrent DF relaying protocol, although currently we only have a lower bound and an upper bound of the DMT performance in a two-source network, these performance bounds have already shown the benefit of the protocol. Generalizing it to a network with M sources is thus interesting and important future work.

Chapter 5
Conclusions and Future Work

In this chapter, we summarize the contents we have presented in each of the previous chapters. We also provide some potential directions that can be treated as extensions of the work in this thesis.

5.1 Conclusions

In Chapter 1, along with a brief introduction on the history of wireless communications, we stated the major concerns (i.e. the fading phenomenon and inter-user interference) and demands (i.e. higher spectral efficiency and higher link reliability) of wireless communications. Although MIMO technology has been considered as one of the most promising approaches to facilitate high-speed and high-quality transmission, we noted that it is difficult to adopt them in current cellular systems since mobile terminals may not be able to support a multiple-antenna structure. This led the focus of this thesis on the so-called cooperative diversity techniques using multi-hop relaying concept in single-antenna systems to emulate multiple-antenna systems.

Chapter 2 offered the background and motivation of the thesis. Specifically, we started with a detailed introduction of the small-scale frequency-flat Rayleigh fading propagation model, which is one of the major issues in wireless communications that lead to decoding errors at receivers. For a fast fading environment, we mentioned that if the transmitted codeword length is sufficiently long such that the codeword experiences many fading realizations, it is possible to drive the decoding error probability arbitrarily close to zero as long as the transmission rate is below a certain positive threshold, the ergodic capacity. However, for slow fading environments, a fixed positive reliable rate does not exist and the probability that the channel cannot support the transmission rate (i.e. the outage probability) decays slowly with increasing SNR. To improve the performance, we described two commonly adopted approaches: time diversity obtained by using a repetition-coding scheme across different fading blocks as well as spatial diversity obtained by deploying multiple antennas at the transmitter and/or the receiver of the communication link. We chose DMT as the major performance metric so that different schemes can be compared from both spectral efficiency and communication reliability aspects at the same time. In addition, for multiple-source networks, we introduced an orthogonal multiple access technique, TDMA, and studied its efficiency on inter-user interference elimination as well as its inefficiency with respect to DMT performance compared with requiring the sources to transmit simultaneously (i.e. non-orthogonally). The last part of Chapter 2 focused on uplink cooperative diversity transmissions in cellular systems, where multiple-antenna structures are not affordable at mobiles. We highlighted both the diversity improvement and the multiplexing loss of the standard transmission protocols compared with direct transmissions. Some advanced protocols aimed at resolving the multiplexing problem were also introduced. However, when considering a simple DF relaying network where there is no destination-source feedback and relays can only use a repetition-coding strategy, those protocols may not be applicable. Our investigations presented in the following two chapters were triggered by such observations.

In Chapter 3, we concentrated on recovering the multiplexing loss induced by the standard repetition-coded DF relaying transmission protocols. This chapter began with the simplest single-source network. For a single-relay scenario, although spatial diversity gain can be realized by the standard protocol since the source information is protected by the relay, the spectral efficiency is significantly reduced because the orthogonal channel allocation requirement allows the source to use only half of the total channel to send information. When there exist more relays, they are conventionally considered as a means to enhance diversity gain but with even more multiplexing loss. To solve the problem, in this chapter, we relaxed the orthogonal channel allocation requirement and adopted a novel way to use multiple relays. More specifically, we assumed that the source continuously transmits L independent codewords using L time slots. From the second to the $(L+1)$th time slot, two relays are activated successively in each individual time slot to repeat one codeword to the destination (the protocol was termed repetition-coded successive DF relaying and was initially proposed by Dr. Yijia Fan for the case of one source). We considered an isolated-relay scenario (where the inter-relay interference is sufficiently weak and each relay directly decodes the source) and a strong-interference scenario (where the inter-relay interference is sufficiently strong and each relay decodes the source after decoding the interference and subtracting it from its received signal) so that the inter-relay interference does not affect the system performance. An upper bound of the reliable transmission rate was derived and was shown to be dramatically better than the reliable transmission rates of the standard protocols. However, this was not a concrete evidence of the superiority of the repetition-coded successive DF relaying protocol. We solved this problem by providing a detailed analysis of the achievable DMT. It was proved theoretically that the repetition-coded successive DF relaying protocol effectively improves the maximal achievable multiplexing gain from $\frac{1}{2}$ ($\frac{1}{3}$) for the single-relay (two-relay) standard DF relaying protocol to $\frac{L}{L+1}$ while still achieving a higher maximal diversity gain than direct source-destination transmission. Although a larger frame length L leads to a larger decoding delay at the destination, to achieve a good multiplexing performance in fact does not necessarily demand a sufficiently large value of L. In general, only good source-relay channel conditions guarantee good diversity performance for DF relaying protocols. We considered a simple adaptive form of the protocol, in which relays transmit/not transmit according to source-relay transmissions, so that the good DMT performance can also be obtained in general source-relay channels. These results were confirmed by simulations. After this, we extended the single-source system model to multiple-source networks. By requiring the sources to communicate with the destination through TDMA, all the aforementioned advantages of using two relays to take turns helping direct transmissions are maintained.

It was proved in Chapter 3 that for a general M-source system, the maximal achievable multiplexing gain of the proposed protocol is $\frac{L}{ML+1}$. An important observation was that for large L, $\frac{L}{ML+1}$ approaches $\frac{1}{M}$, which matches the result for direct source-destination transmission. This means that the proposed protocol can actually obtain the optimal multiplexing gain the system can provide. Therefore, our attention was drawn back to diversity performance in Chapter 4. Specifically, we considered a simple multiple-source network where $M = 2$. The purpose of Chapter 4 was to more efficiently exploit the advantages of the multiple-source multiple-relay structure of an unlink transmission to provide better diversity performance without significantly losing multiplexing performance compared with the previously discussed protocol. We first studied the strong-interference scenario and focused on the multiple-relay structure. For the protocol proposed in Chapter 3 (termed repetition-coded concurrent DF relaying), the inter-relay interference is neglected when each relay forwards the source information. However, since the interference is actually useful information for the destination, in this chapter we required each relay to repeat the sum of the interference and the relay's desired signal (the protocol was termed superposition-coded concurrent DF relaying). In this way, each codeword is protected by both relays instead of previously only one of them so that the new protocol enhances diversity performance. Since a maximal multiplexing gain $\frac{L}{2L+2}$ can be obtained, compared with $\frac{L}{2L+1}$ for the repetition-coded concurrent DF relaying protocol, no significant multiplexing loss is induced, especially when L is large. In addition, Chapter 2 showed that

TDMA does not achieve the optimal system DMT. So for the isolated-relay scenario, we permitted the two sources to transmit simultaneously to take advantage of the multiple-source structure. Under general source-relay channel conditions, we provided an upper bound and a lower bound of the achievable DMT for the so-called multiple-access concurrent DF relaying protocol. The maximal achievable multiplexing gain $\frac{L}{2L+2}$ that we derived indicates that the multiplexing performance of the repetition-coded concurrent DF relaying protocol is not reduced significantly. With large L, the DMT performance of the multiple-access concurrent DF relaying protocol outperforms that of the repetition-coded concurrent DF relaying protocol for almost all possible multiplexing gains. Finally, since a multiple-antenna setup is affordable in base stations, we also studied the impact of using multiple antennas at the destination on the diversity performance of the repetition-coded concurrent DF relaying protocol (for the isolated-relay scenario) and the superposition-coded concurrent DF relaying protocol (for the strong-interference scenario). Through DMT analysis, it was proved that a substantial improvement of diversity gain can be obtained under general source-relay channel conditions by using a relay selection scheme in a cooperative network where a large number of terminals can act as potential relays for the sources. Since all the protocols discussed in this chapter still use a repetition-coding based relaying strategy, the diversity performance of the protocol discussed in the previous chapter is improved without using complex coding strategies or significantly losing multiplexing gains.

5.2 Future work

In Chapter 4, we showed that when the destination is equipped with N receive antennas, both the repetition-coded and superposition-coded concurrent DF relaying protocols can be adopted in a single-source network. The expression of the achievable DMT is known and it is not difficult to extend the results to a general M-source network. However, for the multiple-access concurrent DF relaying protocol, even in a single-antenna system, we can only have DMT bounds rather than an exact expression. Furthermore, when more than two sources are considered, the achievable DMT performance is currently unknown. Consequently, deriving the achievable DMT performance for a general network with M sources simultaneously communicating with an N-antenna destination can be treated as a direct extension of the work presented in the thesis.

For all proposed protocols, we have assumed two specific scenarios such that inter-relay interference does not negatively affect the system performance. Clearly, how these protocols behave under general relay-relay channel conditions is of more importance in practical systems. In addition, we presented the relay selection scheme in Chapter 4 to show that it is possible to achieve the full DMT performance. How to select the relay pair in practice, and further how to select the most appropriate relays to obtain, for example, even better diversity performance are beyond the scope of this thesis but are considered as interesting future work.

In this thesis, we have constrained our focus within current cellular systems so that terminals (at least the sources and relays) are equipped with only a single antenna. Although this is the most commonly adopted system model, many investigations have already moved on to multiple-antenna cooperative systems (e.g. [65, 76, 80, 81]). These researches have shown interesting and promising results on adopting cooperative diversity techniques in ad hoc networks, cellular systems with fixed relays (which may afford a relatively complex structure than mobiles), or future cellular systems where advanced technology permits a multiple-antenna setup at mobiles. Therefore, studying the achievable DMT performance of our spectrally efficient protocols when all terminals are equipped with multiple antennas and further applying practical MIMO techniques to attain the predicted performance would serve as important directions for future investigation.

The DMT performance studied in this thesis has been under the assumption that the SNR approaches infinity. However, recently more and more attention has been drawn to a more practical performance metric, the finite-SNR DMT, where the multiplexing gain r and the diversity gain d are respectively defined as [74, 75]

$$r = \frac{R}{\log(1+g\rho)}, \text{ and } d = -\frac{\partial \ln P_{\text{out}}(r,\rho)}{\partial \ln \rho} \tag{5.1}$$

in which g denotes an array gain. For the same scheme, the use of the finite-SNR DMT may lead to an entirely different result from that shown by the infinite-SNR DMT (the two results merge when the finite-SNR DMT considers high SNR). For instance, reference [76] proved that when the SNR approaches infinity, the maximal achievable multiplexing gain of direct transmission cannot be enhanced even though full-duplex relays with complex coding strategies are used. However, it is shown in [81, 82] that by the use of an independent-coding strategy at relays, such an enhancement is possible for the finite-SNR region. This result is important as it may point out a new direction in future wireless networking and coding design: a direction towards *cooperative multiplexing* rather than cooperative diversity. Hence, we conjecture that investigations on the finite-SNR DMT performance of the protocols presented in this thesis may also be interesting and important.

Appendix A
Proof of Theorems and Corollaries

A.1 Proof of Theorem 1, Corollaries 1 and 6

A.1.1 Proof of Theorem 1

To characterize the DMT achieved by each rate constraint from (3.17) to (3.20), we consider an $m \times (m+1)$ MIMO channel ($1 \le m \le L$)

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{m+1} \end{bmatrix} = \sqrt{\rho} \underbrace{\begin{bmatrix} g_0 & 0 & 0 & \cdots & 0 \\ g_1 & g_0 & 0 & \cdots & 0 \\ 0 & g_2 & g_0 & \cdots & 0 \\ 0 & 0 & g_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & g_0 \\ 0 & 0 & 0 & \cdots & g_k \end{bmatrix}}_{\mathbf{G}_m} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ \vdots \\ s_m \end{bmatrix} + \mathbf{n} \tag{A.1}$$

where s_i and y_i denote the transmitted and received signals respectively, g_i ($i \in \{0, 1, 2\}$) denotes a Rayleigh fading channel coefficient, and $k = 1$ when m is odd and $k = 2$ when m is even. For infinite-SNR, we assume the multiplexing gain of such a MIMO system is mr'. By setting

$$r' = \frac{T_L}{L} r, \tag{A.2}$$

where r is the multiplexing gain of the considered transmission protocol, the task of finding the smallest DMT achieved by each constraint from (3.17) to (3.20) is equivalent to finding the smallest DMT (with respect to r) achieved by the system (A.1) for every $1 \le m \le L$.

When $m = 1$, the system model in (A.1) is a 1×2 SIMO system. The achievable DMT is clearly

$$d(r') = 2\left(1 - r'\right)$$

When $m > 1$, the proof follows the DMT calculation for the ISI channel in [83]. We assume s_i is chosen from a quadrature amplitude modulation (QAM) constellation[1] and ML decoding at the destination is applied. Define

$$\mathbf{g} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \end{bmatrix}.$$

It can be proved that the error probability at the destination is upper bounded by[2]

$$P_e \dot{\leq} c \cdot \bar{\lambda}^{-3} \rho^{-3(1-r')},$$

where c is a constant,

$$\bar{\lambda} = \inf_{\mathbf{g} \in \mathcal{C}^4} \lambda_{\min}\left(\frac{\mathbf{G}_m}{||\mathbf{g}||}\right),$$

$\mathcal{C}^4$ is the 4-dimensional complex space, and $\lambda_{\min}(\mathbf{X})$ denotes the minimum singular value of $\mathbf{X}$.

When m is even and odd, define the following matrices for these two cases:

$$\mathbf{A}_e = \begin{bmatrix} a_1 & 0 & 0 \\ a_2 & a_1 & 0 \\ a_3 & 0 & a_2 \\ a_4 & a_3 & 0 \\ \vdots & \vdots & \vdots \\ a_{m-1} & 0 & a_{m-2} \\ a_m & a_{m-1} & 0 \\ 0 & 0 & a_m \end{bmatrix}, \mathbf{A}_o = \begin{bmatrix} a_1 & 0 & 0 \\ a_2 & a_1 & 0 \\ a_3 & 0 & a_2 \\ a_4 & a_3 & 0 \\ \vdots & \vdots & \vdots \\ a_{m-1} & a_{m-2} & 0 \\ a_m & 0 & a_{m-1} \\ 0 & a_m & 0 \end{bmatrix}$$

in which a_i is defined as that in the proof of Theorem 3.4 in [83]. Then using a similar method as the proof of Lemma 4.1 in [83], it can be proved that $\bar{\lambda} > 0$. As $\bar{\lambda}$ is not a function of ρ, it can be concluded

$$P_e \dot{\leq} \rho^{-3(1-r')}.$$

[1]The QAM modulation is only for the proof. If the source codewords are chosen from Gaussian random codebooks, the performance would be at least the same as that of where codewords are chosen from QAM constellations.

[2]In the following, we use $\doteq$ to denote *exponential equality* [36] such that $f(\rho) \doteq \rho^b$ denotes $b = \lim_{\rho \to \infty} \frac{\log f(\rho)}{\log \rho}$ ($\dot{\geq}$ and $\dot{\leq}$ are defined similarly).

The achievable diversity gain of the considered protocol is dominated by the smallest diversity gain achieved by the system (A.1) for all m. Regarding $T_L = L + 1$, we have

$$d(r) = \min\left\{2\left(1 - r'\right), 3\left(1 - r'\right)\right\} = 2\left(1 - \frac{L+1}{L}r\right). \tag{A.3}$$

The proof is complete.

A.1.2 Proof of Corollary 1

For the two-source system, we consider an $m \times (m + 1)$ MIMO system with input-output relation

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{m+1} \end{bmatrix} = \sqrt{\rho} \underbrace{\begin{bmatrix} g_0 & 0 & 0 & \cdots & 0 \\ g_1 & g_2 & 0 & \cdots & 0 \\ 0 & g_3 & g_0 & \cdots & 0 \\ 0 & 0 & g_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & g_{k_1} \\ 0 & 0 & 0 & \cdots & g_{k_2} \end{bmatrix}}_{\mathbf{G}_m} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ \vdots \\ s_m \end{bmatrix} + \mathbf{n}, \tag{A.4}$$

where $k_1 = 0$, $k_2 = 1$ when m is odd and $k_1 = 2$, $k_2 = 3$ when m is even. Assuming for infinite-SNR the multiplexing gain of the MIMO system is mr' and $r' = \frac{2L+1}{L}r$, the smallest DMT achieved by the $(2^{2L} - 1)$ rate constraints (3.30)-(3.37) can be calculated by finding the smallest achievable DMT (with respect to r) of the MIMO system (A.4) for every $1 \leq m \leq 2L$.

Following the analysis in the proof of Theorem 1, when $m \geq 2$, we define

$$\mathbf{g} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix}, \tag{A.5}$$

$$\mathbf{A}_e = \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_1 & a_2 & 0 \\ a_3 & 0 & 0 & a_2 \\ 0 & a_3 & a_4 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & a_{m-1} & a_m & 0 \\ 0 & 0 & 0 & a_m \end{bmatrix}, \text{ and } \mathbf{A}_o = \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_1 & a_2 & 0 \\ a_3 & 0 & 0 & a_2 \\ 0 & a_3 & a_4 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ a_m & 0 & 0 & a_{m-1} \\ 0 & a_m & 0 & 0 \end{bmatrix}.$$

It can be proved that the error probability is upper bounded by

$$P_e \dot{\leq} c \cdot \bar{\lambda}^{-4} \rho^{-4(1-r')} \doteq \rho^{-4(1-r')}.$$

Since the achievable DMT is only $d(r') = 2\,(1 - r')$ when $m = 1$, the DMT achieved by the repetition-coded concurrent DF relaying protocol thus is $d(r) = 2\left(1 - \frac{2L+1}{L} r\right)$. The proof is complete.

For Corollary 3, when all source-relay channels are sufficiently good, the analysis is straightforward.

A.1.3 Proof of Corollary 6 (for isolated-relay scenario)

When the destination is equipped with N antennas, the equivalent system we considered becomes an $m \times (m+1)N$ MIMO system (with multiplexing gain $mr' = \frac{2L+1}{L} mr$) with channel matrix

$$\mathbf{G}_m = \begin{bmatrix} \mathbf{g}_0 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{g}_1 & \mathbf{g}_2 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{g}_3 & \mathbf{g}_0 & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{g}_1 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{g}_{k_1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{g}_{k_2} \end{bmatrix}, \tag{A.6}$$

where $\mathbf{g}_i$ is an $N \times 1$ Rayleigh channel fading vector, when m is odd $k_1 = 0$, $k_2 = 1$, and when m is even $k_1 = 2$, $k_2 = 3$.

When $m = 1$, $d(r') = 2N\,(1 - r')$. When $m > 1$, we define matrices $\mathbf{A}_e$ and $\mathbf{A}_o$ for even and odd m as following respectively

$$\mathbf{A}_e = \begin{bmatrix} \mathbf{D}_1 & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{D}_1 & \mathbf{D}_2 & \mathbf{O} \\ \mathbf{D}_3 & \mathbf{O} & \mathbf{O} & \mathbf{D}_2 \\ \mathbf{O} & \mathbf{D}_3 & \mathbf{D}_4 & \mathbf{O} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{O} & \mathbf{D}_{m-1} & \mathbf{D}_m & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{D}_m \end{bmatrix}, \text{ and } \mathbf{A}_o = \begin{bmatrix} \mathbf{D}_1 & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{D}_1 & \mathbf{D}_2 & \mathbf{O} \\ \mathbf{D}_3 & \mathbf{O} & \mathbf{O} & \mathbf{D}_2 \\ \mathbf{O} & \mathbf{D}_3 & \mathbf{D}_4 & \mathbf{O} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{D}_m & \mathbf{O} & \mathbf{O} & \mathbf{D}_{m-1} \\ \mathbf{O} & \mathbf{D}_m & \mathbf{O} & \mathbf{O} \end{bmatrix}.$$

where matrix $\mathbf{D}_i$ is an $N \times N$ diagonal matrix in which all the main diagonal entries equal to a_i. Defining vector $\mathbf{g}$ as

$$\mathbf{g} = \begin{bmatrix} \mathbf{g}_0 \\ \mathbf{g}_1 \\ \mathbf{g}_2 \\ \mathbf{g}_3 \end{bmatrix}, \tag{A.7}$$

it can be proved that now $P_e \dot{\leq} \rho^{-4N(1-r')}$. Therefore the achievable DMT is the same as that when $m = 1$, i.e. $d(r) = 2N\left(1 - \frac{2L+1}{L} r\right)$. The proof is complete.

A.2 Proof of Theorem 2

As mentioned in Section 3.3.4, the overall system outage probability of the adaptive repetition-coded successive DF relaying protocol can be expressed by (3.25), i.e.

$$\begin{aligned} P_{\text{out}} &= P_{\mathcal{R}_1} P_{\mathcal{R}_2} P_{\mathcal{SD}} + (1 - P_{\mathcal{R}_1}) P_{\mathcal{R}_2} P_{\mathcal{SR}_1\mathcal{D}} \\ &\quad + P_{\mathcal{R}_1}(1 - P_{\mathcal{R}_2}) P_{\mathcal{SR}_2\mathcal{D}} + (1 - P_{\mathcal{R}_1})(1 - P_{\mathcal{R}_2}) P_{\mathcal{SR}_1\mathcal{R}_2\mathcal{D}}. \end{aligned} \tag{A.8}$$

According to the rate constraints (3.14) and (3.15), it is not difficult to prove that

$$P_{\mathcal{R}_1} = P_{\mathcal{R}_2} \doteq \rho^{-(1-\frac{L+1}{L} r)}.$$

If both relays cannot successfully decode the source, the source communicate with the destination without the help of any relay (the transmission still uses $(L+1)$ time slots) so that

$$P_{\mathcal{SD}} \doteq \rho^{-(1-\frac{L+1}{L}r)}.$$

In addition, when only one relay can correctly decode the source (e.g. when only $\mathcal{R}_1$ is activated, the equivalent channel matrix is expressed by (3.24)), it can be proved that the system error probability is always dominated by the decoding errors of those codewords that are not protected by the silent relay. As a result, the overall system diversity gain is actually the same as that of direct source-destination transmission. We have

$$P_{\mathcal{SR}_1\mathcal{D}} = P_{\mathcal{SR}_2\mathcal{D}} \doteq \rho^{-(1-\frac{L+1}{L}r)}.$$

Since it is known from Theorem 1 that $P_{\mathcal{SR}_1\mathcal{R}_2\mathcal{D}} \doteq \rho^{-2\left(1-\frac{L+1}{L}r\right)}$, the overall outage probability can be calculated as

$$\begin{aligned} P_{\text{out}} &\doteq \rho^{-(1-\frac{L+1}{L}r)}\rho^{-(1-\frac{L+1}{L}r)}\rho^{-(1-\frac{L+1}{L}r)} + \rho^{-(1-\frac{L+1}{L}r)}\rho^{-(1-\frac{L+1}{L}r)} \\ &\quad + \rho^{-(1-\frac{L+1}{L}r)}\rho^{-(1-\frac{L+1}{L}r)} + \rho^{-2\left(1-\frac{L+1}{L}r\right)} \\ &\doteq \rho^{-2\left(1-\frac{L+1}{L}r\right)}. \end{aligned} \tag{A.9}$$

The proof is complete.

For multiple-source cases (i.e. Corollaries 2 and 3), the analysis is similar.

A.3 Proof of Theorem 3 and Corollary 6

A.3.1 Proof of Theorem 3

Assuming $1 \leq m \leq 2L$, we consider an $m \times (m+2)$ MIMO channel (with multiplexing gain mr' in which $r' = \frac{2L+2}{L}r$)

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{m+1} \\ y_{m+2} \end{bmatrix} = \sqrt{\rho} \underbrace{\begin{bmatrix} g_0 & 0 & 0 & \cdots & 0 \\ g_1 & g_2 & 0 & \cdots & 0 \\ g_3 & g_3 & g_0 & \cdots & 0 \\ 0 & g_1 & g_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & g_{k_1} \\ 0 & 0 & 0 & \cdots & g_{k_2} \\ 0 & 0 & 0 & \cdots & g_{k_3} \end{bmatrix}}_{\mathbf{G}_m} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ \vdots \\ s_m \end{bmatrix} + \mathbf{n}, \tag{A.10}$$

where $k_1 = 0$, $k_2 = 1$, $k_3 = 3$ when m is odd, or $k_1 = 2$, $k_2 = 3$, $k_3 = 1$ when m is even. For infinite SNR, the task of finding the smallest diversity gain achieved by each constraint in (4.6) is the same as finding the smallest diversity gain achieved by the system (A.10) for every $1 \leq m \leq 2L$.

When $m = 1$, the system model in (A.10) is a 1×3 SIMO system so that $d(r') = 3(1 - r')$.

When $m > 1$, following [83] and the proof of Theorem 1, we define $\mathbf{g}$ as the same form of (A.5). It can be proved that the error probability is upper bounded by $P_e \dot{\leq} c \cdot \bar{\lambda}^{-4} \rho^{-4(1-r')}$.

Defining

$$\mathbf{A}_e = \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_1 & a_2 & 0 \\ a_3 & 0 & 0 & \xi_1^2 \\ 0 & \xi_2^3 & a_4 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \xi_{m-2}^{m-1} & a_m & 0 \\ 0 & 0 & 0 & \xi_{m-1}^m \\ 0 & a_m & 0 & 0 \end{bmatrix}, \; \mathbf{A}_o = \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_1 & a_2 & 0 \\ a_3 & 0 & 0 & \xi_1^2 \\ 0 & \xi_2^3 & a_4 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ a_m & 0 & 0 & \xi_{m-2}^{m-1} \\ 0 & \xi_{m-1}^m & 0 & 0 \\ 0 & 0 & 0 & a_m \end{bmatrix},$$

in which $\xi_\alpha^\beta = \gamma_i a_\alpha + \gamma_d a_\beta$, it can be proved that $\bar{\lambda} > 0$. Therefore, $P_e \dot{\leq} \rho^{-4(1-r')}$.

Since the smallest diversity gain achieved by all possible values of m dominates the overall achievable diversity of the considered system, by comparing the cases when $m = 1$ and $m > 1$, the system achievable DMT is clearly $d(r) = 3\,(1 - r') = 3\left(1 - \frac{2L+2}{L} r\right)$. The proof is complete.

A.3.2 Proof of Corollary 6 (for strong-interference scenario)

When considering an N-antenna destination, the equivalent $m \times (m+2)N$ MIMO channel matrix is expressed as

$$\mathbf{G}_m = \begin{bmatrix} \mathbf{g}_0 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{g}_1 & \mathbf{g}_2 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{g}_3 & \mathbf{g}_3 & \mathbf{g}_0 & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{g}_1 & \mathbf{g}_1 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{g}_{k_1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{g}_{k_2} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{g}_{k_3} \end{bmatrix}. \tag{A.11}$$

When $m = 1$, $d(r') = 3N\,(1 - r')$. When $m > 1$, we define $\mathbf{g}$ as (A.7), and matrices $\mathbf{A}_e$ and $\mathbf{A}_o$ for even and odd m as follows

$$\mathbf{A}_e = \begin{bmatrix} \mathbf{D}_1 & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{D}_1 & \mathbf{D}_2 & \mathbf{O} \\ \mathbf{D}_3 & \mathbf{O} & \mathbf{O} & \Xi_1^2 \\ \mathbf{O} & \Xi_2^3 & \mathbf{D}_4 & \mathbf{O} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{O} & \Xi_{m-2}^{m-1} & \mathbf{D}_m & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{O} & \Xi_{m-1}^{m} \\ \mathbf{O} & \mathbf{D}_m & \mathbf{O} & \mathbf{O} \end{bmatrix}, \mathbf{A}_o = \begin{bmatrix} \mathbf{D}_1 & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{D}_1 & \mathbf{D}_2 & \mathbf{O} \\ \mathbf{D}_3 & \mathbf{O} & \mathbf{O} & \Xi_1^2 \\ \mathbf{O} & \Xi_2^3 & \mathbf{D}_4 & \mathbf{O} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{D}_m & \mathbf{O} & \mathbf{O} & \Xi_{m-2}^{m-1} \\ \mathbf{O} & \Xi_{m-1}^{m} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{D}_m \end{bmatrix},$$

where $\Xi_\alpha^\beta = \gamma_i \mathbf{D}_\alpha + \gamma_d \mathbf{D}_\beta$. Similarly, it can be proved that $P_e \dot{\leq} \rho^{-4N(1-r')}$. Since $r' = \frac{2L+2}{L} r$, the achievable DMT for the considered protocol is expressed as $d(r) = 3N\left(1 - \frac{2L+2}{L} r\right)$.

A.4 Proof of Theorem 4, Corollaries 4, 5, and 7

A.4.1 Proof of Theorem 4

The overall outage probability is expressed by (4.10), i.e.

$$P_{\text{out}} = P_{\mathcal{R}} P_{\mathcal{SD}} + (1 - P_{\mathcal{R}}) P_{\mathcal{SRD}}. \tag{A.12}$$

Clearly, $P_{\mathcal{SD}} \dot{=} \rho^{-\left(1-\frac{2L+2}{L}r\right)}$ and $P_{\mathcal{SRD}} \dot{=} \rho^{-3\left(1-\frac{2L+2}{L}r\right)}$. The probability that the relays are not used, $P_{\mathcal{R}}$, can be expressed as

$$\begin{aligned} P_{\mathcal{R}} &= P(A_1 \cup A_2 \cup A_3) \\ &= \underbrace{P(A_1)}_{\to \rho^{-2\left(1-\frac{2L+2}{L}r\right)}} + \underbrace{P(A_2)}_{\to \rho^{-2\left(1-\frac{2L+2}{L}r\right)}} - \underbrace{P(A_1 \cap A_2)}_{\to \rho^{-4\left(1-\frac{2L+2}{L}r\right)}} + \underbrace{P(A_3)}_{\to \rho^{-2\left(1-\frac{2L+2}{L}r\right)}} \end{aligned}$$

in which A_1 and A_2 denote the events that no relay can correctly decode $\mathcal{S}_1$ and $\mathcal{S}_2$ respectively, and A_3 denotes the event that one relay can decode both $\mathcal{S}_1$ and $\mathcal{S}_2$ but the other relay can decode neither of them. We have

$$\begin{aligned} P_{\text{out}} &\dot{=} \left(2\rho^{-2(1-\frac{2L+2}{L}r)} - \rho^{-4(1-\frac{2L+2}{L}r)} + \rho^{-2(1-\frac{2L+2}{L}r)}\right)\rho^{-(1-\frac{2L+2}{L}r)} + \rho^{-3(1-\frac{2L+2}{L}r)} \\ &\dot{=} \rho^{-3(1-\frac{2L+2}{L}r)}. \end{aligned}$$

The proof is complete.

A.4.2 Proof of Corollary 4

The overall outage probability is expressed by

$$P_{\text{out}} = P_{\mathcal{R}} P_{\mathcal{SD}} + (1 - P_{\mathcal{R}}) P_{\mathcal{SRD}}, \tag{A.13}$$

where $P_{\mathcal{SRD}} \dot{=} \rho^{-2(1-2r)}$. If the relays are not used, the two sources and the destination form a multiple-access channel during the broadcasting phase. Since $2L$ time slots are used to transmit L codewords from each source, the average transmission rate from each source $R_i = \frac{1}{2}\bar{R}_i$ BPCU. Assuming for infinite-SNR $R_i \dot{=} r \log \rho$, it can be simply shown that

$$P_{\mathcal{SD}} \dot{=} \rho^{-\min\{1-2r, 2(1-4r)\}}.$$

Similarly, the outage probability at each relay is $\rho^{-\min\{1-2r,2(1-4r)\}}$. As a result, the probability that the relays are not used, $P_{\mathcal{R}}$, is expressed as

$$\begin{aligned} P_{\mathcal{R}} &= P(A_1 \cup A_2) \\ &= \underbrace{P(A_1)}_{\to\rho^{-\min\{1-2r,2(1-4r)\}}} + \underbrace{P(A_2)}_{\to\rho^{-\min\{1-2r,2(1-4r)\}}} - \underbrace{P(A_1 \cap A_2)}_{\to\rho^{-\min\{2(1-2r),4(1-4r)\}}} \\ &\doteq \rho^{-\min\{1-2r,2(1-4r)\}} \end{aligned}$$

in which A_1 and A_2 denote the events that the decoding is not successful at $\mathcal{R}_1$ and $\mathcal{R}_2$ respectively.

We have

$$\begin{aligned} P_{\text{out}} &\doteq \rho^{-\min\{1-2r,2(1-4r)\}}\rho^{-\min\{1-2r,2(1-4r)\}} + \rho^{-2(1-2r)} \\ &\doteq \rho^{-\min\{2(1-2r),4(1-4r)\}}. \end{aligned} \tag{A.14}$$

The proof is complete.

A.4.3 Proof of Corollary 5

For the adaptive multiple-access concurrent DF relaying protocol, since $L+1$ time slots are used to transmit L codewords from each source, the outage probability at each relay is thus $P_{\mathcal{R}_i} \doteq \rho^{-\min\{1-\frac{L+1}{L}r,2\left(1-\frac{2L+2}{L}r\right)\}}$. The probability that relays are not used to assist the sources (i.e. the probability that any relay cannot correctly decode both sources), $P_{\mathcal{R}}$, is expressed as

$$\begin{aligned} P_{\mathcal{R}} &\doteq 2\rho^{-\min\{1-\frac{L+1}{L}r,2\left(1-\frac{2L+2}{L}r\right)\}} - \rho^{-\min\{2\left(1-\frac{L+1}{L}r\right),4\left(1-\frac{2L+2}{L}r\right)\}} \\ &\doteq \rho^{-\min\{1-\frac{L+1}{L}r,2\left(1-\frac{2L+2}{L}r\right)\}}. \end{aligned}$$

Since $P_{\mathcal{SD}} \doteq \rho^{-\min\{1-\frac{L+1}{L}r,2\left(1-\frac{2L+2}{L}r\right)\}}$, the overall outage probability is calculated by

$$\begin{aligned} P_{\text{out}} &\doteq \rho^{-\min\{1-\frac{L+1}{L}r,2\left(1-\frac{2L+2}{L}r\right)\}}\rho^{-\min\{1-\frac{L+1}{L}r,2\left(1-\frac{2L+2}{L}r\right)\}} + \rho^{-\bar{d}(r)} \\ &= \rho^{-\min\{2(1-\frac{L+1}{L}r),4(1-\frac{2L+2}{L}r)\}} + \rho^{-\bar{d}(r)} \end{aligned} \tag{A.15}$$

where $\bar{d}(r)$ denotes the exact full DMT expression of the multiple-access concurrent DF relaying protocol. Since $\min\{2(1-\frac{L+1}{L}r), 4(1-\frac{2L+2}{L}r)\} \geq \min\{2(1-\frac{L+1}{L}r), \frac{7}{2}(1-\frac{2L+2}{L}r)\}$, it can be seen that the DMT of the adaptive multiple-access concurrent DF relaying protocol is lower bounded by (4.21) and is upper bounded by (4.22). The proof is complete.

A.4.4 Proof of Corollary 7

Following the analysis in Theorem 4, when the destination is equipped with N antennas, for the repetition-coded concurrent DF relaying protocol, we have $P_{\mathcal{SD}} \doteq \rho^{-N\left(1-\frac{2L+1}{L}r\right)}$ and $P_{\mathcal{SRD}} = \rho^{-2N\left(1-\frac{2L+1}{L}r\right)}$. Since now we have K terminals which can act as potential relays for the source, the probability that none of them can correctly decode $\mathcal{S}_1$ is expressed as $P(A_1) \doteq \rho^{-K\left(1-\frac{2L+1}{L}r\right)}$. Similarly, the probability that no relay can correctly decode $\mathcal{S}_2$ is expressed as $P(A_2) \doteq \rho^{-K\left(1-\frac{2L+1}{L}r\right)}$. The probability that neither source can be decoded by any potential relay is calculated by $P(A_1 \cap A_2) \doteq \rho^{-2K\left(1-\frac{2L+1}{L}r\right)}$. Finally, the probability that one potential relay can correctly decode both sources but none of other potential relays can correctly decode any of the two sources is calculated by

$$P(A_3) \doteq K\rho^{-(2K-2)\left(1-\frac{2L+1}{L}r\right)}\left(1-\rho^{-\left(1-\frac{2L+1}{L}r\right)}\right)^2 \doteq \rho^{-(2K-2)\left(1-\frac{2L+1}{L}r\right)}.$$

Therefore, the probability that relays are not used is expressed by

$$\begin{aligned} P_{\mathcal{R}} &= P(A_1)+P(A_2)-P(A_1 \cap A_2)+P(A_3) \\ &\doteq 2\rho^{-K\left(1-\frac{2L+1}{L}r\right)} - \rho^{-2K\left(1-\frac{2L+1}{L}r\right)} + \rho^{-(2K-2)\left(1-\frac{2L+1}{L}r\right)} \\ &\doteq \rho^{-\min\{K,2K-2\}\left(1-\frac{2L+1}{L}r\right)}. \end{aligned}$$

The overall system outage probability is calculated by

$$\begin{aligned} P_{\text{out}} &\doteq \rho^{-N\left(1-\frac{2L+1}{L}r\right)}\rho^{-\min\{K,2K-2\}\left(1-\frac{2L+1}{L}r\right)} + \rho^{-2N\left(1-\frac{2L+1}{L}r\right)} \\ &\doteq \rho^{-\min\{N+\min\{K,2K-2\},2N\}\left(1-\frac{2L+1}{L}r\right)}. \end{aligned} \tag{A.16}$$

For the superposition-coded concurrent DF relaying protocol with K potential relays, the overall outage probability can be calculated by

$$\begin{aligned} P_{\text{out}} &\doteq \rho^{-N\left(1-\frac{2L+2}{L}r\right)}\rho^{-\min\{K,2K-2\}\left(1-\frac{2L+2}{L}r\right)} + \rho^{-3N\left(1-\frac{2L+2}{L}r\right)} \\ &\doteq \rho^{-\min\{N+\min\{K,2K-2\},3N\}\left(1-\frac{2L+2}{L}r\right)}. \end{aligned} \tag{A.17}$$

When $K > 1$, $\min\{K, 2K-2\} = K$. The proof is complete.

A.5 Proof of Theorem 5

Considering an $mn \times (m+1)$ matrix

$$\mathbf{Q} = \begin{bmatrix} p_{1,1} & \cdots & p_{1,n} & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ q_{1,1} & \cdots & q_{1,n} & p_{2,1} & \cdots & p_{2,n} & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & q_{2,1} & \cdots & q_{2,n} & p_{3,1} & \cdots & p_{3,n} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & p_{m,1} & \cdots & p_{m,n} \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & q_{m,1} & \cdots & q_{m,n} \end{bmatrix},$$

since the matrix $\left(\mathbf{I} + \rho\mathbf{Q}\mathbf{Q}^H\right)$ is a tridiagonal matrix, using the determinant calculation formula of tridiagonal matrices in [84], it can be proved that its determinant is lower bounded by

$$\det\left(\mathbf{I} + \rho\mathbf{Q}\mathbf{Q}^H\right) \geq 1 + \prod_{i=1}^{m}\left(\sum_{j=1}^{n}\rho|q_{i,j}|^2\right) + \prod_{i=1}^{m}\left(\sum_{j=1}^{n}\rho|q_{i,j}|^2\right). \tag{A.18}$$

To calculate the achievable DMT of the multiple-access concurrent DF relaying protocol, we consider a codeword set $\mathbf{x} = [\mathbf{x}[1]\ \mathbf{x}[2]\ \ldots\ \mathbf{x}[U]]^T$, where $\mathbf{x}[j] = [x_1[j]\ x_2[j]]^T$ or $\mathbf{x}[j] = [x_1[j]\ 0]^T$ or $\mathbf{x}[j] = [0\ x_2[j]]^T$ and there are V codewords in total. Assuming the number of $\mathbf{x}[j] = [x_1[j]\ x_2[j]]^T$ is λ_1 and the number of other two cases is λ_2, we have $\lambda_1 + \lambda_2 = U$ and $2\lambda_1 + \lambda_2 = V$. Denoting the channel matrix for such a codeword set as $\mathbf{H}_{U,V}$, the task of finding the achievable DMT of the multiple-access concurrent DF relaying protocol is the same as finding the smallest DMT that can be achieved by a MIMO system with channel fading matrix $\mathbf{H}_{U,V}$ and multiplexing gain Vr', in which $r' = \frac{L+1}{L}r$.

According to the above analysis, the determinant of the matrix $D = \det\left(\mathbf{I} + \rho\mathbf{H}_{U,V}\mathbf{H}_{U,V}^H\right)$ can be expressed as

$$\begin{aligned} D &= \det\left(\mathbf{I} + \rho\mathbf{H}_{U,V}\mathbf{H}_{U,V}^H\right) \\ &\dot{\geq} 1 + (\rho|h_{\mathcal{S}_1}|^2 + \rho|h_{\mathcal{S}_2}|^2)^{\lambda_1}(\rho|h_{\mathcal{S}_1}|^2)^a(\rho|h_{\mathcal{S}_2}|^2)^{\lambda_2-a} \\ &\quad + (\rho|h_{\mathcal{R}_1}|^2)^c(\rho|h_{\mathcal{R}_2}|^2)^{V-c}, \end{aligned} \tag{A.19}$$

where $a \leq \lambda_2$, $c = \frac{1}{2}V$ when V is even, and $c = \frac{V+1}{2}$ or $c = \frac{V-1}{2}$ when V is odd. Clearly,

$$a \leq \lambda_2 = 2U - V.$$

Define v_1, v_2,v_3, and v_4 as the exponential orders of $\frac{1}{|h_{\mathcal{S}_1}|^2}$, $\frac{1}{|h_{\mathcal{S}_2}|^2}$, $\frac{1}{|h_{\mathcal{R}_1}|^2}$, and $\frac{1}{|h_{\mathcal{R}_2}|^2}$ respectively. Following the analysis in reference [34] (Proof of Theorem 3), the system diversity gain can be calculated by

$$d(r') = \min\{v_1 + v_2 + v_3 + v_4\}, \tag{A.20}$$

where

$$\min\{av_1 + (U-a)v_2, cv_3 + (U-c)v_4\} \geq U - Vr' \tag{A.21}$$

when $v_1 \geq v_2$, or

$$\min\{(a+\lambda_1)v_1 + (U-\lambda_1-a)v_2, cv_3 + (U-c)v_4\} \geq U - Vr' \tag{A.22}$$

when $v_1 \leq v_2$. In addition, if we set $b = U - \lambda_1 - a$, (A.22) can be written as $\min\{(U - b)v_1 + bv_2, cv_3 + (U-c)v_4\} \geq U - Vr'$ so that we only need to consider the case $v_1 \geq v_2$ and (A.21).

For $v_1 \geq v_2$, using

$$av_1 + (U-a)v_2 \geq U - Vr',$$

it is easy to see that if $a \leq U - a$ (i.e. $a \leq \frac{U}{2}$), when $v_1 = v_2$, $v_1 + v_2$ approaches its minimum value. Therefore,

$$\min\{v_1 + v_2\} = 2\left(1 - \frac{V}{U}r'\right). \tag{A.23}$$

And $2(1 - \frac{V}{U}r')$ approaches its minimum value when choosing minimum U.

When $a \geq \frac{U}{2}$, we can have the minimum value of $v_1 + v_2$ as

$$\min(v_1 + v_2) = \begin{cases} 2 - \frac{V}{U-a}r' & 0 \leq r' \leq \frac{U-a}{V} \\ \frac{U-Vr'}{a} & \frac{T-a}{V} \leq r' \leq \frac{V}{U} \end{cases} \tag{A.24}$$

Since $a \leq 2U - V$, we have

$$\frac{U - Vr'}{a} \geq \frac{U - Vr'}{2U - V} = \frac{1}{2} + \frac{1}{2}\frac{1}{\frac{2U}{V} - 1}(1 - 2r'),$$

which approaches its minimum value when the maximum T is chosen.

Regarding $v_3 + v_4$, if U is even, $c = \frac{U}{2}$ so that

$$v_3 + v_4 \geq 2\left(1 - \frac{V}{U}r'\right). \tag{A.25}$$

Otherwise, if T is odd, $c = \frac{T+1}{2}$ and

$$\min(v_3 + v_4) = \begin{cases} 2 - \frac{2V}{U-1}r' & 0 \leq r' \leq \frac{U-1}{2V} \\ \frac{2(U-Vr')}{U+1} & \frac{U-1}{2V} \leq r' \leq \frac{V}{U} \end{cases} \tag{A.26}$$

Smaller U leads to smaller $\min(v_3 + v_4)$.

In the following, we will separatively discuss the impact of choosing different values of U and V on the achievable DMT.

1. $U = 1$ and $V = 1$

   In this simplest case, the matrix

   $$\mathbf{H}_{U,V} = \begin{bmatrix} h_{\mathcal{S}_1} \\ h_{\mathcal{R}_1} \end{bmatrix}$$

   so that the achievable DMT is

   $$d(r') = 2\left(1 - r'\right) \tag{A.27}$$

2. $U = 1$ and $V = 2$

   When $U = 1$ and $V = 2$,

   $$\mathbf{H}_{U,V} = \begin{bmatrix} h_{\mathcal{S}_1} & h_{\mathcal{S}_2} \\ h_{\mathcal{R}_1} & h_{\mathcal{R}_1} \end{bmatrix}.$$

   We can have

   $$\det\left(\mathbf{I} + \mathbf{H}_{U,V}\mathbf{H}_{U,V}^H\right) \doteq 1 + |h_{\mathcal{S}_1}|^2 + |h_{\mathcal{S}_2}|^2 + 2|h_{\mathcal{R}_1}|^2 + |h_{\mathcal{R}_1}|^2(h_{\mathcal{S}_1} - h_{\mathcal{S}_2})^2.$$

   Applying a similar method as that in the Proof of Theorem 1 in [64] and letting $\theta = \frac{h_{\mathcal{S}_1} - h_{\mathcal{S}_2}}{\sqrt{2}}$ and $\psi = \frac{h_{\mathcal{S}_1} + h_{\mathcal{S}_2}}{\sqrt{2}}$, we have $\varepsilon\{\theta\} = \varepsilon\{\psi\} = 0$, $\varepsilon\{|\theta|^2\} = \varepsilon\{|\psi|^2\} = 1$, and

$\varepsilon\{\theta\psi^*\} = 0$, which means θ and ψ are uncorrelated. Since $|\theta|^2 + |\psi|^2 = |h_{\mathcal{S}_1}|^2 + |h_{\mathcal{S}_2}|^2$, we have

$$\det\left(\mathbf{I} + \rho\mathbf{H}\mathbf{H}^H\right) = 1 + \rho|\theta|^2 + \rho|\psi|^2 + \rho 2|h_{\mathcal{R}_1}|^2 + \rho^4|h_{\mathcal{R}_1}|^2|\theta|^2$$

Thus, defining $(z)^+ = \max\{z, 0\}$, it is not difficult to have

$$d(r') = (1 - 2r')^+ + 2(1 - r')^+ \tag{A.28}$$

which is always larger than or equal to (A.27).

3. $U = 2$ and $V = 2$

   In this case,

$$\mathbf{H}_{U,V} = \begin{bmatrix} h_{\mathcal{S}_1} & 0 \\ h_{\mathcal{R}_1} & h_{\mathcal{S}_1} \\ 0 & h_{\mathcal{R}_2} \end{bmatrix}, \text{ or } \mathbf{H}_{U,V} = \begin{bmatrix} h_{\mathcal{S}_1} & 0 \\ h_{\mathcal{R}_1} & h_{\mathcal{S}_2} \\ 0 & h_{\mathcal{R}_2} \end{bmatrix}.$$

   They are the equivalent channel matrices for the repetition-coded successive DF relaying protocol (with $L = 2$) and the repetition-coded concurrent DF relaying protocol (with $L = 1$), respectively. Following the analysis in the Proof of Theorem 1 and Corollary 1, it can be seen that the two cases result in the achievable DMTs

$$d(r') = 3(1 - r'), \tag{A.29}$$

   and

$$d(r') = 4(1 - r'), \tag{A.30}$$

   respectively.

4. $V > 2$

   (a) $a \leq \frac{U}{2}$

   When U is even, $v_1 + v_2 + v_3 + v_4 = 2(1 - \frac{V}{U}r') + 2(1 - \frac{V}{U}r') = 4(1 - \frac{V}{U}r')$ decreases if U decreases (when V is fixed). Since $U \geq \frac{V}{2}$,

$$d(r) = 4\left(1 - \frac{V}{U}r'\right) \geq 4(1 - 2r') \tag{A.31}$$

On the other hand, if U is odd,

$$\begin{aligned} v_1 + v_2 + v_3 + v_4 &\geq 2\left(1 - \frac{V}{U}r'\right) + \frac{2(U - Vr')}{U+1} \\ &= 2\left(1 + \frac{U}{U+1}\right)\left(1 - \frac{V}{U}r'\right), \end{aligned}$$

which decreases if U decreases (when V is fixed). Therefore, by choosing the smallest U (which is odd), it can be seen

$$d(r) \geq \frac{7}{2}\left(1 - 2r'\right), \tag{A.32}$$

the RHS of which is realized when $V = 6$ and $U = 3$.

From (A.27) to (A.32), it can be seen that the lower bound of the DMT is

$$d(r') = \begin{cases} 2(1 - r') & 0 \leq r' \leq \frac{3}{10} \\ \frac{7}{2}(1 - 2r') & \frac{3}{10} \leq r' \leq \frac{1}{2} \end{cases} \tag{A.33}$$

And clearly, when $0 \leq r' \leq \frac{3}{10}$, $d(r') = 2(1 - r')$ is tight.

(b) $a > \frac{U}{2}$

If U is even,

$$v_1 + v_2 + v_3 + v_4 \geq \frac{T - mr'}{2T - m} + 2\left(1 - \frac{m}{T}r'\right). \tag{A.34}$$

Otherwise,

$$v_1 + v_2 + v_3 + v_4 \geq \frac{T - mr'}{2T - m} + \frac{2(T - mr')}{T+1}. \tag{A.35}$$

It is not difficult to prove (through simple mathematical calculations) that both (A.34) and (A.35) are no less than $2(1-r')$ when $r' \leq \frac{3}{10}$ and no less than $\frac{7}{2}(1-2r')$ otherwise.

Based on the above analysis, it can be concluded that assuming perfect decoding at the relays, the achievable DMT for each source of the multiple-access concurrent DF relaying protocol is lower-bounded by (4.17). The proof is complete.

Appendix B
Publication List

Publications relevant to the thesis can be found at the end of the thesis.

Journal Articles

1. C. Wang, Y. Fan, J. S. Thompson, and H. V. Poor, "A comprehensive study of repetition-coded protocols in multi-user multi-relay networks", *IEEE Transactions on Wireless Communications*, accepted for publication.

2. Y. Fan, C. Wang, H. V. Poor, J. S. Thompson, "Cooperative multiplexing in a half duplex relay network: Performance and constraints", submitted to *IEEE Transactions on Wireless Communications*, July 2008.

3. Y. Fan, C. Wang, H. V. Poor, J. S. Thompson, "Cooperative multiplexing: Toward higher spectral efficiency in multi-antenna relay networks", *IEEE Transactions on Information Theory*, accepted for publication.

4. C. Wang, Y. Fan, J. S. Thompson, "Recovering multiplexing loss through concurrent decode-and-forward (DF) relaying", *Wireless Personal Communications*, vol. 48, no. 1, pp. 193-213, January 2009

5. Y. Fan, C. Wang, J. S. Thompson and H. V. Poor, "Recovering multiplexing loss through successive relaying using repetition coding", *IEEE Transactions on Wireless Communications*, vol. 6, no. 12, pp. 4484-4493, December 2007.

Conference Proceedings

1. C. Wang, Y. Fan, I. Krikidis, J. S. Thompson, and H. V. Poor, "Superposition-coded concurrent decode-and-forward relaying", *2008 IEEE International Symposium on Information Theory (ISIT '08)*, Toronto, Canada, July 2008.

2. C. Wang, Y. Fan, J. S. Thompson, and H. V. Poor, "On the diversity-multiplexing tradeoff of concurrent decode-and-forward relaying", *2008 IEEE Wireless Communications & Networking Conference (WCNC '08)*, Las Vegas, US, 31 March - 3 April, 2008.

3. C. Wang, Y. Fan, and J. S. Thompson, "Recovering multiplexing loss through concurrent decode-and-forward (DF) relaying", *2007 IET Workshop on Smart Antennas and Cooperative Communications*, London, UK, October 2007

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[3] C. Wang, Y. Fan, and J. S. Thompson, "Recovering multiplexing loss through concurrent decode-and-forward (DF) relaying," in *2007 IET Workshop on Smart Antennas and Cooperative Communications*, (London, UK), October 2007.

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Recovering Multiplexing Loss Through Successive Relaying Using Repetition Coding

Yijia Fan, *Member, IEEE,* Chao Wang, *Student Member, IEEE,* John Thompson, *Member, IEEE,* and H. Vincent Poor, *Fellow, IEEE*

***Abstract*— In this paper, a transmission protocol is studied for a two relay wireless network in which simple repetition coding is applied at the relays. Information-theoretic achievable rates for this transmission scheme are given, and a space-time V-BLAST signalling and detection method that can approach them is developed. It is shown through the diversity multiplexing tradeoff analysis that this transmission scheme can recover the multiplexing loss of the half-duplex relay network, while retaining some diversity gain. This scheme is also compared with conventional transmission protocols that exploit only the diversity of the network at the cost of a multiplexing loss. It is shown that the new transmission protocol offers significant performance advantages over conventional protocols, especially when the interference between the two relays is sufficiently strong.**

***Index Terms*— Multiple-input multiple-output (MIMO), relay, capacity, wireless networks, cooperative diversity.**

I. Introduction

A. Background

IN the past few years, cooperative diversity protocols [1]–[4], [7]–[12] have been studied intensively to improve the diversity of relay networks. In most of the prior work, a time-division-multiple-access (TDMA) half-duplex transmission is assumed and the most popular transmission protocol (e.g. [2]) can be described in two steps: In the first step, the source broadcasts the information to all the relays. The relays process the information and forward it to the destination (in either the same or a different time slot) in the second step, while the source remains *silent*. The destination performs decoding based on the message it received in both steps. We refer to this protocol as the *classic protocol* throughout the paper.

For digital relaying, where the relay decodes, re-encodes and forwards the message, the simplest coding method is *repetition coding* [2], [3], where the source and all the relays use the *same* codebook. This scheme can achieve full diversity and is also practically implementable. Any capacity achieving AWGN channel codes can be used to approach the performance limit of such schemes. The disadvantage of this scheme is that it requires the relays to transmit in orthogonal time slots in the second step in order for the destination to combine effectively the relays' signals. This will result in a significant *multiplexing loss* compared with direct transmission. Space-time codes, which were originally applied in multiple-input multiple-output (MIMO) systems, have been suggested for use in relay networks (e.g. [3], [24]). Here all the relays can transmit the signals simultaneously to the destination in the second step and the multiplexing factor is recovered to $1/2$. However, this still causes spectral inefficiency for the high signal to noise ratio (SNR) region. In fact, the network capacity in this scenario will become only *half* of the non-relay network capacity for high SNR, even assuming that the message is always correctly decoded at the relays.

To fully recover the multiplexing loss, much more complicated protocols and coding strategies have been proposed in [9], where *new independent random codebooks* are used at the relays to transmit the same information as they received from the source, while the relays can adjust their listening times dynamically in the first step. Those approaches, which are based on Shannon's random coding theory, are currently theoretical and extremely difficult to realize in reality. Practical coding design for relay networks often follows a quite different approach from these theoretical investigations (see [25]–[28] for example).

Instead of using complicated coding schemes, a protocol using *repetition coded* relaying was proposed in [4](see also [29]) to avoid multiplexing loss for single relay channels. In this protocol, denoted as protocol I in [4], the source transmits a different message in the second time slot, so that the destination sees a collision of messages from both the relay and the source in the second time slot. Although multiplexing loss is recovered due to the continuous transmission of the source, diversity gain is lost due to the fact that the source transmission in time slot two is not relayed to the destination.

B. Contribution of the Paper

In this paper, we study a transmission protocol based on protocol I in [4] for digital relaying. By adding an additional relay in the network and making the two relays transmit in turn, we show that multiplexing loss can be effectively recovered while diversity/combining gain can still be obtained. Specifically, L codewords can be transmitted in $(L+1)$ time

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Y. Fan was with the Institute for Digital Communications, University of Edinburgh. He is now with the Department of Electrical Engineering, Princeton University, Princeton, NJ, 08544, USA (e-mail: yijiafan@princeton.edu).

C. Wang and J. Thompson are with the Institute for Digital Communications, University of Edinburgh, Edinburgh, EH9 3JL, UK (e-mail: {chao.wang, john.thompson}@ed.ac.uk).

H. V. Poor is with Department of Electrical Engineering, Princeton University, Princeton, NJ 08544 USA (email: poor@princeton.edu).

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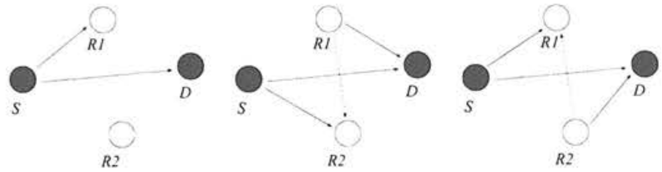
slots with joint decoding at the destination. Our analysis is based on two different scenarios: (a) The instantaneous channel state information (CSI) is known to the receiver and can be fed back to the transmitter; (b) The instantaneous CSI is known to the receiver but is not available to the transmitter. We make the following observations in this paper for the proposed protocol:

- For scenario (a), we derive the achievable rates for this protocol when repetition coding is assumed to be used at the relays. We show that in certain scenarios, the capacity for the network becomes that for a MIMO system with L inputs and $L+1$ outputs and has a multiplexing gain of $L/(L+1)$. Assuming that the relays correctly decode the signal, we show that the proposed protocol offers significant capacity performance advantages over the classic protocol due to its improved multiplexing gain.
- We also discuss the source-relay channel conditions and the interference that arises between the relays. We derive the required channel constraints for the optimal performance of such a protocol as a function of SNR, as well as the achievable rates for different channel conditions. We believe these analyses offer strong insights for *adaptive protocol design*, where relaying and direct transmission can be combined. Based on our network models we show that the proposed protocol, combined with the direct transmission protocol, can also give a significant capacity performance advantage over the classic protocol, especially when the two relays are located close to each other.
- We propose a practical low-rate feedback V-BLAST decoding algorithm which approaches the theoretical achievable rates for a slow fading environment.
- For scenario (b), we analyze the diversity multiplexing tradeoff for such a network when L is large, conditioned on the signals being correctly decoded at the relays. We show that in this scenario the network mimics a multiple-input single-output (MISO) system with two transmit and one receive antennas. This means it can offer a maximal diversity gain of two with almost no multiplexing loss.

C. Relations to Previous and Concurrent Work

The idea for successive relaying first appeared in [30]. This study was focused on amplify-and-forward relaying and did not offer insight into the achievable rates and diversity multiplexing tradeoff for such relaying methods. The scheme has been further analyzed for amplify-and-forward relaying in [6], [31] and [32]. In [6] capacity analysis was performed assuming that the direct link is ignored. Hence no diversity can be obtained at the destination. The analysis in very recent papers [31] and [32] make the assumption that the relays are isolated, when there are more than one relay. Also the relay-to-relay link in [31] and [32] acts in a different way from that in our work due to a different relaying mode.

A further work [5] also analyzes the capacity for such schemes when digital relaying is used. One major difference between [5] and our work is that the direct link is ignored in [5] while it is considered in this paper. Therefore the analysis becomes different for the following reasons. First, the scheme in [5] does not offer any *cooperative diversity* gain, which is a very important benefit that the relay can offer. We will show in this paper that a diversity gain of 2 can be obtained by considering the direct link. Secondly, this also results in very different characteristics in terms of achievable rates and signalling methods due to the additional interference and diversity that the direct link introduces. In this paper we *specifically* analyze the network capacity under different channel and interference constraints, which were not given in [5]. Also the use of the V-BLAST decoder is unique to our paper. Finally, we note that the capacity analysis discussed in our paper in fact *contains* the scenario in [5] as a special case, i.e. the same capacity values as in [5] is obtained on assuming that the channel coefficient for the direct link is zero in our model. Therefore our analysis is more general, and the adaptive protocols introduced here fit better in the context of previous work on this topic [2].



(a) Time slot 1. (b) Time slot 2. (c) Time slot 3.

Fig. 1. Transmission schedule for the proposed protocol.

II. Protocol Design

We assume a four-node network model, where one source, one destination and two relays exist in the network. For simplicity, we denote the source as S, the destination as D, and the two relays as $R1$ and $R2$. We split the source transmission into different frames, each containing L codewords denoted as s_l. These L codewords are transmitted continuously by the source, and are decoded and forwarded by two relays successively in turn. Before decoding L codewords, the destination waits for $L+1$ transmission time slots until all L codewords are received, from both direct link and the relay links. It then performs joint decoding of all L codewords. The specific steps for each transmission (reception) time slot for every frame are described as follows:

Time slot 1: S transmits s_1. $R1$ listens to s_1 from S. $R2$ remains silent. D receives s_1.

Time slot 2: S transmits s_2. $R1$ decodes, re-encodes and forwards s_1. $R2$ listens to s_2 from S while being interfered with by s_1 from $R1$. D receives s_1 from $R1$ and s_2 from S.

Time slot 3: S transmits s_3. $R2$ decodes, re-encodes and forwards s_2. $R1$ listens to s_3 from S while being interfered with by s_2 from $R2$. D receives s_2 from $R2$ and s_3 from S. The progress repeats until *Time slot L*.

Time slot L+1: $R1$ (or $R2$) decodes, re-encodes and forwards s_L. D performs a joint decoding algorithm to decode all L codewords received from the $L+1$ transmission time slots.

The transmission schedule for the first three time slots for each frame is shown in Fig. 1. Compared with direct transmission, the multiplexing ratio for this protocol is clearly $L/(L+1)$, which approaches 1 for large frame lengths L.

Unlike protocol III in [4], the destination always receives two copies of each codeword, from both the direct and relay link (a delayed version). This implies that diversity gain can still be realized by this protocol.

The major issue for this protocol to be effectively implemented is to tackle the co-channel interference at the relays and the destination. As described above, except for the first and last time slot, the relays and the destination always observe collisions from different transmitters (i.e. the source or the relays). Suppression of the interference thus becomes a major problem. We will discuss this problem further in the next two sections.

III. Achievable Rates

When the channel information can be fed back to the transmitter, one can implement adaptive coding and modulation to achieve the system performance limit. Thus capacity is a key measurement in this scenario. We assume a slow, flat, block fading environment, where the channel remains static for each message frame transmission (i.e. $L+1$ time slots). Note that while this assumption is made for presentation simplicity, the capacity analysis can also be applied to a more relaxed flat block fading scenario, e.g. fast fading where *each channel coefficient changes for each time slot.* We also assume that each transmitter transmits with equal power (i.e. no power allocation or saving among the source and relays). We denote the channel coefficient between node a and b by $h_{a,b}$, which may contain path-loss, Rayleigh fading, and lognormal shadowing. For simplicity, we denote the capacity function $\log_2(1+x)$ by $C(x)$, in which the parameter SNR denotes the ratio of signal power to the noise variance at the receiver.

A. Source-Relay Link

In order for the relays to decode the signals correctly, the source transmission rate should be below the Shannon capacity of the source-relay channels. We express this constraint as

$$R_i \le C\left(|h_{S,r_i}|^2 SNR\right), 1 \le i \le L \tag{1}$$

where r_i is the ith element in the L dimensional relay index vector

$$\mathbf{r} = [R1R2R1R2R1\cdots], \tag{2}$$

and R_i denotes the achievable rate for s_i.

B. Interference Cancellation Between Relays

One major defect of the protocol is the interference generated among the relays when one relay is listening to the message from the source, while the other relay is transmitting the message to the destination. This situation mimics a two user Gaussian interference channel [13], where two transmitters (the source and one of the relays) are transmitting messages each intended for one of the two receivers (the other relay and the destination). The optimal solution for this problem is still open. We concern ourselves only with suppressing the interference at the relays at this stage (interference suppression at the destination will be left until all L signals are transmitted). We give a very simple decoding criterion for the relays: if the interference between relays is stronger than the desired signal, we decode the interference and subtract it from the received signals before decoding the desired signal. Otherwise, we decode the signal directly while treating the interference as Gaussian noise.

The achievable rate is therefore based on different channel conditions between the source to relay and the relay to destination links. For example, when $R1$ transmits s_1 while $R2$ is receiving s_2, if $|h_{R1,R2}| \succ |h_{S,R2}|$, $R2$ first decodes s_1, subtracts it (as the interference), then decodes s_2 (as the desired signal). Therefore, besides the rate constraint proposed in the previous subsection, there will be an additional rate constraint for s_1 to be correctly decoded at $R2$, which can be expressed as follows:

$$R_1 \le C\left(\frac{|h_{R1,R2}|^2 SNR}{1+|h_{S,R2}|^2 SNR}\right). \tag{3}$$

Otherwise if s_2 is decoded directly, treating s_1 as noise, the achievable rate for s_2 is further constrained and can be expressed as

$$R_2 \le C\left(\frac{|h_{S,R2}|^2 SNR}{1+|h_{R1,R2}|^2 SNR}\right). \tag{4}$$

Note that this decoding criterion applies from the second time slot to the Lth time slot when transmitting each frame. In slot i, inequality (3) can be adapted to a constraint on R_{i-1} and inequality (4) can be adapted to a constraint on R_i.

C. Space-Time Processing at the Destination

If the transmission rate is below the Shannon capacity proposed by the previous two subsections, the relays can successfully decode and retransmit the signals for all the $L+1$ time slots. The input output channel relation for the relay network is equivalent to a multiple access MIMO channel, which can be expressed as

$$\mathbf{y} = \sqrt{SNR}\underbrace{\begin{bmatrix} h_{S,D} & 0 & 0 & 0 & 0 \\ h_{r_1,D} & h_{S,D} & 0 & 0 & 0 \\ 0 & h_{r_2,D} h_{S,D} & & 0 & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & h_{r_{L-1},D} & h_{S,D} \\ 0 & 0 & 0 & 0 & h_{r_L,D} \end{bmatrix}}_{\mathbf{H}}\mathbf{s}+\mathbf{n}, \tag{5}$$

where $\mathbf{y}$ is the $(L+1)\times 1$ received signal vector, $\mathbf{s}$ is the $L\times 1$ transmitted signal vector and $\mathbf{n}$ is an $(L+1)\times 1$ complex circular additive white Gaussian noise vector at the destination. Unlike conventional multiple access MIMO channels, the dimensions of $\mathbf{y}$, $\mathbf{s}$ and $\mathbf{n}$ are expanded in the time domain rather than the space domain. However, the capacity region should be the same, which can be expressed as follows [14]:

$$R_k \le \log_2\left(\det\left(\mathbf{I}+\mathbf{h}_k\mathbf{h}_k^H SNR\right)\right), \tag{6}$$

$$R_{k_1}+R_{k_2} \le \log_2\left(\det\left(\mathbf{I}+SNR\left(\mathbf{h}_{k_1}\mathbf{h}_{k_1}^H+\mathbf{h}_{k_2}\mathbf{h}_{k_2}^H\right)\right)\right) \tag{7}$$

$$\cdots$$

$$\sum_{k=1}^{L} R_k \le \log_2\left(\det\left(\mathbf{I}+\mathbf{H}\mathbf{H}^H SNR\right)\right), \tag{8}$$

where $\mathbf{h}_k$ denotes the kth column of $\mathbf{H}$. As it is extremely complicated to give an exact description for the rate region of each signal when $L > 2$, we will concentrate only on inequalities (6) and (8) to give a sum capacity upper bound for the network in the next subsection. However, as will be shown later in the paper, this bound is extremely tight and is achievable when a space-time V-BLAST algorithm is applied at the destination to decode the signals in a slow fading scenario.

D. Network Achievable Rates

Combining the transmission rate constraints proposed by the previous three subsections, we provide a way of calculating the network capacity upper bound for the proposed protocol. First, we impose a rate constraint R_i for each transmitted codeword s_i. In the first time slot (initialization), we write

$$R_{S,r_1} \leq C\left(|h_{S,r_1}|^2\, SNR\right). \tag{9}$$

For $(i+1)$th time slot (for $1 \leq i \leq L-1$), we calculate the rate constraints based on the decoding criterion at the relays. The calculation can be written as a logical `if` statement as follows:

`if` $h_{R1,R2} \succ h_{S,r_{i+1}}$,

$$\begin{aligned} R_i \leq \min\Bigg(& C\left(\frac{|h_{R1,R2}|^2\, SNR}{1+\left|h_{S,r_{i+1}}\right|^2 SNR}\right), R_{S,r_i}, \\ & C\left(|h_{S,D}|^2\, SNR + |h_{r_i,D}|^2 SNR\right)\Bigg), \\ R_{S,r_{i+1}} \leq & C\left(\left|h_{S,r_{i+1}}\right|^2 SNR\right); \end{aligned} \tag{10}$$

`else`

$$\begin{aligned} R_i \leq & \min\left(R_{S,r_i}, C\left(|h_{S,D}|^2\, SNR + |h_{r_i,D}|^2\, SNR\right)\right), \\ R_{S,r_{i+1}} \leq & C\left(\frac{\left|h_{S,r_{i+1}}\right|^2 SNR}{1+|h_{R1,R2}|^2\, SNR}\right); \end{aligned} \tag{11}$$

end.

Note that the term $C\left(|h_{S,D}|^2\, SNR + |h_{r_i,D}|^2\, SNR\right)$ represents the constraint expressed by (6). The purpose of the `if` statement is to select the decoding order at the relay and to decide whether equation (3) or (4) is the correct constraint to apply.

In the $(L+1)$th time slot, we have

$$R_L \leq \min\left(R_{S,r_L}, C\left(|h_{S,D}|^2\, SNR + |h_{r_L,D}|^2\, SNR\right)\right). \tag{12}$$

Combining these constraints with the sum capacity constraint expressed by (8), an achievable rate per time slot can then be written as

$$\begin{aligned} C_{ach} = \frac{1}{L+1} \min\Bigg(& \max_{R_1 \cdots R_L} \left\{\sum_{i=1}^{L} R_i\right\}, \\ & \log_2\left(\det\left(\mathbf{I} + \mathbf{H}\mathbf{H}^H SNR\right)\right)\Bigg) \end{aligned} \tag{13}$$

The first term in the min function comes from the calculation described above, the second one comes from equation (8).

E. Interference Free Transmission

From the above discussion of the proposed protocol, it is clear that the interference between relays is one major and obvious factor that can significantly degrade the network capacity performance. However, it has been shown that for a Gaussian interference network, if the interference is sufficiently strong, the network can perform the same as an interference free network [15]. Specifically, for the scenario discussed in our model, if the interference between relays (i.e. the value of $|h_{R1,R2}|$) is so large that the following inequality holds

$$\begin{aligned} \frac{|h_{R1,R2}|^2\, SNR}{1+\left|h_{S,r_{i+1}}\right|^2 SNR} \geq \min\Big(& |h_{S,r_i}|^2 SNR, \\ & \left(|h_{S,D}|^2 + |h_{r_i,D}|^2\right) SNR\Big), i = 1 \cdots L, \end{aligned} \tag{14}$$

then the relay can always correctly decode the interference and subtract it before decoding the desired message, without affecting the overall network capacity. In this situation, the capacity analysis for the ith ($1 \leq i \leq L$) transmitted signal as expressed by (10)-(12) can be simplified to

$$\begin{aligned} R_i \leq \min\Big(& C\left(|h_{S,r_i}|^2\, SNR\right), \\ & C\left(\left(|h_{S,D}|^2 + |h_{r_i,D}|^2\right) SNR\right)\Big). \end{aligned} \tag{15}$$

It is obvious that the rate bounds provided by (15) are significantly larger than those provided by (10)-(12).

From the above capacity analysis, it can also be seen that the quality of the source to relay link (i.e. h_{S,r_i}) is also an important factor that may constrain the network capacity. This has also been justified and discussed in many papers (e.g. [2], [3], [7], [9], [12]). Similar to this previous work, we suggest that h_{S,r_i} should be compared with $h_{S,D}$ or $h_{r_i,D}$ before deciding to relay or not. For the interference free scenario discussed here, the constraint becomes

$$|h_{S,r_i}|^2 \geq |h_{S,D}|^2 + |h_{r_i,D}|^2, 1 \leq i \leq L. \tag{16}$$

The capacity expressed by (13) can be simplified to

$$\begin{aligned} C_{ach} = \frac{1}{L+1} \min\Bigg(& \sum_{i=1}^{L} C\left(\left(|h_{S,D}|^2 + |h_{r_i,D}|^2\right) SNR\right), \\ & \log_2\left(\det\left(\mathbf{I} + \mathbf{H}\mathbf{H}^H SNR\right)\right)\Bigg). \end{aligned} \tag{17}$$

By Jensen's inequality [15] it is clear that

$$\begin{aligned} & \sum_{i=1}^{L} C\left(\left(|h_{S,D}|^2 + |h_{r_i,D}|^2\right) SNR\right) \geq \\ & \log_2\left(\det\left(\mathbf{I} + \mathbf{H}\mathbf{H}^H SNR\right)\right). \end{aligned}$$

Therefore the rate is equal to the MIMO channel capacity equation with a multiplexing scaling factor:

$$C_{ach} = \frac{1}{L+1} \log_2\left(\det\left(\mathbf{I} + \mathbf{H}\mathbf{H}^H SNR\right)\right). \tag{18}$$

This result shows that the proposed protocol can offer the best capacity performance conditioned on (14) and (16), which guarantees that the relays will correctly decode the message without affecting the network capacity. To summarize, we have the following theorem.

Theorem 1: Conditioned on (14) and (16), the capacity for the successive relaying scheme can be expressed as

$$C_{ach} = \frac{1}{L+1} \log_2 \left(\det \left(\mathbf{I} + \mathbf{H}\mathbf{H}^H SNR \right) \right)$$

where $\mathbf{H}$ denotes the channel matrix in (5).

It should be noted that this high interference scenario (i.e. condition (14)) is *not uncommon* in reality. A practical example is when the two relays (e.g. mobiles) are located close to each other. If the routing techniques are developed to choose these relays, the capacity performance can be significantly improved by applying the proposed protocol. To satisfy condition (16), an adaptive protocol can be developed from the proposed protocol to guarantee that the relays are used only when (16) holds, otherwise direct transmission is assumed. However, for a large dense network of relays, it is even not difficult to find two relays satisfying both (14) and (16). A simple example is a fixed relay network scenario [16], where the source to relay links are often assumed to be significantly better than the corresponding relay to destination links and the direct link. Therefore both (14) and (16) can be met by choosing the two nearby fixed relays. Furthermore, studies have shown that for a large relay network where many relays exist, choosing the best one or few relays will be preferable to using all the relays in many situations (e.g. [10], [17], [18], [20]). Therefore it is possible that the proposed relay protocol can be combined with relay selection techniques to achieve an even higher capacity gain over the classic multicast relay protocol, especially for high SNR conditions.

F. The V-BLAST Algorithm

In this section we apply the low-rate feedback V-BLAST minimum mean squared error (MMSE) algorithm for detecting the signals at the destination. The V-BLAST algorithm was initially designed for spatial multiplexing MIMO systems [21]. For a system with M transmit and N receive antennas, the message at the transmitter is multiplexed into M different signal streams, each independently encoded and transmitted to the receiver. The receiver uses N antennas to detect and decode each signal stream by a V-BLAST MMSE detector [22]. The V-BLAST MMSE detection consists of M iterations, each aimed at decoding one signal stream. For each iteration, the receiver applies the MMSE algorithm to detect and decode the *strongest* signal while treating the other signals as interference, then subtracts it from the received signal vector. The detection continues until all M signal streams are decoded. The Shannon capacity of this system can be achieved if we assume that each signal is correctly decoded [23]:

$$\begin{aligned} C =& \log_2 \det \left(\mathbf{I} + \mathbf{H}\mathbf{H}^H SNR \right) \\ =& \sum_{i=1}^{M} \log_2 \left(1 + SINR_i \right), \end{aligned} \tag{19}$$

where $SINR_i$ is the output signal to interference plus noise ratio (SINR) for signal s_i in the V-BLAST detector. In order for each signal to be correctly decoded, a low-rate feedback channel can be used to feed the value of $SINR_i$ back to the transmitter. Adaptive modulation and coding should be applied to make the transmission rate for s_i lower than $\log_2 (1 + SINR_i)$.

Unlike traditional MIMO systems, when we apply this V-BLAST MMSE detector at the destination for the proposed protocol, each signal stream is independently encoded along the *time dimension* rather than the space dimension. When considering the rate bound R_i, the same analysis should be made as in Section III. The initialization step is the same as (9). For the $(i+1)$th time slot (for $1 \le i \le L-1$), based on the same interference cancellation criterion as in Section III, the rate calculation can be performed as follows:

`if` $h_{R1,R2} \succ h_{S,r_{i+1}}$

$$\begin{aligned} R_i \le \min \Bigg(& C \left(\frac{|h_{R1,R2}|^2 SNR}{1 + \left| h_{S,r_{i+1}} \right|^2 SNR} \right), R_{S,r_i}, \\ & \log_2 \left(1 + SINR_{r_i} \right) \Bigg), \\ R_{S,r_{i+1}} \le & C \left(\left| h_{S,r_{i+1}} \right|^2 SNR \right); \end{aligned} \tag{20}$$

`else`

$$R_i \le \min \left(R_{S,r_i}, \log_2 \left(1 + SINR_{r_i} \right) \right), \tag{21}$$

$$R_{S,r_{i+1}} \le C \left(\frac{\left| h_{S,r_{i+1}} \right|^2 SNR}{1 + |h_{R1,R2}|^2 SNR} \right); \tag{22}$$

`end.`

In the $(L+1)$th time slot, we have

$$R_L \le \min \left(R_{S,r_L}, \log_2 \left(1 + SINR_{r_L} \right) \right). \tag{23}$$

The $SINR_{r_i}$ denotes the SINR for s_i, which is decoded, encoded and forwarded by relay r_i. The network capacity is therefore

$$C_{ach_{BLAST}} = \frac{1}{L+1} \max_{R_1 \cdots R_L} \left\{ \sum_{i=1}^{L} R_i \right\}. \tag{24}$$

The condition for interference free transmission discussed in Section III-E can be expressed as

$$\begin{aligned} C \left(\frac{|h_{R1,R2}|^2 SNR}{1 + \left| h_{S,r_{i+1}} \right|^2 SNR} \right) \ge \min \Big(& C \left(\left| h_{S,r_i} \right|^2 SNR \right), \\ & \log_2 (1 + SINR_{r_i}) \Big). \end{aligned} \tag{25}$$

The rate for the ith ($1 \le i \le L$) signal under this condition can be expressed as

$$R_i \le \min \left(C \left(\left| h_{S,r_i} \right|^2 SNR \right), \log_2 \left(1 + SINR_{r_i} \right) \right). \tag{26}$$

Similar to the discussion in Section III-E, we can further apply adaptive protocols or make relay selections in the network to enhance the source to relay links; i.e.,

$$C \left(\left| h_{S,r_i} \right|^2 SNR \right) \ge \log_2 \left(1 + SINR_{r_i} \right), \tag{27}$$

and it is clear from (19) that (24) equals (18) under conditions (27) and (25). This implies that the V-BLAST algorithm can achieve rate (18) for the protocol if the interference channel between relays and source to relay channels are sufficiently strong.

It can be seen that the conditions in (25) and (27) have a higher probability of being fulfilled than those in (14) and (16) due to the following observation:

$$SINR_{r_i} \leq \left(|h_{S,D}|^2 + |h_{r_i,D}|^2\right) SNR. \quad (28)$$

This further implies that the conditions in (25) and (27) are better suited to assist the VBLAST algorithm to achieve the rate in (18), than those in (14) and (16). We note that in practice these conditions also imply a signalling overhead among the source, relays and destination in order to obtain the required SINR information. Furthermore, we note that V-BLAST might be applied only to a slow fading scenario in which the channel remains unchanged at least in every $L+1$ transmission time slots. This is due to the fact that *SINR* has to be fed back to the transmitters *before* the source starts transmitting at the beginning of the *L+1* time slots.

G. Comparison with Classic Protocols

1) Classic Protocol I: The first classic protocol was presented by Laneman and Wornell [3], where each message transmission is divided into three time slots. In the first time slot, the source broadcasts the message to the two relays and the destination. In the next two time slots, each relay retransmits the message to the destination in turn after decoding and re-encoding it by repetition coding. The destination combines the signals it receives in the three time slots. The network capacity for this protocol can be written as:

$$C = \frac{1}{3}\times\min\Big(C\left(|h_{S,R1}|^2 SNR\right), C\left(|h_{S,R2}|^2 SNR\right), C\Big(\left(|h_{S,D}|^2 + |h_{R1,D}|^2 + |h_{R2,D}|^2\right) SNR\Big)\Big), \quad (29)$$

where the term $\frac{1}{3}$ denotes the multiplexing loss compared with direct transmission.

2) Classic Protocol II: A simple improvement of Classic Protocol I is to apply distributed Alamouti codes at the relays [8]. The system uses four time slots to transmit two signals. In the first two time slots the source broadcasts s_1 and s_2 to both the relays and the destination. In the next two time slots $R1$ transmits $[s_1, -s_2^*]$ and $R2$ transmits $[s_2, s_1^*]$. The destination uses maximal ratio combining to combine the signals received from all four time slots in order to detect and decode them. The capacity achieved by this protocol can be written as

$$C = \frac{1}{2}\times\min\Big(C\left(|h_{S,R1}|^2 SNR\right), C\left(|h_{S,R2}|^2 SNR\right), C\Big(\left(|h_{S,D}|^2 + |h_{R1,D}|^2 + |h_{R2,D}|^2\right) SNR\Big)\Big). \quad (30)$$

It is clear that (30) outperforms (29) as it has the same diversity gain but reduced multiplexing loss compared with direct transmission.

In practice, both protocols can be combined with relay selection or adaptive relaying protocols to make sure that

$$\min\left(C\Big(|h_{S,R1}|^2 SNR\Big), C\left(|h_{S,R2}|^2 SNR\right)\right) \geq C\left(\left(|h_{S,D}|^2 + |h_{R1,D}|^2 + |h_{R2,D}|^2\right) SNR\right) \quad (31)$$

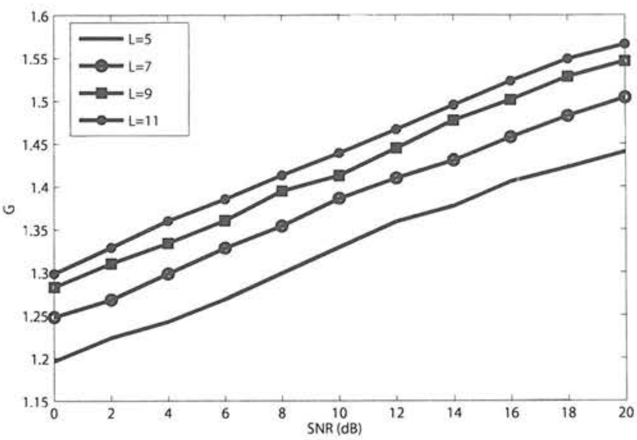


Fig. 2. Capacity gain of the proposed protocol over classic protocol II.

when relaying is used. The network under this condition can achieve the best capacity performance (i.e. the third term in (29) and (30)). This result clearly mimics the performance of a 3×1 single-input multiple-output (SIMO) or multiple-input single-output (MISO) system.

3) Performance Comparison: It can be seen that if the two relays are close to each other so that (14) holds, then condition (16) is more likely to hold than (31). This implies that the best capacity (18) for the proposed protocol can be achieved with a higher probability than that for the classic protocols. We now simply compare the best capacities can be achieved by both proposed protocol and Classic Protocol II:

$$G \triangleq \frac{\frac{1}{L+1}\mathrm{E}\left[\log_2\left(\det\left(\mathbf{I}+\mathbf{H}\mathbf{H}^H SNR\right)\right)\right]}{0.5\times \mathrm{E}\left[C\left(\left(|h_{S,D}|^2 + |h_{R1,D}|^2 + |h_{R2,D}|^2\right) SNR\right)\right]}, \quad (32)$$

where $\mathrm{E}[\bullet]$ denotes the expectation and we assume $\{h_{a,b}\}$ is a set of identically, independent distributed (i.i.d), complex, zero mean Gaussian random variables with unit variances. G is plotted as a function of SNR in Fig. 2 for different values of L. It is clear that the capacity gain increases as the value of SNR increases. Larger values of L lead to reduced multiplexing loss and offer higher capacity gains.

IV. Diversity Multiplexing Tradeoff

When the instantaneous CSI is not known to the transmitter, outages will occur. In this scenario diversity-multiplexing tradeoff is a powerful tool to measure the balance between the rates and error probability. In this section we study further the diversity multiplexing tradeoff [33] for such a protocol. For simplicity our analysis is based on the assumption that the signals are correctly decoded at the relays. We note that this analysis can provide insights on the best possible performance this scheme can offer. We summarize our results in the following.

Theorem 2: Define the diversity gain d and multiplexing gain r as those in [33]. Conditioned on the relays correctly decoding the signals (i.e, (14) and (16)[1]), the diversity multiplexing tradeoff for the successive relaying scheme in a slow

[1]Note that the probability that (14) holds decreases as the SNR increases. Therefore, the theorem offers an upper bound on the performance of such a system at high SNR

TABLE I

COMPARISON OF THE DIFFERENT TRANSMISSION SCHEMES FOR THE TWO RELAY CASE

Schemes/Maximum Gain	Multiplexing	Diversity
Direct transmission	1	1
Classic I	1/3	3
Classic II	1/2	3
Proposed scheme	$L/(L+1)$	2

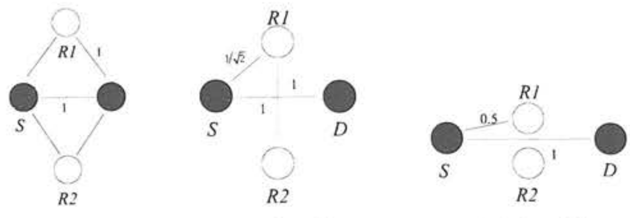


Fig. 3. Network models for different geometries.

fading scenario, where the channel coefficients remain the same for $L+1$ time slots, can be expressed as:

$$d(r) = 2\left(1 - \frac{L+1}{L}r\right)^{+}. \tag{33}$$

Proof: See Appendix. ■

As predicted in the previous section, we can see from this theorem that a maximal diversity gain of 2 can be obtained, while the multiplexing gain can be recovered to nearly 1 for large L. This will offer a significant advantage in terms of spectral efficiency, which will be shown through simulations in the next section. Table I compares the maximal diversity and multiplexing gains between the successive relaying protocol and the classic protocols in a slow fading scenario. Note that for a faster fading scenario where the channel coefficient changes in every transmission time slot, the same theorem still holds if the signal transmitted in each time slot is independently encoded. Furthermore, we note that it is possible to obtain the same diversity-multiplexing tradeoff performance if proper adaptive protocols similar to those in [2] are used to consider the conditions of the source to relay links [36].

V. Simulation Results

In this section we make further comparison of the above protocols for different network geometries in terms of achievable rates. We compare only Classic Protocol II with the proposed protocol. As mentioned previously, to achieve better capacity performance in practice, the classic protocols should be combined with adaptive protocols so that relaying is applied only if the source to relay channels are good. There are a number of ways to enable adaptive protocols. Three examples are to base adaptation on one of the following conditions: (a) $\min(|h_{S,R1}|, |h_{S,R2}|) \geq |h_{S,D}|$, i.e. the source to relay link is better than the direct link; (b) condition (16) holds; or (c) condition (31) holds. Although (b) and (c) fits better with the analysis in this paper, condition (a) is the *simplest* since it does not require knowledge of the relay to destination links. In the following we will only adopt (a) in the simulations. I.e, if condition (a) is not met, the system will use direct transmission. Similar results would be obtained if condition (b) or (c) were to be adopted instead.

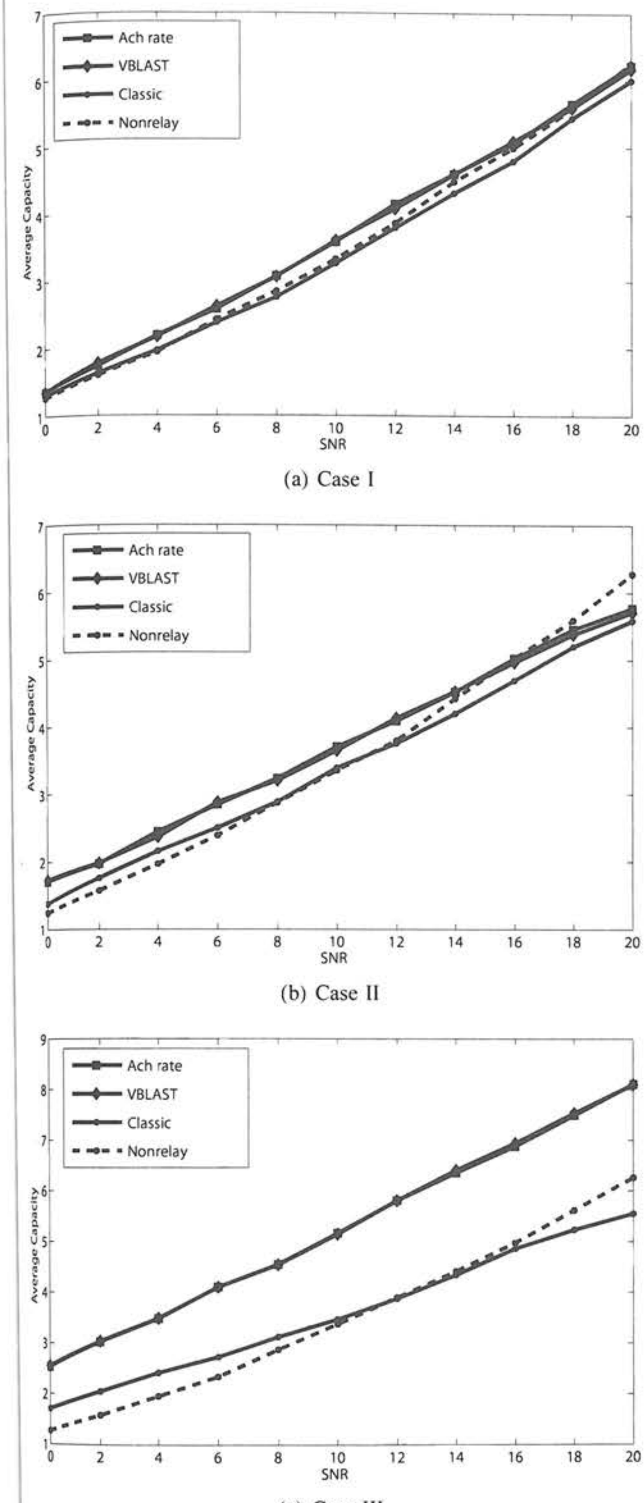
Our simulations are based on three network geometries: cases I, II and III, which are shown in Fig.3. We assume that each $h_{a,b}$ contains Rayleigh fading, pathloss and independent lognormal shadowing terms. These terms can be written as $h_{a,b} = v_{a,b}\sqrt{x_{a,b}^{-\gamma}10^{\zeta_k/10}}$, where $\{v_{a,b}\}$ is a set of i.i.d. complex Gaussian random variables with unit variances, and $x_{a,b}$ is the distance between the nodes a and b. The scalar γ denotes the path loss exponent (in this paper it is always set to 4). The lognormal shadowing term ζ_k is a random variable drawn from a normal distribution with a mean of 0 dB and a standard deviation $\delta = 8$ (dB). We assume that the distance between the source and destination is normalized to unit distance. In case I, the distances between the source to relays and relays to destination are all normalized, so the distance between the two relays is therefore $\sqrt{3}$. In case II, the distance between relays is normalized, while the distance between the source to relays and relays to destinations is $1/\sqrt{2}$. In case III, the relays are located in the middle region between the source and destination, so that the distance between the source and relays is $1/2$ while the distance between the relays is negligible compared with the source to relays links. For the proposed protocol, these three cases represent a meaningful tradeoff between the strength of source to relay channels and the interference channel between the two relays.

We assume $L = 7$ in the simulation, and the performance for the proposed protocol will certainly increase as L increases. Fig.4 shows the achievable rates for the proposed protocols (ach rate), the capacity achieved by V-BLAST MMSE detection (VBLAST), the classic protocols (classic) and direct transmission (direct), all averaged over $10,000$ channel realizations. It can be clearly seen from all three figures that the V-BLAST algorithm approaches the capacity bounds for the protocol proposed in this paper.

Both Fig.4(a) and Fig. 4(b) imply that it is generally not helpful to implement relaying protocols when the source to relay link is about the same quality as the source to destination (direct) link, as the link gain due to relaying is small in this case. However, the proposed protocol still offers a performance gain over direct transmission for both the high and low SNR regions in these cases. Compared with case I and II, in case III the source to relay links are much stronger, and the relays become close to each other so that the interference is sufficiently strong to allow interference free transmission, as discussed in Sections III and IV. It can be clearly seen in Fig. 4(c) that the proposed protocol gives a significant performance advantage over direct transmission for both low and high SNR regions due to its combining gain and negligible multiplexing loss. The classic protocol still performs worse than direct transmission due to its significant multiplexing loss compared with direct transmission, although its performance gain over direct transmission for the low SNR region is improved.

VI. Conclusions

In this paper we have analyzed successive relaying protocol. Our analysis shows that this protocol can maintain combining/diversity gain while recovering the multiplexing



(a) Case I

(b) Case II

(c) Case III

Fig. 4. Average capacity of the network for different network geometries in bits per transmission time slot.

loss associated with the classic protocol. We have proposed the use of a low complexity V-BLAST detection algorithm to help implement this protocol effectively. From the simulation study based on different geometries, we can draw two main conclusions: (a) For both the proposed and classic protocols, the network capacity increases when the source-relay link becomes stronger; (b) in this scenario, while the classic protocol still loses its performance advantage for the high SNR region, the proposed protocol can give significant performance advantages for both the low and high SNR regions.

Note that one very important factor that impairs the capacity performance of the proposed protocol is interference between the two relays. Our capacity analysis does not offer the optimal capacity results for this protocol because the optimal method of suppressing the interference between the relays is not known in general. For the adaptive protocol discussed in the paper, it is also worthwhile to develop alterative forms of the protocol that explicitly account for the impact of interference between relays on the network capacity. Also it should be interesting to extend the analysis into a more than two relay scenario. These are interesting topics for future work.

APPENDIX
PROOF OF *Theorem 1*

As mentioned in Section III.E, conditioned on the event that the relays correctly decode the message, the successive relaying protocol mimics a multiple access MIMO channel (5) with a capacity constraints (6) - (8). For each constraint there is a probability of not meeting it. The probability of outage is the highest among all these probabilities. Therefore there are $(2^L - 1)$ diversity-multiplexing tradeoffs for all those conditions and the lowest curve within the range of multiplexing gain is the optimal tradeoff curve for the system [35]. To characterize the diversity-multiplexing tradeoff achieved by each constraint, we consider an $(m+1) \times m$ MIMO channel matrix $\mathbf{H}_m$ in the same form as in (5). Define v_0 as the exponential order [9] of $1/|h_{S,D}|^2$ and v_k as the exponential order of $1/|h_{r_k,D}|^2$. Furthermore, Let $\mathbf{M}_{m+1} = \mathbf{I} + \frac{1}{2}\Sigma_S \Sigma_n^{-1}$, where Σ_S and Σ_n denote the covariance matrices of the observed signal and noise components *at the receiver*, respectively. We assume that each source message s_i is chosen from a Gaussian random codebook of codeword length l. When $m = 1$, the upper bound on the ML conditional pairwise error probability (PEP) can be calculated by

$$\begin{aligned} P_{PE|v_0,v_1} &\leq \det\left(\mathbf{I} + \frac{1}{2}\Sigma_{S_1}\Sigma_n^{-1}\right)^{-l} \\ &= \left(1 + \frac{1}{2}\rho\,|h_{S,D}|^2 + \frac{1}{2}\rho\,|h_{r_1,D}|^2\right)^{-l} \\ &\doteq \rho^{-l(\max\{1-v_0,1-v_1\})^+} \end{aligned} \tag{34}$$

where $\doteq$ denotes the exponential equality [33] and SNR is replaced by ρ for notational simplicity. We assume each s_i is transmitted with data rate R bits in each transmission time slot. Since the successive relaying protocol uses $(L+1)$ time slots to transmit L different symbols, the average transmission rate is $\overline{R} = \frac{L}{L+1}R$. On assuming that average transmission rate changes as $\overline{R} = r \log \rho$ with respect to ρ, then it is easy to see $R = \frac{L+1}{L} r \log \rho$. Therefore, we have a total of $\rho^{\frac{L+1}{L}rl}$ codewords. Thus, the error probability can be bounded by

$$P_{E|v_0,v_1} \dot{\leq} \rho^{-l((\max\{1-v_0,1-v_1\})^+ - \frac{L+1}{L}r)}. \tag{35}$$

Next, we want to find the set in which the outage event always dominates the error probability performance. The analysis regarding this is similar to that in [9] and is thus omitted here. This set is given by

$$O^+ = \left\{(v_0, v_1) \in R^{2+} \,\middle|\, (\max\{1-v_0, 1-v_1\})^+ \leq \frac{L+1}{L} r\right\}. \tag{36}$$

Then, for any error event which belongs to the non-outage set, we can choose l to make its probability sufficiently small to ensure that the error performance is dominated by the outage probability, which can be expressed as $\rho^{-d_o(r)}$ for $d_o(r) = \inf_{(v_0,v_1)\in O^+} (v_0+v_1)$. Now using (36), $d_o(r)$ can be calculated as

$$d_o(r) = 2\left(1 - \frac{L+1}{L} r\right)^+ \tag{37}$$

which represents the diversity-multiplexing tradeoff in the case $m=1$. When $m \geq 2$, the analysis of the determinant of $\mathbf{M}_{m+1}$ can be conducted in a way similar to that in [32]; so we omit the specific calculation due to limited space. Define $D_k := \det(\mathbf{M}_{(k)})$, where $\mathbf{M}_{(k)}$ denotes a $k \times k$ sub-matrix formed by the first k rows and k columns from the upper left-most corner of $\mathbf{M}$. The coefficients of D_{m+1} can be calculated recursively as

$$D_{m+1}\left(\frac{1}{2}\rho|h_{S,D}|^2\right) = \left(\frac{1}{2}\rho|h_{S,D}|^2\right)^m + \prod_{j=1}^{m}\left(1+\frac{1}{2}\rho|h_{r_j,D}|^2\right) + P\left(\frac{1}{2}\rho|h_{S,D}|^2\right)$$

where $P(\frac{1}{2}\rho|h_{S,D}|^2)$ is a polynomial in $\frac{1}{2}\rho|h_{S,D}|^2$ and is always nonnegative. Thus, we have

$$D_{m+1} \geq \left(\frac{1}{2}\rho|h_{S,D}|^2\right)^m + \prod_{k=1}^{m}\left(1+\frac{1}{2}\rho|h_{r_k,D}|^2\right). \tag{38}$$

Since we assume a slow fading environment, $v_1 = v_3 = \ldots$ and $v_2 = v_4 = \ldots$. On setting $v = \max\{v_1, v_2\}$, it can be seen that

$$\det(\mathbf{I} + \Sigma_{S_m}\Sigma_n^{-1}) \dot{\geq} \rho^{\max\{m(1-v_0)^+, m(1-v)^+\}}. \tag{39}$$

If we define $\det(\mathbf{I} + \Sigma_{S_m}\Sigma_n^{-1}) \doteq \rho^{f(v_0,v_1,v_2)}$ and

$$\rho^{\max\{m(1-v_0)^+, m(1-v)^+\}} \doteq \rho^{g(v_0,v_1,v_2)}, \tag{40}$$

then we have

$$f(v_0,v_1,v_2) \dot{\geq} g(v_0,v_1,v_2), \qquad \forall (v_0,v_1,v_2) \in R^{3+}. \tag{41}$$

Similarly to the analysis for $m=1$, O_f^+ should be defined as

$$O_f^+ = \left\{(v_0,v_1,v_2) \in R^{3+} \,\middle|\, f(v_0,v_1,v_2) \leq \frac{L+1}{L} mr\right\} \tag{42}$$

where m denotes that m symbols are transmitted and the equivalent data rate $R = \frac{L+1}{L} mr \log\rho$. We define

$$O_g^+ = \left\{(v_0,v_1,v_2) \in R^{3+} \,\middle|\, f(v_0,v_1,v_2) \leq \frac{L+1}{L} mr\right\}. \tag{43}$$

Because of (41), it can be seen that $O_f^+ \subseteq O_g^+$. Therefore

$$\inf_{(v_0,v_1,v_2)\in O_f^+} (v_0+v_1+v_2) \geq \inf_{(v_0,v_1,v_2)\in O_g^+} (v_0+v_1+v_2),$$

which means that the diversity gain calculated from O_f^+ is always larger than that calculated from O_g^+.

From (40) and (43), it is not difficult to show that

$$\inf_{(v_0,v_1,v_2)\in O_g^+} (v_0+v_1+v_2) \geq 2\left(1-\frac{L+1}{L} r\right)^+. \tag{44}$$

Comparing (37) and (44), we can see that the diversity gain achieved by a multiple access MIMO channel with channel matrix $\mathbf{H}_m$ $(m > 1)$is always larger than that for $\mathbf{H}_1$.

Now we consider the product of the determinants of n matrices $\prod_{i=1}^{n} \det(\mathbf{I} + \frac{1}{2}\Sigma_{S_{m_i}}\Sigma_n^{-1})$, which is related to all other rate constraints from (6) - (8). Using (39), it is easy to obtain

$$\prod_{i=1}^{n}\det\left(\mathbf{I} + \frac{1}{2}\Sigma_{S_{m_i}}\Sigma_n^{-1}\right) \dot{\geq} \rho^{\max\{(\sum_{i=1}^{n} m_i)(1-v_0)^+, (\sum_{i=1}^{n} m_i)(1-v)^+\}}$$

Define

$$\rho^{f_n(v_0,v_1,v_2)} \doteq \prod_{i=1}^{n}\det(\mathbf{I} + \frac{1}{2}\Sigma_{S_{m_i}}\Sigma_n^{-1})$$

and

$$\rho^{g_n(v_0,v_1,v_2)} \doteq \rho^{\max\{(\sum_{i=1}^{n} m_i)(1-v_0)^+, (\sum_{i=1}^{n} m_i)(1-v)^+\}}.$$

It can be seen that

$$f_n(v_0,v_1,v_2) \dot{\geq} g_n(v_0,v_1,v_2), \qquad \forall (v_0,v_1,v_2) \in R^{3+}. \tag{45}$$

Similarly, applying

$$O_{g_n}^+ = \{(v_0,v_1,v_2) \in R^{3+} | g(v_0,v_1,v_2) \leq \frac{L+1}{L}(\sum_{i=1}^{n} m_i) r\},$$

we have that

$$\inf_{v_0,v_1,v_2 \in O_{f_n}^+} (v_0+v_1+v_2) \geq 2\left(1-\frac{L+1}{L} r\right)^+.$$

The determinant of the matrix $(\mathbf{I} + \frac{1}{2}\Sigma_S\Sigma_n^{-1})$ can always be decomposed into the product of the determinants of several submatrices $(\mathbf{I} + \frac{1}{2}\Sigma_{S_{mi}}\Sigma_n^{-1})$. Therefore the error exponent is always larger than or equal to $2(1-\frac{L+1}{L} r)^+$ and the proof is complete.

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Yijia (Richard) Fan received his BEng degree in electrical engineering from Shanghai Jiao Tong University (SJTU), Shanghai, P.R. China, in July 2003, and PhD degree from the Institute for Digital Communications, University of Edinburgh, March, 2007. His PhD project was fully funded by Engineering and Physical Sciences Research Council (EPSRC), UK. He is currently a postdoctoral research associate in Department of Electrical Engineering, Princeton University. His research interests include signal processing and information theory and their applications in future wireless networks.

Chao Wang (S' 07) received the B.E. degree from University of Science and Technology of China, Hefei, China, in 2003 and the MSc degree (with distinction) from The University of Edinburgh, Edinburgh, UK, in 2005. He is currently a Ph.D. candidate at The University of Edinburgh and participates in the Delivery Efficiency Core Research Programme of the Virtual Centre of Excellence in Mobile and Personal Communications.

His current research projects include Multiple Input Multiple Output (MIMO) wireless systems and cooperative communications.

John S Thompson received his BEng and PhD degrees from the University of Edinburgh in 1992 and 1996, respectively. From July 1995 to August 1999, he worked as a postdoctoral researcher at Edinburgh, funded by the UK Engineering and Physical Sciences Research Council (EPSRC) and Nortel Networks. Since September 1999, he has been a lecturer at the School of Engineering and Electronics at the University of Edinburgh. In October 2005, he was promoted to the position of reader. His research interests currently include signal processing algorithms for wireless systems, antenna array techniques and multihop wireless communications. He has published approximately 100 papers to date including a number of invited papers, book chapters and tutorial talks, as well as co-authoring an undergraduate textbook on digital signal processing. He is currently an editor-in-chief of *IEE Proceedings on Vision, Image and Signal Processing* and was the technical programme co-chair for the IEEE International Conference on Communications (ICC) 2007, which was held in Glasgow.

H. Vincent Poor (S'72, M'77, SM'82, F'77) received the Ph.D. degree in EECS from Princeton University in 1977. From 1977 until 1990, he was on the faculty of the University of Illinois at Urbana-Champaign. Since 1990 he has been on the faculty at Princeton, where he is the Michael Henry Strater University Professor of Electrical Engineering and Dean of the School of Engineering and Applied Science. Dr. Poor's research interests are in the areas of stochastic analysis, statistical signal processing and their applications in wireless networks and related fields. Among his publications in these areas is the recent book *MIMO Wireless Communications* (Cambridge University Press, 2007).

Dr. Poor is a member of the National Academy of Engineering, a Fellow of the American Academy of Arts and Sciences, and a former Guggenheim Fellow. He is also a Fellow of the Institute of Mathematical Statistics, the Optical Society of America, and other organizations. In 1990, he served as President of the IEEE Information Theory Society, and in 2004-2007 he served as the Editor-in-Chief of the *IEEE Transactions on Information Theory*. Recent recognition of his work includes the 2005 IEEE Education Medal and the 2007 IEEE Marconi Prize Paper Award.

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Recovering Multiplexing Loss through Concurrent Decode-and-Forward (DF) Relaying

Chao Wang · Yijia Fan · John S. Thompson

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Abstract In this paper, we develop a novel digital cooperative diversity transmission protocol for a two-source scenario by combining the two sources' two classic decode-and-forward (DF) relaying steps and using $2L + 1$ time slots to transmit L codewords from each source. Assuming the relays can perfectly decode their associated source messages, we give an information-theoretic average achievable capacity region for this transmission scheme. Through diversity-multiplexing tradeoff analysis, we show that our so called concurrent DF relaying protocol can effectively recover the multiplexing loss induced by the half-duplex operation in the relays, while still obtaining some diversity gain. Numerical results reveal our scheme offers significant performance advantages over the classic DF relaying protocols, especially for high signal-to-noise ratio (SNR) and large frame length L regime.

Keywords Cooperative diversity · Concurrent decode-and-forward (DF) relaying · Capacity region · Diversity-multiplexing tradeoff

1 Introduction

In the past few years, cooperative diversity protocols [1–16] have been studied intensively to improve the performance of wireless networks, where nodes help each other by relaying transmissions. In most of the prior work, relaying schemes consist of two steps: in the first step (broadcasting step), the source broadcasts its message to all the relays and the destination. The relays then process and forward their received information to the destination in the

C. Wang (✉) · Y. Fan · J. S. Thompson
Institute for Digital Communications, Joint Research Institute for Signal and Image Processing, School of Engineering and Electronics, University of Edinburgh, Edinburgh EH9 3JL, UK
e-mail: chao.wang@ed.ac.uk

Y. Fan
e-mail: y.fan@ed.ac.uk

J. S. Thompson
e-mail: john.thompson@ed.ac.uk

second step (relaying step), while the source remains silent. Due to the fact that relay terminals can not receive and transmit simultaneously, in the most popular transmission protocols (e.g. [2,3]), a time-division-multiple-access (TDMA) *half-duplex* transmission is assumed and the two steps often take separate TDMA time slots. We refer to these protocols as the *classic protocols* throughout the paper. The protocols may be used in ad hoc networks or as part of a cellular network using either fixed relays or mobile terminals.

For digital relaying, where the relays decode, re-encode and retransmit the source messages, repetition coding [2] (the relays simply repeat their received source message in turn) is often considered due to its low complexity. Diversity gain can be obtained to enhance the link reliability if the repetition-coded protocol is used. However, when compared with TDMA direct source–destination transmission, repetition-coded relaying loses spectral efficiency for the high signal-to-noise ratio (SNR) region because a single signal packet is transmitted to the destination using multiple time slots. Advanced multiple-input multiple-output (MIMO) coding techniques can also be combined with the classic protocols. For instance, the protocols proposed in [3] and [4] permit the relays to utilize distributed space-time codes and retransmit the signals simultaneously in the relaying step. Although these space-time-coded protocols can improve the network capacity over the repetition-coded protocol, they still cause spectral inefficiency for the high SNR region because two time slots are needed to transmit one signal packet. From the diversity-multiplexing tradeoff [17] point of view, when the SNR approaches infinity, the repetition-coded and space-time-coded classic protocols obtain smaller maximum multiplexing gain than TDMA direct source–destination transmission. We refer to this multiplexing gain reduction as *multiplexing loss* and thus we say the classic protocols induce multiplexing loss due to their inefficient use of the degrees of freedom of the channel.

To recover the multiplexing loss, some advanced transmission protocols have been developed. For example, a protocol proposed in [5], which is designed for single–relay networks, permits the source to transmit a different message during the second time slot. Although multiplexing loss is recovered because of the continuous transmission of the source, diversity gain is lost since the source's transmission in time slot two is not relayed to the destination. In a dynamic decode and forward protocol [6], the source messages are re-encoded by the relays using new independent Gaussian codebooks before they are forwarded to the destination. This scheme has much better performance than the classic approaches, but it is extremely difficult to realize in reality.

Most of the current spectrally efficient digital relaying protocols focus on single-source networks. Only a few papers have considered cooperative frameworks with multiple sources. For example, in the two-way relaying protocol presented in [7], a communication link between two terminals, which intend to transmit signals to each other, is established via a single relay terminal. Reference [8] studies a multiaccess relay channel, where multiple sources communicate with one common destination with the help of one single relay.

In this paper, we propose a novel decode-and-forward (DF) relaying transmission protocol to recover the multiplexing loss for a two-source scenario. Specifically, we assume a five-node network with two source terminals, two relay terminals, and one common destination terminal, which differs from those in [7] and [8]. We utilize concurrent transmission in the network by combining one source's relaying step (time slot 2) with the other source's broadcasting step (time slot 1) and let L codewords from each source be transmitted to the common destination using $2L + 1$ time slots. Assuming the relays can always correctly decode their associated source messages, we show that the average achievable capacity region of our so called concurrent DF relaying protocol is equivalent to that of a $2L$-user multiple access single-input multiple-output (SIMO) channel. The capacity scaling factor is improved from

1/4 for the space-time-coded classic protocol (1/6 for the repetition-coded classic protocol) with two relays to $L/(2L+1)$. When L is chosen as a large value, the capacity scaling factor approaches 1/2, which matches the result for TDMA direct source–destination transmission. Through diversity-multiplexing tradeoff analysis, we also show that, for the high SNR regime, the multiplexing loss induced by the classic DF relaying protocols can be effectively recovered while some diversity gain can still be obtained, which makes relaying more beneficial.

The impact of applying concurrent transmission on the throughput of multihop systems has been investigated in [18]. It is shown that further capacity gains over classic point to point transmission can be obtained by permitting concurrent relaying in multihop networks. However, [18] mainly focuses on the downstream capacity enhancement and does not offer insight on the achievable diversity-multiplexing tradeoff for such relaying methods. Combining the two steps of the classic amplify-and-forward (AF) relaying transmission protocol for a single-source scenario has been analyzed in [7] and [10]. When DF relaying is used, Ref. [7] analyzes the capacity for such schemes when the direct link between the source and the destination is ignored. The successive relaying protocol proposed in [16] considers the impact of the direct link and offers the achievable capacity and diversity-multiplexing tradeoff for a slow fading environment.

Our work is mainly based on the successive relaying protocol. The major difference is that the successive relaying protocol considers a four-node network, where two relays take turns helping the transmission of a single source to its destination, while our concurrent DF relaying protocol assumes that there are two sources in the network and each of them is individually served by one relay. Conditioned on the source messages being correctly decoded by the relays, the two-source network mimics a multiple access SIMO channel. As analyzed in [19] and [20], for a multiple access channel, the achievable sum capacity has to be further constrained besides the rate limitation for each source. Therefore, we have different achievable transmission data rate expressions from those in [16]. The other difference is that we assume a fast fading environment while a slow fading scenario is considered in [16]. This difference leads to different channel matrix structures and different protocol performance analysis methods, especially for that of the diversity-multiplexing tradeoff. In fact, the fast fading environment discussed in this paper contains the slow fading environment as a special case, i.e. the same channel matrix might be obtained if all the channel fading coefficients are assumed to be static for the whole transmission of $2L+1$ time slots. Hence, as explained later, the achievable capacity region and diversity-multiplexing tradeoff expressions derived in this paper hold for both fast fading and slow fading environments.

In this paper, we use boldface lowercase letters $\mathbf{x}$ to denote vectors, boldface capital letters $\mathbf{X}$ to denote matrices. $\det(\mathbf{X})$ and $\mathbf{X}^H$ means the matrix determinant and conjugate transpose, respectively. $\mathbf{I}$ denotes the identity matrix. $(x)^+$ means $\max\{0, x\}$. $\log(\cdot)$ denotes the base-2 logarithm. We use $\mathfrak{R}^N$ and $\mathfrak{R}^{N+}$ to express the set of real and nonnegative real N-tuples, respectively. The set O^C denotes the complement set of $O \subseteq \mathfrak{R}^N$, and O^+ means $O \cap \mathfrak{R}^{N+}$. ρ indicates the average receive SNR.

The rest of the paper is organized as follows. In Sect. 2, we review the diversity and multiplexing behavior of the classic DF relaying protocols and highlight their deficiencies compared with direct transmission. The concurrent DF relaying protocol and its related average achievable capacity region calculation are described in Sect. 3. In Sect. 4, we give a detailed analysis of the achievable diversity-multiplexing tradeoff for the concurrent DF relaying protocol. Simulation results are presented in Sect. 5 to illustrate the capacity region improvement over the classic DF relaying protocols. Finally, we offer discussion and conclusions in Sects. 6 and 7, respectively.

2 Classic Protocols

We assume a five single-antenna node network with two source terminals (denoted as S_1 and S_2), two *half-duplex* DF relay terminals (denoted as R_1 and R_2), and one common destination terminal (denoted as D). The transmitted messages from each source are divided into different frames, each containing L codewords denoted as s_i^j, $i = 1, 2$, $j = 1, \ldots, L$. The two sources use two independent Gaussian random codebooks, which are known by both relays. Each signal codeword s_i^j is independently chosen from the associated Gaussian random codebook. We also assume a fast, flat, block Rayleigh fading environment, where the channel remains static for each codeword's transmission but changes independently for the next codeword's transmission, which differs from the slow fading assumption in [16]. The unit power channel fading coefficient between nodes a and b is denoted as $h_{a,b}$ and is modeled as an independent and identically distributed (i.i.d.) complex Gaussian random variable with zero-mean and unit variance. Only the receiver of each link has the channel knowledge and there is no feedback from the receiver to the transmit terminal. We assume perfect synchronization and no power allocation scheme is applied, which means each transmit terminal transmits with equal power. Moreover, there is no cooperation between the two source terminals.

For conventional TDMA direct transmission, as displayed in Fig. 1a, the time-division channel is allocated to the two source terminals. The L codewords from each source are transmitted to the destination during $2L$ time slots. The average achievable transmission rate per time slot for each source then can be written as

$$C_1^D \leq \frac{1}{2L} \sum_{j=1}^{L} \log(1 + \rho |h_{S_1,D}^j|^2) \tag{1}$$

$$C_2^D \leq \frac{1}{2L} \sum_{j=1}^{L} \log(1 + \rho |h_{S_2,D}^j|^2) \tag{2}$$

where the superscript j denotes the transmission of the jth source codeword to the destination.

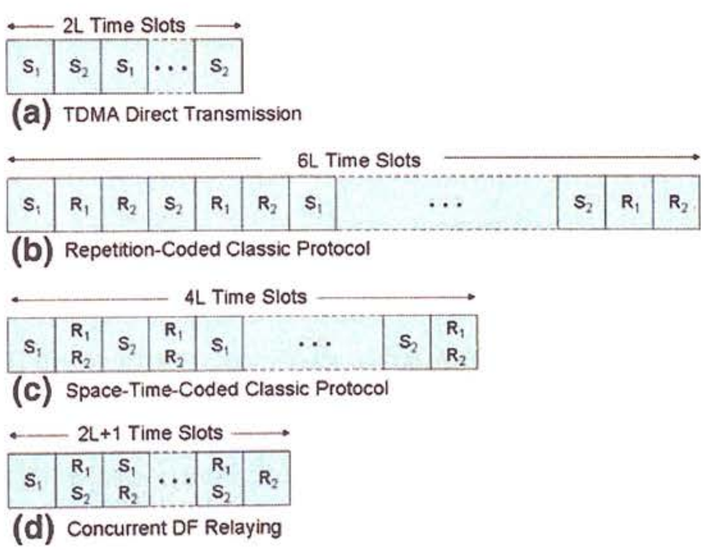

Fig. 1 Time-division channel allocations for different protocols. The terminals displayed in each time slot denote the transmitters during that time slot

From the diversity-multiplexing tradeoff point of view, in the high SNR regime, diversity gain d measures the rate at which the average error probability $P_E(\rho)$ decays and multiplexing gain r measures the rate at which the transmission data rate $R(\rho)$ scales with respect to $\log \rho$. Specifically, d and r are defined as follows respectively [17],

$$d = -\lim_{\rho\to\infty} \frac{\log(P_E(\rho))}{\log \rho} \tag{3}$$

$$r = \lim_{\rho\to\infty} \frac{\log(R(\rho))}{\log \rho} \tag{4}$$

We assume the system is *symmetric* [21], where the two source terminals have identical multiplexing gains r. The diversity-multiplexing tradeoff achieved by each source can be expressed by

$$d^D(r) = (1-2r)^+. \tag{5}$$

To fully exploit the advantage of the two relays, we utilize the classic DF relaying protocols designed for multiple relays. The first classic protocol is proposed in [3]. For this repetition-coded protocol, due to the half-duplex operation in the relays, each codeword's transmission is divided into three time slots. During the first time slot, the source broadcasts message to the two relays and the destination. During the next two time slots, the two relays decode, re-encode, and retransmit the message to the destination in turn by repetition coding. The destination combines the messages received from all the three time slots and performs decoding to recover the transmitted information. Therefore, $6L$ time slots are used to transmit the $2L$ codewords from the two sources. The time-division channel allocation for the repetition-coded classic protocol is illustrated in Fig. 1b.

One major defect of the classic DF relaying protocol is that requiring the relays to fully decode their source information makes the quality of the source–relay channels an important factor that may constrain the system capacity. It has been shown that the classic DF relaying protocol cannot provide diversity gain and the outage error performance can be even worse than TDMA direct transmission due to severe error propagation [2]. However, if we assume the source–relay channels are sufficiently good such that the relay terminals can always successfully decode the source codewords, the system capacity will not be limited by the source–relay links. In this case, following [3], the average achievable transmission rate per time slot for the repetition-coded protocol can be calculated as

$$C_1^{\mathrm{DF}_R} \le \frac{1}{6L} \sum_{j_1=1}^{L} \log(1+\rho|h_{S_1,D}^{j_1}|^2 + \rho|h_{R_1,D}^{j_1}|^2 + \rho|h_{R_2,D}^{j_1}|^2) \tag{6}$$

$$C_2^{\mathrm{DF}_R} \le \frac{1}{6L} \sum_{j_2=1}^{L} \log(1+\rho|h_{S_2,D}^{j_2}|^2 + \rho|h_{R_1,D}^{j_2}|^2 + \rho|h_{R_2,D}^{j_2}|^2) \tag{7}$$

where j_i $(i = 1, 2)$ denotes the transmission of the j_ith codeword for source S_i.

Although third order diversity can be obtained by the repetition-coded classic protocol, compared with the capacity scaling factor 1/2 for direct transmission, the scaling factor 1/6 indicates the capacity reduction for the repetition-coded protocol is nontrivial. When the SNR approaches infinity, the diversity-multiplexing tradeoff achieved by each source is expressed by

$$d^{\mathrm{DF}_R}(r) = 3(1-6r)^+. \tag{8}$$

To avoid two orthogonal channels (i.e. two time slots) being allocated to the relays, for the space-time-coded protocol presented in [4], the two relay terminals utilize distributed space-time codes and retransmit simultaneously during the second time slot (relaying step), which is displayed in Fig. 1c. The destination employs maximum ratio combining to combine the signals received from both direct and relay links in order to detect and decode them. Conditioned on that the relays decode the source messages without any error, the average achievable transmission rate per time slot for each source can be calculated as

$$C_1^{\mathrm{DF}s} \leq \frac{1}{4L} \sum_{j_1=1}^{L} \log(1 + \rho|h_{S_1,D}^{j_1}|^2 + \rho|h_{R_1,D}^{j_1}|^2 + \rho|h_{R_2,D}^{j_1}|^2) \tag{9}$$

$$C_2^{\mathrm{DF}s} \leq \frac{1}{4L} \sum_{j_2=1}^{L} \log(1 + \rho|h_{S_2,D}^{j_2}|^2 + \rho|h_{R_1,D}^{j_2}|^2 + \rho|h_{R_2,D}^{j_2}|^2). \tag{10}$$

The diversity-multiplexing tradeoff achieved by each source is expressed as

$$d^{\mathrm{DF}s}(r) = 3(1 - 4r)^+. \tag{11}$$

It is clear (9) and (10) outperform (6) and (7), respectively. The space-time-coded protocol obtains the same maximum diversity gain but higher multiplexing gain than the repetition-coded protocol since it compensates the capacity scaling factor to 1/4. However, comparing (11) with (5), we can see the space-time-coded protocol still suffers from severe multiplexing loss.

3 Concurrent DF Relaying Protocol

3.1 Protocol Design

To overcome the multiplexing limitation of the classic DF relaying protocols, we develop an improved protocol based on the successive relaying proposed in [16] but for the two-source scenario. Instead of using both relays to help each source's transmission, we require S_1 and S_2 to be served by R_1 and R_2 (using repetition coding) respectively. We combine one source's relaying step (time slot 2) with the other source's broadcasting step (time slot 1) and let one source and one relay communicate with the common destination simultaneously (except the first and the last time slots) until the $2L$ codewords are finished transmitting during $2L + 1$ time slots. The specific steps divided by each transmission (reception) time slot for the transmission of the two frames are described as follows:

Time slot 1: S_1 broadcasts s_1^1 to both R_1 and D; S_2 and R_2 remain silent.

Time slot 2: R_1 forwards s_1^1 to D and S_2 transmits s_2^1. R_2 listens to S_2 while being interfered by s_1^1 from R_1. D receives s_1^1 from R_1 and s_2^1 from S_2.

Time slot 3: R_2 forwards s_2^1 to D and S_1 transmits s_1^2. R_1 listens to S_1 while being interfered by s_2^1 from R_2. D receives s_2^1 from R_2 and s_1^2 from S_1. This process repeats until the $(2L)$th time slot.

Time slot $2L + 1$: R_2 decodes, re-encodes and retransmits s_2^L, the last codeword from S_2, to D.

After all the $2L$ codewords are received via both direct and relay links, D performs joint decoding and tries to recover the information transmitted by the two sources. The time-division channel allocation and the transmission schedule for this protocol are illustrated in

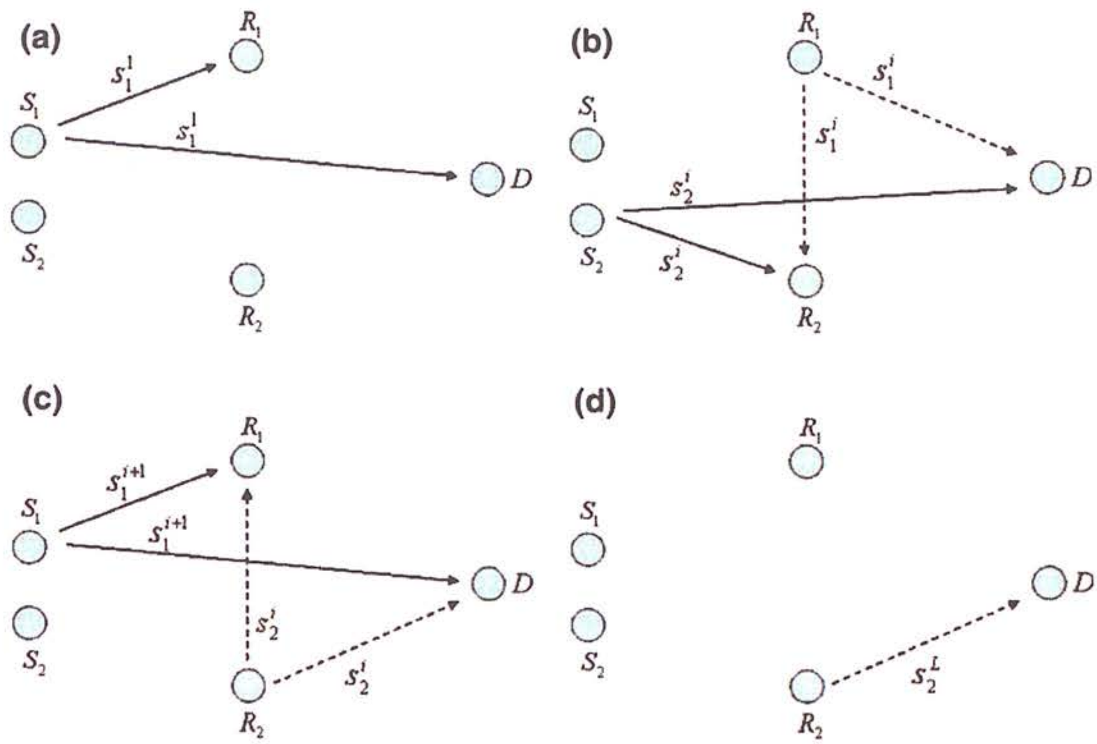

Fig. 2 Transmission schedule for the concurrent DF relaying protocol in (**a**) time slot 1, (**b**) time slot $2i$, $i = 1, \dots, L$, (**c**) time slot $2i + 1$, $i = 1, \dots, L - 1$, and (**d**) time slot $2L + 1$. Solid lines and dashed lines denote the broadcasting step (time slot 1) and relaying step (time slot 2) of each source's classic DF relaying process, respectively

Figs. 1d and 2, respectively. This transmission process allows concurrent transmission [18] among the cooperation network and is similar to letting two sources simultaneously communicate with the common destination by alternately applying the repetition-coded classic protocol, hence we name this protocol *concurrent DF relaying*.

The major issue of the concurrent DF relaying protocol is the interference generated among the relays when one relay is listening to its associated source, while the other relay is forwarding its source message to the destination. This situation mimics a Gaussian interference channel [19] with S_2 and R_1 (S_1 and R_2) acting as two senders which intend to communicate with two receivers R_2 and D (R_1 and D) respectively, in even (odd) time slots. We consider a simple decoding criterion for the relays to suppress the interference [16]: if the interference between relays is stronger than the desired signal, the relay decodes the interference signal and subtracts it from the received signal before decoding the desired signal. Otherwise, the relay decodes the desired signal directly while treating the interference as Gaussian noise.

3.2 Capacity Analysis

The reliable transmission rate region for the concurrent DF relaying protocol depends on different source–relay, relay–relay, source–destination, and relay–destination channel conditions. The detailed calculation of the achievable capacity region follows that in [16]. However, in this paper, we mainly focus on the case where the relays are assumed to be able to always perfectly decode their associated source messages. In order to reach this situation, there are two difficulties need to be solved: interference cancelation and source–relay transmission.

3.2.1 Interference Free Transmission

It is obvious that the interference between the relays is the major factor that can significantly degrade the network capacity performance. However, it has been shown in [19] that for a Gaussian interference network, if the interference is sufficiently strong, the network can perform the same as an interference free network. Specifically, following the analysis in [16], if the interference between relays is sufficiently large such that the following inequality holds

$$\log\left(1+\frac{\rho|h^k_{R_i,R_j}|^2}{1+\rho|h^k_{S_j,R_j}|^2}\right)$$
$$\geq \min\left\{\log(1+\rho|h^k_{S_i,R_i}|^2),\log(1+\rho|h^k_{S_i,D}|^2+\rho|h^k_{R_i,D}|^2)\right\} \tag{12}$$

where $i, j = 1, 2,\ i \neq j,\ k = 1, \ldots, L$, the network capacity region will no longer be affected by the inter-relay interference. The left hand side (LHS) term of (12) denotes the rate constraint for the case where the relays decode their desired signals after decoding and subtracting the interference signals from the received signals. The first term in min{ } represents the maximum rate at which the relays can reliably decode their source messages, while the second term represents the maximum rate at which the destination can correctly decode the source messages using the received signals from both direct and relay links.

3.2.2 Good Source–Relay Channel

Similar to the classic DF relaying protocols, the quality of the source–relay links (i.e. $h_{S_i,D}$) may also limit the network capacity performance. However, conditioned on (12), if the source–relay links have sufficiently high SNR such that the following inequality is met [16]

$$\log(1+\rho|h^k_{S_i,R_i}|^2)\geq\log(1+\rho|h^k_{S_i,D}|^2+\rho|h^k_{R_i,D}|^2) \tag{13}$$

where $i = 1, 2,\ k = 1, \ldots, L$, they are never the limiting factors on the system capacity.

3.2.3 Achievable Capacity Region

If the assumption of both interference free transmission (i.e. condition (12)) and sufficiently good source–relay links (i.e. condition (13)) can be satisfied and the source messages are assumed to be successfully decoded and forwarded by the relays to the destination for all the $2L + 1$ time slots, our concurrent DF relaying protocol mimics a $2L$-user multiple access SIMO channel. The associated input–output channel relation for the relay network can be written as,

$$\mathbf{y}=\begin{bmatrix} h^1_{S_1,D} & 0 & 0 & \cdots 0 & 0 \\ h^1_{R_1,D} & h^1_{S_2,D} & 0 & \cdots 0 & 0 \\ 0 & h^1_{R_2,D} & h^2_{S_1,D} & \cdots 0 & 0 \\ \vdots & \vdots & \vdots & \ddots \vdots & \vdots \\ 0 & 0 & 0 & \cdots h^L_{R_1,D} & h^L_{S_2,D} \\ 0 & 0 & 0 & \cdots 0 & h^L_{R_2,D} \end{bmatrix}\begin{bmatrix} s^1_1 \\ s^1_2 \\ s^2_1 \\ \vdots \\ s^L_1 \\ s^L_2 \end{bmatrix}+\mathbf{n} \tag{14}$$

where $\mathbf{y}$ is the $(2L+1)\times 1$ receive signal vector and $\mathbf{n}$ is the $(2L+1)\times 1$ complex circular additive white Gaussian noise (AWGN) vector at the destination. Unlike conventional multiple access SIMO channels, the dimensions of the channel matrix, the input signal, output signal, and noise vectors are expanded in the time domain rather than the space domain. Following the capacity calculation for multiple access MIMO channels in [20], there are $(2^{2L} - 1)$ constraints for the source transmission rates for a given realization of the channel, which can be expressed as,

$$R_1^j \le \log\left(\det\left(\mathbf{I} + \rho \mathbf{h}_{2j-1}\mathbf{h}_{2j-1}^H\right)\right), \quad j = 1, \ldots, L \tag{15}$$

$$R_2^j \le \log\left(\det\left(\mathbf{I} + \rho \mathbf{h}_{2j}\mathbf{h}_{2j}^H\right)\right), \quad j = 1, \ldots, L \tag{16}$$

$$R_1^i + R_1^j \le \log\left(\det\left(\mathbf{I} + \rho \mathbf{h}_{2i-1}\mathbf{h}_{2i-1}^H + \rho \mathbf{h}_{2j-1}\mathbf{h}_{2j-1}^H\right)\right), \quad i, j = 1, \ldots, L, \quad i \ne j \tag{17}$$

$$R_2^i + R_2^j \le \log\left(\det\left(\mathbf{I} + \rho \mathbf{h}_{2i}\mathbf{h}_{2i}^H + \rho \mathbf{h}_{2j}\mathbf{h}_{2j}^H\right)\right), \quad i, j = 1, \ldots, L, \quad i \ne j \tag{18}$$

$$R_1^i + R_2^j \le \log\left(\det\left(\mathbf{I} + \rho \mathbf{h}_{2i-1}\mathbf{h}_{2i-1}^H + \rho \mathbf{h}_{2j}\mathbf{h}_{2j}^H\right)\right), \quad i, j = 1, \ldots, L \tag{19}$$

$$\cdots\cdots$$

$$\sum_{i=1}^{L} R_1^i \le \log\left(\det\left(\mathbf{I} + \rho \mathbf{H}_1\mathbf{H}_1^H\right)\right) \tag{20}$$

$$\sum_{j=1}^{L} R_2^j \le \log\left(\det\left(\mathbf{I} + \rho \mathbf{H}_2\mathbf{H}_2^H\right)\right) \tag{21}$$

$$\sum_{i=1}^{L} R_1^i + \sum_{j=1}^{L} R_2^j \le \log\left(\det\left(\mathbf{I} + \rho \mathbf{H}\mathbf{H}^H\right)\right) \tag{22}$$

where

$$\mathbf{H} = \begin{bmatrix} h_{S_1,D}^1 & 0 & 0 & \cdots & 0 & 0 \\ h_{R_1,D}^1 & h_{S_2,D}^1 & 0 & \cdots & 0 & 0 \\ 0 & h_{R_2,D}^1 & h_{S_1,D}^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & h_{R_1,D}^L & h_{S_2,D}^L \\ 0 & 0 & 0 & \cdots & 0 & h_{R_2,D}^L \end{bmatrix}$$

$$\mathbf{H}_1 = \begin{bmatrix} h_{S_1,D}^1 & 0 & \cdots & 0 \\ h_{R_1,D}^1 & 0 & \cdots & 0 \\ 0 & h_{S_1,D}^2 & \cdots & 0 \\ 0 & h_{R_1,D}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{S_1,D}^L \\ 0 & 0 & \cdots & h_{R_1,D}^L \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \quad \mathbf{H}_2 = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ h_{S_2,D}^1 & 0 & \cdots & 0 \\ h_{R_2,D}^1 & 0 & \cdots & 0 \\ 0 & h_{S_2,D}^2 & \cdots & 0 \\ 0 & h_{R_2,D}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{S_2,D}^L \\ 0 & 0 & \cdots & h_{R_2,D}^L \end{bmatrix}$$

$\mathbf{h}_j$ denotes the jth column of $\mathbf{H}$, and R_i^j denotes the achievable transmission data rate for the jth codeword of source S_i.

It is extremely complicated to give an exact description for the achievable rate of each codeword when $L \geq 2$. We only concentrate on inequalities (20–22) and give the upper bounds of the average achievable transmission rate per time slot for each source as

$$C_1^{CDF} \leq \frac{1}{2L+1} \log(\det(\mathbf{I} + \rho \mathbf{H}_1 \mathbf{H}_1^H)) \tag{23}$$

$$C_2^{CDF} \leq \frac{1}{2L+1} \log(\det(\mathbf{I} + \rho \mathbf{H}_2 \mathbf{H}_2^H)) \tag{24}$$

$$C_1^{CDF} + C_2^{CDF} \leq \frac{1}{2L+1} \log(\det(\mathbf{I} + \rho \mathbf{H} \mathbf{H}^H)). \tag{25}$$

Since $2L + 1$ time slots are used to complete the transmission of L codewords from each source, our concurrent DF relaying protocol has a capacity scaling factor $L/(2L + 1)$, which is obviously larger than the scaling factors 1/6 in (6) and (7) and 1/4 in (9) and (10) for the classic DF relaying protocols. As will be shown through simulations in Sect. 5, the larger scaling factor leads to better capacity performance than the classic protocols for both low SNR and high SNR regimes. When L is increased to a large value, the capacity scaling factor $L/(2L + 1)$ approaches 1/2. Therefore, we expect that, for a symmetric system in the high SNR regime, our concurrent DF relaying protocol can achieve maximum multiplexing gain $L/(2L + 1)$ for each user. With large frame length L, the maximum multiplexing gain can approach 1/2, which is achieved by direct transmission. The multiplexing loss is thus fully recovered. Unlike the protocol presented in [5], since each codeword is transmitted via two independent paths, we also expect that the maximum diversity gain achieved by this protocol is 2.

The above analysis is based on a fast fading environment assumption. However, the results are also suitable for the slow fading environment, where the channel remains static for the whole transmission of $2L + 1$ time slots. In fact, the fast fading assumption contains slow fading as a special case, i.e. the equivalent multiple access SIMO channel matrix for the slow fading scenario can be obtained if we simply assume $h_{S_i,D}^1 = h_{S_i,D}^j$, $h_{R_i,D}^1 = h_{R_i,D}^j$, $i = 1, 2, \forall j = 2, \ldots, L$. Consequently, the transmission rate constraints (15–22) and the calculation for the average achievable transmission rate per time slot (23–25) can also be applied.

4 Diversity-Multiplexing Tradeoff Analysis

As mentioned in the previous section, we expect that in the very high SNR regime, our concurrent DF relaying protocol can improve the maximum multiplexing gain over the space-time-coded (repetition-coded) classic DF relaying protocol from 1/4 (1/6) to $L/(2L+1)$ and obtain the maximum diversity gain 2. But is this expectation feasible? The answer is positive. In this section we analyze the character of the achievable diversity-multiplexing tradeoff for the concurrent DF relaying protocol for both fast fading and slow fading environments and summarize the results as follows:

Theorem 1 *In a symmetric scenario, conditioned on* (12) *and* (13)*, the diversity-multiplexing tradeoff achieved by each source for the concurrent DF relaying protocol, for both fast fading and slow fading environments, can be expressed by*

$$d^{\text{CDF}}(r) = 2\left(1 - \frac{2L+1}{L} r\right)^+. \tag{26}$$

Proof See Appendix. □

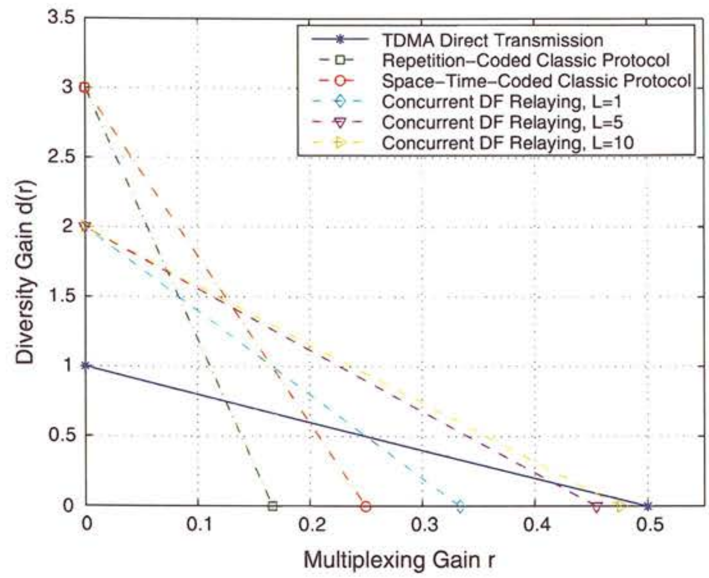

Fig. 3 Diversity-multiplexing tradeoff comparison

Figure 3 illustrates the diversity-multiplexing tradeoff comparison. It is clear that when SNR approaches infinity and L is large, our concurrent DF relaying protocol can effectively compensate the multiplexing loss of classic DF relaying protocols. This will offer a significant advantage in terms of spectral efficiency. Since diversity gain can still be obtained, the concurrent DF relaying protocol makes relaying more beneficial.

5 Simulation Results

In this section, we compare our concurrent DF relaying protocol with TDMA direct source–destination transmission and the classic DF relaying protocols in terms of achievable transmission rate. Since the space-time-coded protocol outperforms the repetition-coded protocol, we only display the former. To further reveal the advantage of multihop relaying over conventional point to point transmission, the channel fading coefficient $h_{a,b}$ in the simulations captures the effects of path-loss, lognormal shadowing, and frequency nonselective fading and is modeled as

$$h_{a,b} = \overline{h} \cdot \sqrt{x_{a,b}^{-\gamma} \cdot 10^{\zeta_k/10}}$$

where $\overline{h}$ is an i.i.d. circularly symmetric complex Gaussian random variable with zero mean and unit variance, $x_{a,b}$ is the distance between the nodes a and b, γ denotes the path loss exponent which is set to 4, and the lognormal shadowing term ζ_k is a random variable drawn from a normal distribution with a mean of 0 dB and a standard deviation of 8 dB. We normalize the distance between each source and the destination to 1. The relays are assumed to be located in the middle between their individual source and destination terminals.

Assuming the relays can perfectly decode their associated source messages, the average maximum transmission rate per time slot for one source (e.g. S_1) versus SNR is illustrated in Fig. 4. As explained in [2], the classic DF relaying protocol can only improve the system capacity for low SNR. For the high SNR regime, the classic protocol performs worse than direct transmission due to the multiplexing loss induced by the half-duplex operation in the

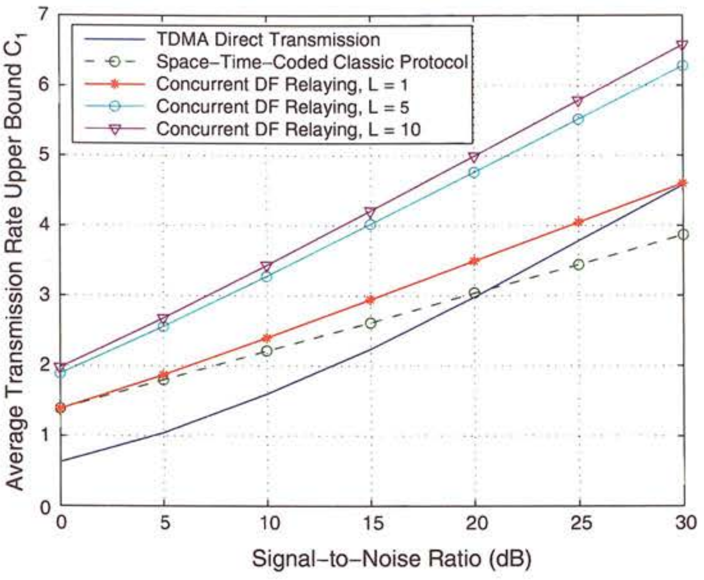

Fig. 4 Average transmission rate upper bound for S_1

relays. However, for our concurrent DF relaying protocol, the source's maximum transmission rate is improved significantly over that of the classic DF relaying protocol (thanks for the capacity scaling factor $L/(2L+1)$). With the increase of L, the maximum transmission rate can be much better than that of direct transmission, for both low SNR and high SNR regimes.

Assuming the average SNR = 25 dB, the average system capacity region, which is the outer boundary of average feasible transmission data rates for both sources, is displayed in Fig. 5. It can be seen that for this relatively high SNR regime, the space-time-coded protocol has a smaller achievable capacity region than the direct transmission without the help of the relays. The concurrent DF relaying protocol, which effectively recovers the multiplexing loss, can have better performance than direct transmission due to the fact that the use of multihop relaying can significantly enhance the system throughput [18]. The performance improves as L increases. Furthermore, since the concurrent DF relaying protocol uses two sources, two relays and one destination to mimic a multiple access SIMO system, besides the rate constraints for each source, the average achievable sum capacity of the two sources has to be further constrained (i.e. condition (25)). This is shown by the diagonal line at the top right of the concurrent DF relaying protocol capacity regions.

6 Discussion

Throughout this paper, we assume both conditions (12) and (13) are satisfied. In this situation, the achievable capacity region of our concurrent DF relaying protocol is not affected by the interference between relays and the quality of the source–relay channels. The overall system outage event is dominated by the outage event caused in the destination given that the relays perfectly decode the source messages. It should be noted that this assumption is not uncommon in reality. A practical example of the high interference scenario (i.e. condition (12)) is when the two relays are *clustered* [14], which means when the two relays are very close to each other, the received signals will have a much stronger line of sight component and the Rayleigh fading assumption is no longer valid. Then the channel between the two relays will

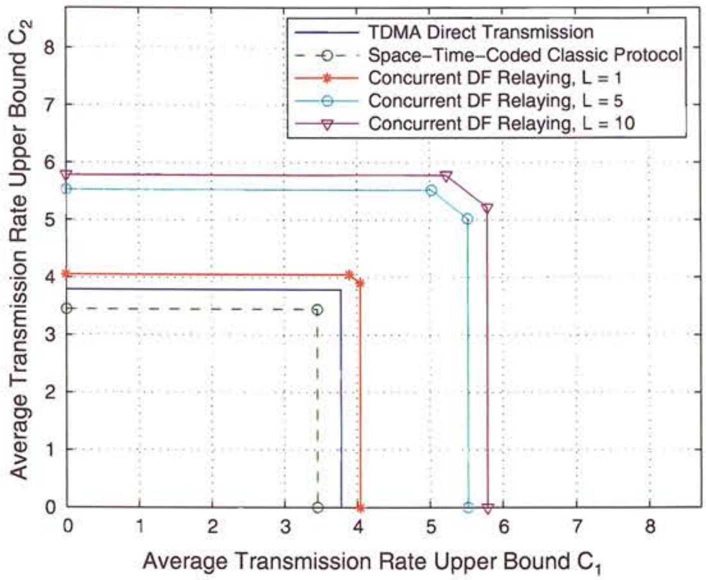

Fig. 5 Average achievable capacity region comparison

be modeled as an AWGN channel. Comparing the inter-relay link with the Rayleigh fading channels between the non-clustered nodes (i.e. source–relay links, source–destination links, and relay-destination links), the condition (12) can be easily satisfied and the good quality of the inter-relay channel will guarantee that the relays can successfully decode the interference signals with very high probability.

Furthermore, due to the fact that the quality of the source–relay links can significantly degrade the system capacity performance, the DF relaying strategy is mainly used when the source–relay links are sufficiently good. An example of this is a fixed relay network scenario [22], where the source–relay links are often assumed to be significantly better than the corresponding direct and relay-destination links. Another example is that by applying relay selection schemes (e.g. [12]) in a dense network of potential relays, one can always find the two best relays which satisfy the condition (13). On the other hand, if the relays cannot decode their source messages with very high probability, adaptive transmission protocols can be employed. In this case, the relays are used only if they can decode their source messages (e.g. [2,3]) or the direct transmission is not successful (e.g. [2,13]). Other relaying strategies such as amplify-and-forward relaying [2] or decode-amplify-forward relaying [15] may also be used to avoid network capacity degradation. The effects of applying adaptive transmission protocols or different relaying strategies on the achievable capacity region and diversity-multiplexing tradeoff are beyond the scope of this paper and thus will be left to future work. Consequently, we can conclude that (12) and (13) can be met by, for example, choosing two closely spaced relays which have sufficiently good source–relay channels.

Besides adaptive transmission protocols and other relaying strategies, there are still many open questions remaining for our concurrent DF relaying protocol which are interesting directions for future investigation. For example, since we assume relay selection schemes can be performed such that the relays always decode their associated source messages without any error, we do not consider the impact of the interference between relays and the quality of source–relay links on the system achievable capacity and diversity-multiplexing tradeoff. However, it can be conjectured, as investigated in many existing papers (e.g. [2,11]), if these two factors are taken into account, the network capacity and diversity-multiplexing tradeoff performance may behave quite differently.

In addition, since our work is based on the assumption that the relays are selected before the beginning of the transmission such that the conditions (12) and (13) hold for all the $2L+1$ time slots, the concurrent DF relaying protocol obtains second order diversity. In fact, in a large network, if relay selection schemes can be utilized during each time slot according to the related instantaneous channel conditions, extra diversity gain can be obtained [12].

We use repetition coding due to its low complexity. Although much more complicated to implement in practice, advanced parallel coding schemes can also be applied to our concurrent DF relaying protocol to further increase the bandwidth efficiency. Finally, when terminals are equipped with multiple antennas, applying MIMO techniques to our protocol will improve the network performance as well.

7 Conclusions

We have proposed a new digital cooperative diversity transmission protocol for a two-source cooperation network. Assuming the relays can perfectly decode their associated source messages, by combining the two sources' two classic DF relaying steps and simply applying repetition coding, our concurrent DF relaying protocol can significantly increase the achievable capacity region over the classic DF relaying protocols for both low SNR and high SNR regions. When the SNR approaches infinity, the concurrent DF relaying protocol can improve the achievable maximum multiplexing gain from 1/4 to $L/(2L+1)$ over the space-time-coded classic protocol and obtain the maximum diversity gain 2. Furthermore, when the frame length L is chosen as a large number, the maximum multiplexing gain will approach 1/2, which means the multiplexing loss induced by the classic DF relaying protocols is totally compensated.

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Appendix

Proof of Theorem 1 We only focus on fast fading environment in our proof. For the slow fading scenario, the identical result can be easily drawn by assuming $h^1_{S_i,D} = h^j_{S_i,D}, h^1_{R_i,D} = h^j_{R_i,D}, i = 1, 2, \forall j = 2, \ldots, L$.

The proof follows that in [6] and [17]. Firstly, we give some definitions and results which will be used in the proof.

Define v as the exponential order of $1/|g|^2$, i.e.

$$v = -\lim_{\rho\to\infty} \frac{\log|g|^2}{\log\rho}.$$

$|g|^2$ is thus said to be exponentially equal to ρ^{-v} and denoted as $|g|^2 \doteq \rho^{-v}$ ($\dot{\geq}$ and $\dot{\leq}$ are defined similarly). If g is a Gaussian random variable with zero mean and unit variance, the probability density function (pdf) of v can be shown to be

$$p_v \doteq \begin{cases} \rho^{-\infty} = 0 & \text{for } v < 0 \\ \rho^{-v} & \text{for } v \geq 0 \end{cases}.$$

For N independent identically distributed random variables $\{v_j\}_{j=1}^{N}$, if O^+ is not empty, the probability P_O that $(v_1, \ldots, v_N)$ belongs to set O only depends on O^+ and can be characterized by

$$P_O \doteq \rho^{-d_o}, \quad \text{for } d_o = \inf_{(v_1,\ldots,v_N)\in O^+} \sum_{j=1}^{N} v_j. \tag{27}$$

For a coherent linear Gaussian channel

$$\mathbf{y} = \mathbf{s} + \mathbf{n},$$

the pairwise error probability (PEP) of an ML decoder P_{PE}, averaged over the ensemble of random Gaussian codes, can be upper bounded by

$$P_{PE} \leq \det\left(\mathbf{I} + \frac{1}{2}\Sigma_{\mathbf{s}}\Sigma_{\mathbf{n}}^{-1}\right)^{-1} \tag{28}$$

where $\Sigma_{\mathbf{s}}$ and $\Sigma_{\mathbf{n}}$ denote the covariance matrices of the observed signal and noise components $\mathbf{s}$ and $\mathbf{n}$ at the receiver, respectively.

In addition, if we denote P_E as the error probability of the ML decoder and O as the outage event, which is chosen such that the outage probability $P_O(R)$ dominates P_{E,O^C}, i.e.

$$P_{E,O^C} \dot{\leq} P_O(R), \tag{29}$$

P_E can be upper bounded by

$$\begin{aligned} P_E &= P_O(R)P_{E|O} + P_{E,O^C} \\ &\leq P_O(R) + P_{E,O^C} \\ &\dot{\leq} P_O(R). \end{aligned} \tag{30}$$

For our concurrent DF relaying protocol, conditioned on (12) and (13), the overall outage event is clearly dominated by the outage event caused in the destination given the source messages perfectly decoded by their relays. As mentioned in Sect. 3, if the relays can correctly decode their source messages, our concurrent DF relaying protocol mimics a $2L$-user multiple access SIMO channel (i.e. (14)). For this multiple access SIMO channel, there are $(2^{2L} - 1)$ transmission data rate constraints for a given realization of the channel, which are expressed from (15) to (22). For each constraint there is a probability of not meeting it. The probability of outage is the highest among all these probabilities [21]. Therefore there are $(2^{2L} - 1)$ diversity-multiplexing tradeoffs for all those conditions and the lowest curve within the range of multiplexing gain is the achievable tradeoff curve for the system.

To characterize the diversity-multiplexing tradeoff achieved by each constraint, we consider an $(m+1) \times m$ MIMO channel matrix

$$\mathbf{H}_m = \begin{bmatrix} h_{s_1} & 0 & 0 & \cdots & 0 \\ h_{r_1} & h_{s_2} & 0 & \cdots & 0 \\ 0 & h_{r_2} & h_{s_3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & h_{s_m} \\ 0 & 0 & 0 & \cdots & h_{r_m} \end{bmatrix}.$$

Define v_{s_i} as the exponential order of $1/|h_{s_i}|^2$, and v_{r_i} as the exponential order of $1/|h_{r_i}|^2$, $i = 1, \ldots, m$. Let $\mathbf{M}_{m+1} = \mathbf{I} + \frac{1}{2}\Sigma_{\mathbf{s}_m}\Sigma_{\mathbf{n}}^{-1}$, where the subscript $\mathbf{s}_m$ denotes that the transmit signal vector has dimension m. It can be seen that $\mathbf{M}_{m+1}$ is a $(m+1) \times (m+1)$ tridiagonal matrix, i.e.

$$\begin{aligned} \mathbf{M}_{m+1} &= \mathbf{I} + \frac{1}{2}\Sigma_{\mathbf{s}_m}\Sigma_{\mathbf{n}}^{-1} \\ &= \begin{bmatrix} 1 + \frac{1}{2}\rho\left|h_{s_1}\right|^2 & \frac{1}{2}\rho h_{s_1} h_{r_1}^* & \cdots 0 \\ \frac{1}{2}\rho h_{r_1} h_{s_1}^* & 1 + \frac{1}{2}\rho\left(\left|h_{r_1}\right|^2 + \left|h_{s_2}\right|^2\right) & \cdots 0 \\ 0 & \frac{1}{2}\rho h_{r_2} h_{s_2}^* & \cdots 0 \\ \vdots & \vdots & \ddots \vdots \\ 0 & 0 & \cdots 1 + \frac{1}{2}\rho\left|h_{r_m}\right|^2 \end{bmatrix}. \end{aligned}$$

We assume each source codeword is chosen from its source's Gaussian random codebook of codeword length l, and is transmitted with data rate R. When $m = 1$, the upper bound of the ML conditional PEP can be calculated by

$$\begin{aligned} P_{PE|v_{s_1},v_{r_1}} &\leq \det\left(\mathbf{I} + \frac{1}{2}\Sigma_{\mathbf{s}_1}\Sigma_{\mathbf{n}}^{-1}\right)^{-l} \\ &= \left(1 + \frac{1}{2}\rho\left|h_{s_1}\right|^2 + \frac{1}{2}\rho\left|h_{r_1}\right|^2\right)^{-l} \\ &\doteq \rho^{-l(\max\{1-v_{s_1},1-v_{r_1}\})^+}. \end{aligned} \tag{31}$$

Since each source uses $2L+1$ time slots to transmit L different codewords, the average transmission rate is $\overline{R} = \frac{L}{2L+1}R$. We assume the average transmission rate changes as $\overline{R} = r\log\rho$ with respect to ρ, and it is easy to see $R = \frac{2L+1}{L}r\log\rho$. Therefore, we have a total of $\rho^{\frac{2L+1}{L}rl}$ distinct codewords that could have been sent. The error probability thus can be bounded by

$$P_{E|v_{s_1},v_{r_1}} \dot{\leq} \rho^{-l\left((\max\{1-v_{s_1},1-v_{r_1}\})^+ - \frac{2L+1}{L}r\right)}. \tag{32}$$

P_{E,O^C} is the average of $P_{E|v_{s_1},v_{r_1}}$ over the set of all possible channel realizations that do not bring outage events. It can be seen that

$$P_{E,O^C} \dot{\leq} \int_{O^{C+}} \rho^{-d_e(r,v_{s_1},v_{r_1})} dv_{s_1} dv_{r_1}$$

for $d_e(r, v_{s_1}, v_{r_1}) = l\left((\max\{1-v_{s_1}, 1-v_{r_1}\})^+ - \frac{2L+1}{L}r\right) + (v_{s_1} + v_{r_1})$.

Because P_{E,O^C} is dominated by the term corresponding to $(v_{s_1}, v_{r_1}) \in O^{C+}$ that minimizes the value of $d_e(r, v_{s_1}, v_{r_1})$, we have

$$P_{E,O^C} \dot{\leq} \rho^{-d_e(r)}$$

for $d_e(r) = \inf_{(v_{s_1}, v_{r_1}) \in O^{C+}} d_e(r, v_{s_1}, v_{r_1})$. Using (27), $P_O(R)$ can be expressed by

$$P_O \doteq \rho^{-d_o(r)} \tag{33}$$

for $d_o(r) = \inf_{(v_{s_1}, v_{r_1}) \in O^{+}} (v_{s_1} + v_{r_1})$.

Comparing P_{E,O^C} with P_O, if we want (29) to be met, the set O^+ should be defined as

$$O^+ = \left\{ (v_{s_1}, v_{r_1}) \in \Re^{2+} | (\max\{1 - v_{s_1}, 1 - v_{r_1}\})^+ \leq \frac{2L+1}{L} r \right\}. \tag{34}$$

Then, for any $(v_{s_1}, v_{r_1}) \in O^{C+}$, we can choose l to make $d_e(r, v_{s_1}, v_{r_1})$ arbitrarily large to guarantee (29). Using (33) and (34), $d_o(r)$ can be calculated as

$$d_o(r) = 2\left(1 - \frac{2L+1}{L} r\right)^+. \tag{35}$$

Because $d_o(r)$ provides a lower bound on the achievable diversity gain [17], the diversity-multiplexing gain achieved in the case $m = 1$ can be expressed as

$$d(r) = 2\left(1 - \frac{2L+1}{L} r\right)^+. \tag{36}$$

When $m \geq 2$, the analysis of the determinant of $\mathbf{M}_{m+1}$ can be conducted in a similar way to that in [10]. Specifically, define $D_k = \det(\mathbf{M}_{(k)})$, where $\mathbf{M}_{(k)}$ denotes a $k \times k$ sub-matrix formed by the first k rows and k columns from the upper left-most corner of $\mathbf{M}_{m+1}$. Applying the determinant calculation for a tridiagonal matrix in [23], we have

$$\begin{aligned} D_{k+1} &= \left(1 + \frac{1}{2}\rho|h_{s_{k+1}}|^2 + \frac{1}{2}\rho|h_{r_k}|^2\right) D_k - \frac{1}{4}\rho^2 |h_{s_k}|^2 |h_{r_k}|^2 D_{k-1} \\ &= \left(1 + \frac{1}{2}\rho|h_{s_{k+1}}|^2\right) D_k + \frac{1}{2}\rho|h_{r_k}|^2 \left(D_k - \frac{1}{2}\rho|h_{s_k}|^2 D_{k-1}\right) \\ &\qquad \text{for } k = 2, \ldots, m-1 \end{aligned} \tag{37}$$

and

$$\begin{aligned} D_{m+1} &= \left(1 + \frac{1}{2}\rho|h_{r_m}|^2\right) D_m - \frac{1}{4}\rho^2 |h_{s_m}|^2 |h_{r_m}|^2 D_{m-1} \\ &= D_m + \frac{1}{2}\rho|h_{r_m}|^2 \left(D_m - \frac{1}{2}\rho|h_{s_m}|^2 D_{m-1}\right). \end{aligned} \tag{38}$$

Define $B_k = D_k - \frac{1}{2}\rho|h_{s_k}|^2 D_{k-1}$ and we can have

$$\begin{bmatrix} D_{k+1} \\ B_{k+1} \end{bmatrix} = \begin{bmatrix} 1 + \frac{1}{2}\rho \left|h_{s_{k+1}}\right|^2 & \frac{1}{2}\rho \left|h_{r_k}\right|^2 \\ 1 & \frac{1}{2}\rho \left|h_{r_k}\right|^2 \end{bmatrix} \begin{bmatrix} D_k \\ B_k \end{bmatrix} \tag{39}$$

where $k = 2, \ldots, m-1$, and

$$D_{m+1} = D_m + \frac{1}{2}\rho|h_{r_m}|^2 B_m. \tag{40}$$

Since $D_2 = \frac{1}{2}\rho|h_{s_1}|^2 \cdot \frac{1}{2}\rho|h_{s_2}|^2 + \frac{1}{2}\rho|h_{s_1}|^2 + \frac{1}{2}\rho|h_{s_2}|^2 + (\frac{1}{2}\rho|h_{r_1}|^2 + 1)$ and $B_2 = \frac{1}{2}\rho|h_{s_1}|^2 + \frac{1}{2}\rho|h_{r_1}|^2 + 1$, from (39), it is not difficult to see D_{k+1}, as a polynomial of $(\frac{1}{2}\rho|h_{s_1}|^2, \ldots, \frac{1}{2}\rho|h_{s_k}|^2, \frac{1}{2}\rho|h_{r_1}|^2, \ldots, \frac{1}{2}\rho|h_{r_k}|^2)$, has nonnegative coefficients for any $k = 2, \ldots, m-1$ [10]. Then, as a polynomial of $(\frac{1}{2}\rho|h_{s_1}|^2, \ldots, \frac{1}{2}\rho|h_{s_k}|^2)$, coefficients of D_m can be calculated recursively using (37) as

$$\begin{aligned} &D_m\left(\frac{1}{2}\rho|h_{s_1}|^2, \ldots, \frac{1}{2}\rho|h_{s_m}|^2\right) \\ &= \prod_{k=1}^{m} \frac{1}{2}\rho|h_{s_k}|^2 + \prod_{k=1}^{m-1}\left(1 + \frac{1}{2}\rho|h_{r_k}|^2\right) + P\left(\frac{1}{2}\rho|h_{s_1}|^2, \ldots, \frac{1}{2}\rho|h_{s_m}|^2\right) \end{aligned}$$

where $P(\frac{1}{2}\rho|h_{s_1}|^2, \ldots, \frac{1}{2}\rho|h_{s_m}|^2) \geq 0$ is a polynomial of $(\frac{1}{2}\rho|h_{s_1}|^2, \ldots, \frac{1}{2}\rho|h_{s_m}|^2)$ and is always nonnegative. Thus,

$$\begin{aligned} &D_{m+1}\left(\frac{1}{2}\rho|h_{s_1}|^2, \ldots, \frac{1}{2}\rho|h_{s_m}|^2\right) \\ &= \prod_{k=1}^{m} \frac{1}{2}\rho|h_{s_k}|^2 + \prod_{k=1}^{m}\left(1 + \frac{1}{2}\rho|h_{r_k}|^2\right) + P'\left(\frac{1}{2}\rho|h_{s_1}|^2, \ldots, \frac{1}{2}\rho|h_{s_m}|^2\right) \\ &\geq \prod_{k=1}^{m} \frac{1}{2}\rho|h_{s_k}|^2 + \prod_{k=1}^{m}\left(1 + \frac{1}{2}\rho|h_{r_k}|^2\right) \end{aligned} \tag{41}$$

where $P'(\frac{1}{2}\rho|h_{s_1}|^2, \ldots, \frac{1}{2}\rho|h_{s_m}|^2) \geq 0$ is also a polynomial of $(\frac{1}{2}\rho|h_{s_1}|^2, \ldots, \frac{1}{2}\rho|h_{s_m}|^2)$ and is always nonnegative.

Define $v_S = \max\{v_{s_1}, \ldots, v_{s_m}\}$ and $v_R = \max\{v_{r_1}, \ldots, v_{r_m}\}$. From (41), it can be seen

$$\begin{aligned} D_{m+1} &= \det\left(\mathbf{I} + \frac{1}{2}\Sigma_{\mathbf{s}_m}\Sigma_{\mathbf{n}}^{-1}\right) \\ &\dot{\geq} \rho^{\max\{\sum_{i=1}^{m}(1-v_{s_i})^+, \sum_{i=1}^{m}(1-v_{r_i})^+\}} \\ &\dot{\geq} \rho^{\max\{m(1-v_S)^+, m(1-v_R)^+\}}. \end{aligned} \tag{42}$$

We define functions $f(v_{s_1}, \ldots, v_{s_m}, v_{r_1}, \ldots, v_{r_m})$ and $g(v_{s_1}, \ldots, v_{s_m}, v_{r_1}, \ldots, v_{r_m})$ such that $\det(\mathbf{I} + \frac{1}{2}\Sigma_{S_m}\Sigma_n^{-1}) \dot{=} \rho^{f(v_{s_1}, \ldots, v_{s_m}, v_{r_1}, \ldots, v_{r_m})}$ and

$$\rho^{\max\{m(1-v_S)^+, m(1-v_R)^+\}} \doteq \rho^{g(v_{s_1}, \ldots, v_{s_m}, v_{r_1}, \ldots, v_{r_m})}, \tag{43}$$

thus we have

$$\begin{aligned} f(v_{s_1}, \ldots, v_{s_m}, v_{r_1}, \ldots, v_{r_m}) &\geq g(v_{s_1}, \ldots, v_{s_m}, v_{r_1}, \ldots, v_{r_m}) \\ &\forall (v_{s_1}, \ldots, v_{s_m}, v_{r_1}, \ldots, v_{r_m}) \in \Re^{2m+}. \end{aligned} \tag{44}$$

Similar to the analysis for the case $m = 1$, O_f^+ should be defined as

$$\begin{aligned} O_f^+ = \Bigg\{ &(v_{s_1}, \ldots, v_{s_m}, v_{r_1}, \ldots, v_{r_m}) \in \Re^{2m+} | f(v_{s_1}, \ldots, v_{s_m}, v_{r_1}, \ldots, v_{r_m}) \\ &\leq \frac{2L+1}{L}mr \Bigg\} \end{aligned} \tag{45}$$

where m denotes m codewords are transmitted and the equivalent data rate $R=\frac{2L+1}{L}mr\log\rho$. We define

$$O_g^+ = \left\{(v_{s_1},\ldots,v_{s_m},v_{r_1},\ldots,v_{r_m}) \in \Re^{2m+} | g(v_{s_1},\ldots,v_{s_m},v_{r_1},\ldots,v_{r_m}) \le \frac{2L+1}{L}mr\right\}. \tag{46}$$

Because of (44), it can be drawn that

$$O_f^+ \subseteq O_g^+.$$

Therefore

$$\inf_{(v_{s_1},\ldots,v_{s_m},v_{r_1},\ldots,v_{r_m})\in O_f^+}\left(\sum_{i=1}^m v_{s_i}+\sum_{i=1}^m v_{r_i}\right) \ge \inf_{(v_{s_1},\ldots,v_{s_m},v_{r_1},\ldots,v_{r_m})\in O_g^+}\left(\sum_{i=1}^m v_{s_i}+\sum_{i=1}^m v_{r_i}\right), \tag{47}$$

which means the diversity gain calculated from O_f^+ is never smaller than that from O_g^+. From (43) and (46), it is not difficult to show that

$$\inf_{(v_{s_1},\ldots,v_{s_m},v_{r_1},\ldots,v_{r_m})\in O_g^+}\left(\sum_{i=1}^m v_{s_i}+\sum_{i=1}^m v_{r_i}\right) = 2\left(1-\frac{2L+1}{L}r\right)^+. \tag{48}$$

The right-hand-side (RHS) of (35) is identical to the RHS of (48). From (47), we can see the diversity gain achieved by a MIMO channel with channel matrix $\mathbf{H}_m$ $(m > 1)$ is always larger than or equal to that by $\mathbf{H}_1$.

Now we consider the product of the determinants of n matrices $\prod_{i=1}^n \det(\mathbf{I}+\frac{1}{2}\Sigma_{\mathbf{s}_{m_i}}\Sigma_{\mathbf{n}}^{-1})$. Using (42), it is easy to get

$$\prod_{i=1}^n \det\left(\mathbf{I}+\frac{1}{2}\Sigma_{\mathbf{s}_{m_i}}\Sigma_{\mathbf{n}}^{-1}\right) \dot{\ge} \rho^{\max\{(\sum_{i=1}^n m_i)(1-v_S)^+,(\sum_{i=1}^n m_i)(1-v_R)^+\}}.$$

Define

$$\rho^{f_n(v_{s_1}^1,\ldots,v_{s_{mn}}^n,v_{r_1}^1,\ldots,v_{r_{mn}}^n)} \doteq \prod_{i=1}^n \det\left(\mathbf{I}+\frac{1}{2}\Sigma_{\mathbf{s}_{m_i}}\Sigma_{\mathbf{n}}^{-1}\right)$$

and

$$\rho^{g_n(v_{s_1}^1,\ldots,v_{s_{mn}}^n,v_{r_1}^1,\ldots,v_{r_{mn}}^n)} \doteq \rho^{\max\{(\sum_{i=1}^n m_i)(1-v_S)^+,(\sum_{i=1}^n m_i)(1-v_R)^+\}}$$

where $v_{s_i}^j$ and $v_{r_i}^j$ denotes the exponential order of $1/|h_{s_i}|^2$ and $1/|h_{r_i}|^2$ in the jth matrix, respectively. It can be seen

$$f_n(v_{s_1}^1,\ldots,v_{s_{mn}}^n,v_{r_1}^1,\ldots,v_{r_{mn}}^n) \ge g_n(v_{s_1}^1,\ldots,v_{s_{mn}}^n,v_{r_1}^1,\ldots,v_{r_{mn}}^n) \quad \forall(v_{s_1}^1,\ldots,v_{s_{mn}}^n,v_{r_1}^1,\ldots,v_{r_{mn}}^n) \in \Re^{2(\sum_{i=1}^n m_i)+}. \tag{49}$$

Similarly, applying

$$O_{g_n}^+ = \left\{(v_{s_1}^1,\ldots,v_{s_{mn}}^n,v_{r_1}^1,\ldots,v_{r_{mn}}^n) \in R^{2(\sum_{i=1}^n m_i)+} | g(v_{s_1}^1,\ldots,v_{s_{mn}}^n,v_{r_1}^1,\ldots,v_{r_{mn}}^n) \le \frac{2L+1}{L}\left(\sum_{i=1}^n m_i\right)r\right\}, \tag{50}$$

we can have

$$\inf_{(v_{s_1}^1,\ldots,v_{s_{m_n}}^n,v_{r_1}^1,\ldots,v_{r_{m_n}}^n)\in O_{fn}^+} \sum_{i=1}^{n}\left(\sum_{j=1}^{m_i} v_{s_j}^{m_i} + \sum_{j=1}^{m_i} v_{r_j}^{m_i}\right) \geq 2\left(1-\frac{2L+1}{L}r\right)^+ .$$

Each rate constraint from (15) to (22) is equivalent to the case where the destination can perfectly decode some of the transmitted codewords and remove their influence from the received signal. Therefore, the task for the destination is to decode the remained signal codewords. The determinant of matrix $(\mathbf{I}+\frac{1}{2}\Sigma_{\mathbf{s}}\Sigma_{\mathbf{n}}^{-1})$ can always be decomposed to the product of the determinants of several submatrices $(\mathbf{I}+\frac{1}{2}\Sigma_{\mathbf{s}_{m_i}}\Sigma_{\mathbf{n}}^{-1})$ so that it is always larger than or equal to the RHS of (35).

From all the above analysis, we can see that the achievable diversity gain for our concurrent DF relaying protocol, which is the minimum diversity gain achieved by all rate constraints from (15) to (22), is expressed by (26). Thus the proof is complete.

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Author Biographies

Chao Wang received the B.E. degree from University of Science and Technology of China (USTC), Hefei, China, in 2003 and the MSc degree (with distinction) from The University of Edinburgh, Edinburgh, UK, in 2005. He is currently a Ph.D. candidate at The University of Edinburgh and participates in the Delivery Efficiency Core Research Programme of the Virtual Centre of Excellence in Mobile and Personal Communications. His current research projects include multiple-input multiple-output (MIMO) wireless systems and cooperative communications.

Yijia Fan finished his B.S. degree in Electrical Engineering from Shanghai Jiao Tong University (SJTU) in June, 2003, and PhD degree in Institute for Digital Communications, University of Edinburgh in March, 2007. His PhD project was fully funded by EPSRC, UK. He is currently working as a Postdoctoral Research Associate in Department of Electrical Engineering, Princeton University. His research interests include information theory and signal processing for future wireless systems.

John S. Thompson received his BEng and PhD degrees from the University of Edinburgh in 1992 and 1996, respectively. From July 1995 to August 1999, he worked as a postdoctoral researcher at Edinburgh, funded by the UK Engineering and Physical Sciences Research Council (EPSRC) and Nortel Networks. Since September 1999, he has been a lecturer at the School of Engineering and Electronics at the University of Edinburgh. In October 2005, he was promoted to the position of reader. His research interests currently include signal processing algorithms for wireless systems, antenna array techniques and multihop wireless communications. He has published approximately 100 papers to date including a number of invited papers, book chapters and tutorial talks, as well as co-authoring an undergraduate textbook on digital signal processing. He is currently editor-in-chief of the IET Signal Processing journal and was recently a technical programme co-chair for the IEEE International Conference on Communications (ICC) 2007, held in Glasgow in June 2007.

A Comprehensive Study of Repetition-coded Protocols in Multi-user Multi-relay Networks

Chao Wang, Yijia Fan, John S. Thompson, and H. Vincent Poor

Abstract—**The diversity-multiplexing tradeoff (DMT) performances of novel decode-and-forward (DF) relaying protocols for multi-source multi-relay cooperative networks are studied in this paper. For a strong-interference scenario and an isolated-relay scenario, the proposed protocols significantly improve diversity performance over direct source-destination transmissions by using a simple repetition coding strategy in relays. This is in addition to enhancing the multiplexing performance over standard DF protocols, which usually suffer from the half-duplex limitation at relays. Although the DMT performance of DF based relaying protocols is limited by the quality of source-relay links in principle, adaptive forms of the proposed protocols at the relays can transmit/not transmit according to source-relay transmissions and still provide link reliability and spectral efficiency advantages in general source-relay channel conditions.**

I. Introduction

The exploitation of cooperation among users has been studied in recent years as a means to improve diversity performance for single-antenna wireless systems. In a multi-user scenario, the so called *cooperative diversity* technique [1] requires users to share their individual antennas and work as relays for each other. Due to the fact that terminals cannot transmit and receive simultaneously in the same frequency band (i.e. the *half-duplex limitation*), standard relaying transmission protocols (e.g. [2] [3]) usually demand orthogonal channels (time-division-multiple-access (TDMA) time slots) for the source and then the relays communicating with the destination. Although they improve the diversity gain over direct source-destination transmission, the standard protocols lose spectral efficiency, especially in the high signal-to-noise ratio (SNR) region.

To overcome the multiplexing limitation of standard protocols, the use of independent coding (i.e. relays re-encode the source information using new independent codebooks) rather than repetition coding (i.e. each relay simply retransmits the same codeword as the source) in relays (e.g. [3] [4]) is considered as a direct extension of multiple-input multiple-output (MIMO) spatial multiplexing techniques to relay networks. However, those information theory based protocols require complex processing in relays and in particular at the destination and are very difficult to realize in reality. Therefore exploiting physical channel reuse is a more practical approach: for example, the concept of successive relaying (independently proposed by [5], [6], and [7] in different contexts). The basic idea behind successive relaying is to use two successively activated relays to mimic a full-duplex relay and provide both multiplexing and diversity improvements over single-relay standard protocols. This is in contrast to other studies where the use of multiple relays is conventionally considered only to increase diversity gain (e.g. [3] [8]). For instance, for amplify-and-forward (AF) relaying, the protocol proposed in reference [6] outperforms a nonorthogonal AF (NAF) protocol (for single-relay scenarios) studied in [4], [9]–[11] in terms of diversity-multiplexing tradeoff (DMT) [12].

For decode-and-forward (DF) relaying systems, a successive relaying protocol proposed in [7] considers a case in which a source terminal intends to transmit a frame with L codewords to its destination. Two relays take turns assisting the source and the transmission is finished using $L+1$ time slots so that all codewords are protected by relays. The single-source system is extended to a two-source network in [13] (termed concurrent DF (CDF) relaying), in which $2L+1$ time slots are used to finish the transmission of L codewords from each source. Assuming perfect decoding at relays, both protocols effectively recover the multiplexing loss induced by standard DF protocols while still providing some diversity gain. Since only repetition coding is used in relays, the two protocols allow simple processing at relays and destination.

Furthermore, when the frame length L is large, the optimal multiplexing gains (i.e. the maximal multiplexing gains obtained by direct source-destination transmissions) [14] are actually achieved. Therefore, the attention is drawn back to diversity performance. The diversity gains derived in references [7] and [13] make the assumption that the source-relay channels are sufficiently good such that the relays can always perfectly decode the source signals. However, in general the high quality of the source-relay links is not guaranteed. Thus one may ask whether it is possible that the same diversity performance can be achieved in general source-relay channel conditions. And, moreover, can even higher diversity gain be achieved without using advanced coding strategies or losing multiplexing performance? The aim of this paper is to use novel protocols to give affirmative answers to these two questions. Throughout the paper, we mainly focus on multiple-source networks; simplifying the analysis to single-source scenarios is straightforward. Note that although we use different names for different protocols, all the proposed

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C. Wang and J. S. Thompson are with the Institute for Digital Communications, The University of Edinburgh, Edinburgh, EH9 3JL, UK. (email: chao.wang@ed.ac.uk, john.thompson@ed.ac.uk)

Y. Fan and H. V. Poor are with the Department of Electrical Engineering, Princeton University, Princeton, NJ, 08544, USA. (email: yijiafan@princeton.edu, poor@princeton.edu)

protocols actually use simple repetition coding in the relays. The contributions of the paper for different protocols are as follows:

1) **Repetition-coded CDF**: We present a simple adaptive protocol for the CDF proposed in [13] (termed repetition-coded CDF) in general source-relay channel conditions. The adaptive protocol can achieve the same DMT performance as that under the assumption of perfect source-relay transmissions (which is expressed in (2)).
2) **Superposition-coded CDF**: For a strong-interference scenario, we make use of the inter-relay interference and permit each relay to use superposition coding to transmit both sources' codewords. The DMT performance is expressed in (5). The diversity gain of the repetition-coded CDF is further improved (in general source-relay channel conditions) with a small multiplexing gain loss, which is negligible for large L.
3) **Multiple-access CDF**: For an isolated-relay scenario in which the inter-relay interference cannot be used, we permit the two sources to transmit simultaneously. We give a lower bound (7) and an upper bound (8) of the achievable DMT (in general source-relay channel conditions) which show the diversity improvement over the repetition-coded CDF within the range of all possible multiplexing gains for large L.
4) **Multiple-antenna scenarios**: Finally, we extend the single-antenna network to a multiple-antenna (at the destination only) scenario. We present the DMT performance for both the repetition-coded CDF (expressed in (11)) and the superposition-coded CDF (expressed in (12)) under the assumption of perfect source-relay transmissions. In general source-relay channel conditions, one relay pair out of K potential relays is selected to assist the sources. It is shown that if K is larger than some specific threshold (related to the number of destination antennas), the system performs the same as the case in which the source-relay transmissions are always successful.

In this paper, we use $\mathbf{I}$ to denote the identity matrix. $\mathbf{0}$ and $\mathbf{O}$ denotes the zero vector and matrix respectively. $\log(\cdot)$ denotes the base-2 logarithm. ρ denotes SNR. $\doteq$, $\dot{>}$, and $\dot{<}$ indicate the exponentially equal, larger, and less than operations respectively [12].

The rest of the paper is organized as follows. In Section II, we present a repetition-coded CDF, a superposition-coded CDF, and a multiple-access CDF for a single-antenna five-node network. The use of multiple antennas at the destination and the associated relay selection schemes for the repetition-coded and superposition-coded CDF protocols are analyzed in Section III. Finally, we provide discussion and conclusions in Sections IV and V respectively.

II. Single-Antenna Scenarios

A. System Model and Standard Protocol

A single-antenna five-node network with two sources S_1 and S_2, two *half-duplex* DF relays R_1 and R_2, and one destination

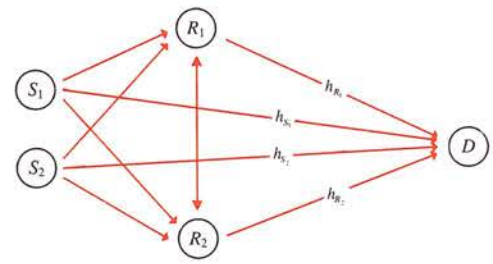

Fig. 1. A five-node network with two sources, two relays and one common destination. The solid lines denote possible transmission routes.

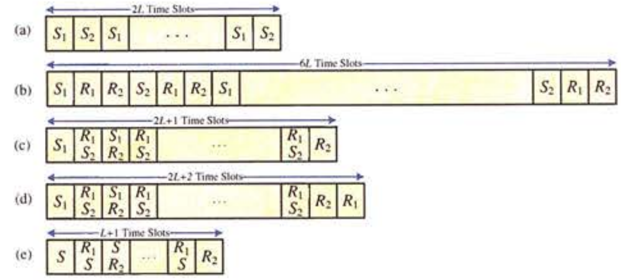

Fig. 2. Time-division channel allocations for (a) TDMA direct transmission, (b) repetition-coded standard DF relaying, (c) repetition-coded concurrent DF relaying, (d) superposition-coded concurrent DF relaying, (e) multiple-access concurrent DF relaying (L is even, and S means the two sources S_1 and S_2 transmit simultaneously). The terminals displayed in each time slot denote the transmitters in that time slot.

D is studied, as displayed in Fig. 1. The transmitted messages from each source are divided into frames, each containing L codewords denoted as x_i^j, $i = 1, 2$, $j = 1, \ldots, L$. The two sources use two independent Gaussian random codebooks, which are known by both relays. Each codeword x_i^j is *independently* chosen from the associated Gaussian random codebook. It is assumed that no cooperation exists between the two sources.

A slow, flat, block Rayleigh fading environment is assumed, where the channel remains static for a coherence interval (two frame periods) and changes independently in different coherence intervals. We assume perfect channel knowledge at the receiver of each link. Moreover, it is assumed that each terminal transmits with equal power.

We define diversity gain d and multiplexing gain r as follows [12],

$$d = -\lim_{\rho\to\infty} \frac{\log(P_E(\rho))}{\log \rho} \quad and \quad r = \lim_{\rho\to\infty} \frac{R(\rho)}{\log \rho}, \qquad (1)$$

where $P_E(\rho)$ and $R(\rho)$ denote the error probability and transmission rate at bits per channel use (BPCU). It is assumed that the system is *symmetric* [15] and each source codeword is transmitted with multiplexing gain $r' = \frac{T}{L}r$, where T denotes the time used to finish the transmission of the $2L$ codewords from the two sources (i.e. two frames), so that the average transmission rate for each source is $\bar{R} \doteq \frac{L}{T} r' \log \rho = r \log \rho$.

For such a two-source network, in conventional TDMA direct source-destination transmission, the time-division channel is allocated to the two sources as displayed in Fig. 2 (a). The achievable DMT for each source can be expressed by

$d(r) = 1 - r' = 1 - 2r$.

Due to the half-duplex limitation, the repetition-coded *standard DF* protocol [3] demands three time slots for each codeword's transmission. During the first time slot, the source broadcasts the codeword to the relays and destination. Then the two relays repeat the codeword successively during the following two time slots while the source remains silent. The destination combines the signals received from the three time slots to decode the transmitted information. $6L$ time slots are used to transmit the $2L$ codewords (i.e. $r' = 6r$) as illustrated in Fig. 2 (b). Assuming perfect decoding at relays, the achievable DMT for each source is equivalent to $d(r) = 3(1 - r') = 3(1 - 6r)$. Although higher diversity gain over TDMA direct transmission can be achieved by the standard DF protocol, the maximal multiplexing gain is dramatically reduced from $\frac{1}{2}$ to $\frac{1}{6}$ (referred as *multiplexing loss*) because such a transmission process inefficiently uses three time slots to transmit only one codeword.

B. Repetition-Coded CDF

In order to compensate the multiplexing gain reduction induced by the standard DF, we have proposed a CDF protocol in [13]. Unlike the standard approach, we require R_1 and R_2 to listen to S_1 and S_2 respectively and permit concurrent transmission [16] among the network nodes. Orthogonal transmission for the standard DF is relaxed so that one source and one relay are permitted to communicate with the destination simultaneously until the $2L$ codewords are transmitted using $2L+1$ time slots (i.e. $r' = \frac{2L+1}{L}r$). The specific transmission process can be briefly described as follows: The two sources communicate with D using TDMA during the first $2L$ time slots (i.e. S_1 broadcasts x_1^k $(k = 1, \ldots, L)$ to R_1 and D at the $(2k-1)$th time slot, and S_2 broadcasts x_2^k to R_2 and D at the $2k$th time slot). From the second to the $(2L+1)$th time slot, R_1 and R_2 take turns forwarding (repeating) the source codeword they decoded during the previous time slot to D (i.e. during the time slot $2k$, R_1 retransmits x_1^k to D, and during the time slot $2k+1$, R_2 retransmits x_2^k to D). After all the $2L$ codewords are received via both direct and relay links, D performs joint decoding to recover the information transmitted by the two sources. The time-division channel allocation is displayed in Fig. 2 (c). We refer to this protocol as *repetition-coded CDF* throughout the paper.

To handle the issue of interference generated among relays when one relay is listening to its source while the other relay is forwarding its source codeword to D, two specific scenarios are considered. For an *isolated-relay scenario* [6], in which the quality of the inter-relay link is much worse than those of the links between the sources and the relays, it is required that each relay directly decodes its source codeword while treating the interference as Gaussian noise. For a *strong-interference scenario* [17], where the channel between the two relays is sufficiently stronger than the source-relay links, each relay is required to decode the interference first while treating its desired codeword as Gaussian noise. Then the relay decodes its desired codeword after subtracting the interference from the received signal. With these approaches, the system DMT performance is *not* affected by the inter-relay interference.

1) Perfect Source-Relay Transmissions: If the source-relay links are sufficiently strong such that the relays can always perfectly decode their source codewords, the DMT achieved by each source can be expressed by [13]

$$d(r) = 2\left(1 - \frac{2L+1}{L}r\right) \qquad 0 \le r \le \frac{L}{2L+1}. \quad (2)$$

The repetition-coded CDF improves the maximal multiplexing gain from $\frac{1}{6}$ for the standard DF to $\frac{L}{2L+1}$, which approaches $\frac{1}{2}$ for large L and matches the result for TDMA direct transmission. This implies that the multiplexing loss induced by the standard DF is totally recovered. However, a major issue with DF based protocols is that the system DMT performance is limited by the quality of source-relay links [2]. Their diversity improvement over direct transmission is attained only when the source-relay links are sufficiently good, which is however difficult to be guaranteed in general. We refer to the DMT under the assumption of perfect source-relay transmissions as *full DMT*. In the following, we use a simple adaptive protocol to achieve the full DMT (2) under general source-relay channel conditions, which is not studied in [13].

2) Adaptive Protocol: If perfect decoding at relays cannot be guaranteed, an adaptive protocol is used. Specifically, the sources take turns broadcasting one codeword during each of the first $2L$ time slots. Each relay listens to and tries to decode its individual source. If the decodings at *both* relays are successful, the relays are used to assist the sources[1]. Otherwise, both relays remain *silent*. It is assumed that the sources are not aware of whether their relays are used to assist them so that the whole transmission process always takes $2L+1$ time slots (i.e. the multiplexing gain is fixed at $r' = \frac{2L+1}{L}r$). Regarding the achievable DMT, we have the following theorem.

Theorem 1: The adaptive repetition-coded CDF achieves the full DMT (2).

Proof: The overall outage probability can be expressed by

$$P^{out} = P_C P^o_{s,d} + (1 - P_C) P^o_{sr,d} \quad (3)$$

in which P_C denotes the probability that the relays are not used, $P^o_{s,d} \doteq \rho^{-(1-r')}$ denotes the associated outage probability at D, $P^o_{sr,d} \doteq \rho^{-2(1-r')}$ denotes the outage probability at D given that the source codewords are correctly decoded at both relays. Clearly, $P_C \doteq \rho^{-(1-r')} + \rho^{-(1-r')} - \rho^{-2(1-r')} \doteq \rho^{-(1-r')}$. Overall, $P^{out} \doteq \rho^{-2(1-r')} = \rho^{-2\left(1-\frac{2L+1}{L}r\right)}$. The proof is complete. ■

Theorem 1 implies that the assumption of sufficiently good source-relay links in reference [13] can be relaxed without affecting DMT performance. The protocol can thus actually be considered in practical systems to improve transmission reliability over TDMA direct transmission. The repetition-coded CDF uses the same simple repetition coding strategy in relays to enhance diversity gain as the standard DF but

[1]For the strong-interference scenario, we assume that the inter-relay link is sufficiently stronger than the source-relay links so that each relay always correctly decodes the inter-relay interference codewords. As a result, for both isolated-relay and strong-interference scenarios, whether the relays are used in the adaptive protocol is related only to the source-relay transmissions. For the other protocols that will be discussed later, the situation is the same.

no longer significantly loses spectral efficiency. Therefore, the repetition-coded CDF makes relaying more beneficial.

Nevertheless, we still argue that the advantages of the multi-user multi-relay structure are not fully exploited by the repetition-coded CDF. In the following, two advanced repetition coding based protocols (i.e. each codeword is identically encoded at the relays and the source) are presented such that the system diversity can be further improved without multiplexing sacrifice.

C. Superposition-Coded CDF

For the strong-interference scenario, the repetition-coded CDF requires that each relay subtracts the inter-relay interference and forwards only its desired source codeword. In fact, the interference is the transmitted codeword from the other source and thus can be made use of. Therefore, we permit the relays to use superposition coding [17] to retransmit both sources' messages, i.e. instead of retransmitting its desired source codeword, each relay transmits the sum of the interference codeword (with a power weighting factor γ_i, $1 > \gamma_i > 0$) and the desired codeword (with a power weighting factor γ_d, $\gamma_d = \sqrt{1-\gamma_i^2}$). To guarantee each codeword being transmitted via three independent paths, $2L+2$ time slots are used to finish the transmission of the $2L$ codewords (i.e. $r' = \frac{2L+2}{L}r$). The transmission process can be described as follows:

Time slot 1: S_1 broadcasts x_1^1 to both R_1 and D; S_2 and R_2 remain silent.

Time slot 2: R_1 forwards x_1^1 to D and S_2 transmits x_2^1. R_2 listens to S_2 while being interfered by x_1^1 from R_1. D receives x_1^1 from R_1 and x_2^1 from S_2.

Time slot 3: R_2 forwards $(\gamma_d x_2^1 + \gamma_i x_1^1)$ to D. S_1 transmits x_1^2. R_1 listens to S_1 while being interfered by $(\gamma_d x_2^1 + \gamma_i x_1^1)$ from R_2. D receives $(\gamma_d x_2^1 + \gamma_i x_1^1)$ from R_2 and x_1^2 from S_1.

Time slot 4: R_1 forwards $(\gamma_d x_1^2 + \gamma_i x_2^1)$ to D. S_2 transmits x_2^2. R_2 listens to S_2 while being interfered by $(\gamma_d x_1^2 + \gamma_i x_2^1)$ from R_1. D receives $(\gamma_d x_1^2 + \gamma_i x_2^1)$ from R_1 and x_2^2 from S_2.

This process repeats until the $(2L)$th time slot.

Time slot $2L+1$: R_2 retransmits $(\gamma_d x_2^L + \gamma_i x_1^L)$ to R_1 and D.

Time slot $2L+2$: R_1 decodes, re-encodes and retransmits x_2^L to D.

Unlike the repetition-coded case, from the 3rd to the $(2L+1)$th time slot, the interference signal received by each relay is not only the other relay's desired source codeword, but also a codeword transmitted by the relay itself (i.e. its desired codeword) during the previous time slot. Because each relay has the full knowledge of its own transmitted codewords, it subtracts that codeword from the received signal before decoding. After all the $2L$ codewords are received, D performs joint decoding. We refer to this protocol as *superposition-coded CDF* and its time-division channel allocation is illustrated in Fig. 2 (d).

1) Perfect Source-Relay Transmissions: If all source codewords are correctly decoded by the relays, the superposition-coded CDF mimics a $2L$-user multiple-access single-input multiple-output (SIMO) channel (except that the signal dimensions are expanded in the time domain):

$$\mathbf{y} = \sqrt{\rho}\underbrace{\begin{bmatrix} h_{S_1} & 0 & 0 & \cdots & 0 & 0 \\ h_{R_1} & h_{S_2} & 0 & \cdots & 0 & 0 \\ \gamma_i h_{R_2} & \gamma_d h_{R_2} & h_{S_1} & \cdots & 0 & 0 \\ 0 & \gamma_i h_{R_1} & \gamma_d h_{R_1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \gamma_d h_{R_1} & h_{S_2} \\ 0 & 0 & 0 & \cdots & \gamma_i h_{R_2} & \gamma_d h_{R_2} \\ 0 & 0 & 0 & \cdots & 0 & h_{R_1} \end{bmatrix}}_{\mathbf{H}} \mathbf{x} + \mathbf{n} \tag{4}$$

in which $\mathbf{x} = [x_1^1 \; x_2^1 \; x_1^2 \; \ldots \; x_1^L \; x_2^L]^T$ is the $2L \times 1$ transmit signal vector, $\mathbf{y} = [y_1 \; y_2 \; \ldots \; y_{2L+2}]^T$, y_i is the received signal at the ith time slot, h_a is the channel fading coefficient between node a and D, and $\mathbf{n}$ is a $(2L+2) \times 1$ unit power complex circular additive white Gaussian noise (AWGN) vector at D. Regarding the DMT, we have the following theorem.

Theorem 2: On assuming that the relays correctly decode the sources, the achievable DMT for each source of the superposition-coded CDF (i.e. the system model in (4)) is

$$d(r) = 3\left(1 - \frac{2L+2}{L}r\right) \qquad 0 \le r \le \frac{L}{2L+2}. \tag{5}$$

Proof: See Appendix A. ∎

Equation (5) indicates that maximal diversity gain 3 and maximal multiplexing gain $\frac{L}{2L+2}$ can be achieved. This implies that the diversity performance of the repetition-coded CDF is further improved by making use of the inter-relay interference. Therefore, unlike the repetition-coded CDF, whose diversity gain is larger than that of the standard DF only for the high r region, the superposition-coded CDF strictly outperforms the standard DF in terms of DMT. Although there exists a slight difference for the maximal multiplexing gains $\frac{L}{2L+1} - \frac{L}{2L+2} = \frac{L}{(2L+1)(2L+2)}$ between the repetition-coded and superposition-coded CDF protocols (due to the extra transmission time slot), when L is large this difference is negligible and the maximal multiplexing gains for both protocols approach $\frac{1}{2}$. It is worth noting that, for the strong-interference scenario, the superposition-coded CDF does not require more complex encoding strategy than the repetition-coded CDF (i.e. after decoding and re-encoding both the desired and interference codewords, the superposition-coded CDF transmits the sum of them but the repetition-coded CDF only transmits the former). The diversity improvement comes from taking advantage of the multi-relay structure and efficiently using both relays rather than just one to assist each source.

2) Adaptive Protocol: If the source-relay links are not good enough, following the analysis in Theorem 1, it can be seen that full DMT cannot be achieved by requiring R_1 (R_2) to listen only to S_1 (S_2). This is because such a requirement only provides maximal diversity gain 2. However, since the transmission of each source can actually be overheard by both relays, if R_1 cannot decode S_1 or R_2 cannot decode S_2 but R_1 and R_2 can decode S_2 and S_1 respectively, the two relays can still be used to assist the sources. Therefore, for the adaptive

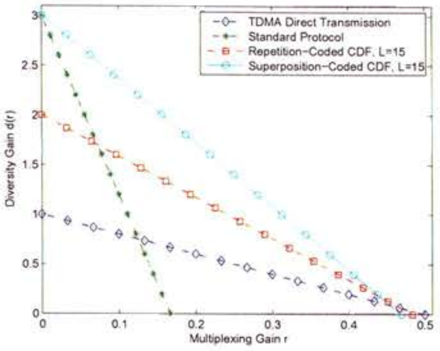

Fig. 3. DMT performance of different protocols for the strong-interference scenario under general source-relay channel conditions. Each terminal is equipped with a single-antenna. The two sources communicate with the destination using TDMA.

superposition-coded CDF, we assume the relays can be configured to assist the sources if one relay can decode one source and the other relay can decode the other source (each source codeword has multiplexing gain $r' = \frac{2L+2}{L}r$). Otherwise, both relays remain silent. We generalize the achievable DMT result to the following corollary to Theorem 1.

Corollary 1: The adaptive superposition-coded CDF achieves the full DMT (5).

Proof: The overall outage probability can also be expressed by (3) with $P^o_{s,d} \dot{=} \rho^{-(1-r')}$, $P^o_{sr,d} = \rho^{-3(1-r')}$, and

$$\begin{aligned} P_C &= P(A_1 \cup A_2 \cup A_3) \\ &= \underbrace{P(A_1)}_{\to \rho^{-2(1-r')}} + \underbrace{P(A_2)}_{\to \rho^{-2(1-r')}} - \underbrace{P(A_1 \cap A_2)}_{\to \rho^{-4(1-r')}} + \underbrace{P(A_3)}_{\to \rho^{-2(1-r')}} \end{aligned}$$

in which A_1 and A_2 denote the events that no relay can correctly decode S_1 and S_2 respectively, and A_3 denotes the event that one relay can decode both S_1 and S_2 but the other relay can decode neither of them. We have

$$P^{out} \dot{=} \rho^{-2(1-r')}\rho^{-(1-r')} + \rho^{-3(1-r')} \dot{=} \rho^{-3(1-r')}.$$

The proof is complete. ■

An example ($L = 15$) of the DMT performance of the superposition-coded CDF is displayed in Fig. 3. In the next subsection, we will move on to discuss how to improve the diversity performance of the repetition-coded CDF for the isolated-relay scenario in which no inter-relay interference can be used.

D. Multiple-Access CDF

For the isolated-relay scenario, we permit the two sources to transmit simultaneously. We refer to this protocol as *multiple-access CDF* and $L+1$ time slots are used for completing the transmission (i.e. $r' = \frac{L+1}{L}r$). More specifically, the two sources simultaneously broadcast their codewords during the first L time slots (i.e. S_1 broadcasts x_1^k and S_2 broadcasts x_2^k during the kth time slot, $1 \le k \le L$). From the second time slot, the two relays take turns to decode both sources' codewords and retransmit the sum of them to D (i.e. during the $(k+1)$th time slot, R_1 forwards $(\gamma_i x_1^k + \gamma_d x_2^k)$ if k is odd, or R_2 forwards $(\gamma_i x_2^k + \gamma_d x_1^k)$ if k is even, where $0 < \gamma_i < 1$, and $\gamma_d = \sqrt{1-\gamma_i^2}$). The time-division channel allocation is illustrated in Fig. 2 (e)

1) Perfect Source-Relay Transmissions: With perfect decoding at relays, the multiple-access CDF mimics a $2L$-user multiple-access SIMO channel

$$\mathbf{y} = \sqrt{\rho} \underbrace{\begin{bmatrix} h_{S_1} & h_{S_2} & 0 & 0 & \cdots & 0 & 0 \\ \gamma_i h_{R_1} & \gamma_d h_{R_1} & h_{S_1} & h_{S_2} & \cdots & 0 & 0 \\ 0 & 0 & \gamma_d h_{R_2} & \gamma_i h_{R_2} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & h_{S_1} & h_{S_2} \\ 0 & 0 & 0 & 0 & \cdots & \gamma_a h_{R_\alpha} & \gamma_b h_{R_\alpha} \end{bmatrix}}_{\mathbf{H}} \mathbf{x} + \mathbf{n} \tag{6}$$

where $\mathbf{y} = [y_1\ y_2\ \ldots\ y_{L+1}]^T$, $\alpha = 1$, $\gamma_a = \gamma_i$ and $\gamma_b = \gamma_d$ if L is odd, $\alpha = 2$, $\gamma_a = \gamma_d$ and $\gamma_b = \gamma_i$ if L is even. Regarding the DMT, we have the following theorem.

Theorem 3: On assuming that the relays correctly decode the sources, the achievable DMT for each source of the multiple-access CDF (i.e. the system model in (6)) is *lower bonded* by

$$d(r) = \begin{cases} 2(1 - \frac{L+1}{L}r) & 0 \le r \le \frac{3L}{10L+10} \\ \frac{7}{2}(1 - \frac{2L+2}{L}r) & \frac{3L}{10L+10} \le r \le \frac{L}{2L+2} \end{cases} \tag{7}$$

Proof: See Appendix B. ■

Unlike the superposition-coded CDF, which takes advantage of the multi-relay structure, the multiple-access CDF takes advantage of the multi-source structure of the considered system. For large L, the multiple-access CDF strictly outperforms the repetition-coded CDF in terms of diversity gain without using any advanced coding strategy in the relays[2].

Note that the DMT (7) is a lower bound. This is because the exact expression of the determinant of the matrix $(\mathbf{I} + \rho\mathbf{H}\mathbf{H}^H)$ is difficult to obtain. From the proof part in Appendix B, it can be seen that the bound is tight for $0 \le r \le \frac{3L}{10L+10}$. Moreover, the nature of CDF is to use two half-duplex relays to mimic a full-duplex relay when the frame length L is large. It has been proved in [14] that even the use of full-duplex relays cannot improve the maximal multiplexing gain for direct source-destination transmission. Since the maximal multiplexing gain $\frac{L}{2L+2}$ approaches $\frac{1}{2}$ for large L, which matches the result for the direct transmission between the sources and the destination (i.e. a two-user multiple-access channel, which achieves the DMT $d(r) = \min\{1-r, 2(1-2r)\}$ [15]). The point $(r, d(r)) = (\frac{1}{2}, 0)$ is also tight.

In the following, we will give an upper bound of the achievable DMT using an adaptive form of the multiple-access CDF under the general source-relay channel conditions. Together with the lower bound (7), it can be seen that the diversity and multiplexing advantages of the multiple-access CDF are still valid if the perfect source-relay transmissions are not guaranteed.

[2]The multiple-access CDF requires higher complexity at relays than the repetition-coded CDF since two codewords instead of only one need to be decoded by relays during each time slot. But since each relay re-encodes each source codeword using the same codebook as the source, the multiple-access CDF is still a repetition coding based protocol.

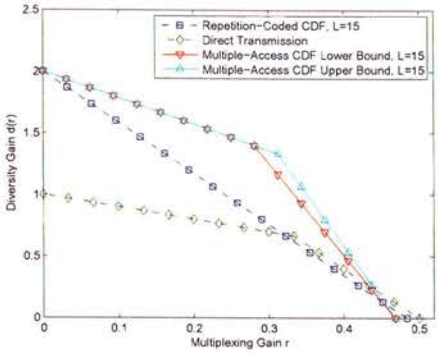

Fig. 4. DMT performance of different protocols for the two-source isolated-relay scenario under general source-relay channel conditions. Each terminal is equipped with a single-antenna. Except for the repetition-coded CDF, both sources communicate with the destination simultaneously.

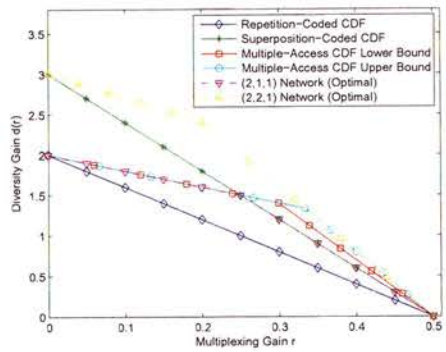

Fig. 5. DMT performance of different protocols for the two-source network. Each terminal is equipped with a single-antenna.

2) Adaptive protocol: For the adaptive multiple-access CDF, the two relays listen to and try to decode the two sources. The relays are used to assist the sources *only if* the signals are correctly decoded at both relays; otherwise, both relays remain silent for the whole $L+1$ time slots. Regarding the DMT, we have the following result.

Corollary 2: The achievable DMT of the adaptive multiple-access CDF is lower bounded by (7) and is *upper bounded* by

$$d(r) = \begin{cases} 2(1-\frac{L+1}{L}r) & 0 \le r \le \frac{L}{3L+3} \\ 4(1-\frac{2L+2}{L}r) & \frac{L}{3L+3} \le r \le \frac{L}{2L+2} \end{cases} \quad (8)$$

Proof: The diversity gains of the source-relay and source-destination channels can be expressed by $d(r) = \min\{1-r', 2(1-2r')\}$. Following the proof of Theorem 1, it can be seen $P_C \doteq \rho^{-\min\{1-r', 2(1-2r')\}}$. The overall outage probability can be expressed by

$$\begin{aligned} P^{out} &= P_C P^o_{s,d} + (1-P_C)P^o_{sr,d} \\ &\doteq \rho^{-\min\{2(1-r'),4(1-2r')\}} + \rho^{-\bar{d}(r')} \end{aligned} \quad (9)$$

where $\bar{d}(r')$ denotes the exact full DMT expression of the multiple-access CDF. Since $\min\{2(1-r'), 4(1-2r')\} \ge \min\{2(1-r'), \frac{7}{2}(1-2r')\}$, it can be seen from (9) that the DMT of the adaptive multiple-access CDF is lower bounded by (7) and is upper bounded by (8). ■

An example of the DMT bounds ($L=15$) is displayed in Fig. 4. Clearly, the point $(r, d(r)) = (\frac{L}{2L+2}, 0)$ is tight for the adaptive protocol. When L is very large, the multiple-access CDF strictly outperforms the repetition-coded CDF for the whole region of possible r. This means the diversity performance is increased without sacrificing multiplexing performance.

To highlight the advantages of our protocols, we also compare the asymptotic DMT performance of the proposed protocols (assuming the frame length $L \to \infty$) with the optimal DMT for a symmetric four-node network with two sources, one *full-duplex* relay, and one destination (a.k.a. a multiple-access relay channel (MARC) [18]) in Fig. 5. We denote such a network as a $(2,1,1)$ network and the optimal DMT is given in [18] as $d(r) = \min\{2(1-r), 3(1-2r)\}$. To (partially) achieve the optimal DMT in half-duplex relay systems conventionally demands the use of complex coding strategies at the relay (e.g. a dynamic DF (DDF) protocol or a compress-and-forward (CF) protocol) [14]. However, we fulfill the task by adding a new half-duplex relay into the network. Clearly, if the inter-relay link is either sufficiently weak or sufficiently strong, the repetition-coded CDF obtains the optimal diversity and multiplexing gains of the $(2,1,1)$ network when L approaches infinity. In addition, it can be seen that the optimal DMT of the $(2,1,1)$ network can be fully achieved by our superposition-coded CDF (in a strong-interference scenario) and multiple-access CDF (in an isolated-relay scenario) protocols. Furthermore, when $0 \le r \le \frac{1}{4}$ and $\frac{1}{4} \le r \le \frac{1}{2}$, the superposition-coded and multiple-access CDF protocols respectively outperform the optimal DMT. This means in these two protocols the two half-duplex relays behave better than a full-duplex relay even using the simple repetition-coding strategy (thanks to the extra data delivery links introduced by the extra relay). For fair comparison, we also plot the optimal DMT for the five-node network considered in this paper (i.e. the network displayed in Fig. 1). The network is denoted as a $(2,2,1)$ network and the optimal DMT for each source is expressed as $d(r) = \min\{3(1-r), 4(1-2r)\}$[3]. It can be seen that the maximal diversity and multiplexing gains for such a $(2,2,1)$ network can be achieved by the superposition-coded CDF. When $\frac{1}{3} \le r \le \frac{1}{2}$, the DMT upper bound of the multiple-access CDF is the same as the optimal DMT. These imply that, in the two specific scenarios, our protocols can use two half-duplex relays to partially mimic two full-duplex relays. Such observations raise interest in future investigation of applying more complex coding strategies at relays to improve performance towards the optimal DMT.

III. Multiple-Antenna Scenarios

So far, all the nodes are assumed to be single-antenna terminals. In fact, using multiple antennas at the terminals is an effective way to improve system performance. In this paper, we concentrate on uplink transmission for practical

[3]Due to space limitations, we omit the detailed derivation, which follows a cut-set bound analysis on the considered network.

cooperative systems, in which mobile terminals (i.e. sources and relays) cannot afford multiple antennas due to hardware or cost limitations, while the base station (i.e. the destination) can be equipped with N antennas ($N \geq 1$). In the following, we first study the full DMT performances of the repetition-coded and superposition-coded CDF protocols. And then adaptive protocols, which select two relays from K potential relays ($K > 1$) to assist the sources, are followed. Note that when we consider repetition-coded CDF, we assume the K potential relays are isolated to each other. On the other hand, for the superposition-coded CDF, it is assumed that the interference between each relay pair is sufficiently strong such that each relay can always correctly decode the interference before decoding its desired signal.

A. Perfect Source-Relay Transmissions

With perfect decoding at relays, the input-output relation of the repetition-coded CDF can be expressed by

$$\mathbf{y} = \sqrt{\rho}\underbrace{\begin{bmatrix} \mathbf{h}_{S_1} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{h}_{R_1} & \mathbf{h}_{S_2} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{h}_{R_2} & \mathbf{h}_{S_1} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{h}_{R_1} & \mathbf{h}_{S_2} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h}_{R_2} \end{bmatrix}}_{\mathbf{H}} \mathbf{x} + \mathbf{n} \quad (10)$$

where $\mathbf{y} = [\mathbf{y}_1^T \ \mathbf{y}_2^T \ \ldots \ \mathbf{y}_{2L+1}^T]^T$, $\mathbf{y}_i$ is the $N \times 1$ received signal vector at the ith time slot, and $\mathbf{h}_a$ is the $N \times 1$ channel fading vector between node a and D. For the superposition-coded CDF, the input-output relation is expressed as (4) by replacing h_a with $\mathbf{h}_a$ and y_i with $\mathbf{y}_i$. We summarize the DMT results as the following corollary.

Corollary 3: In a symmetric scenario in which destination is equipped with N antennas, on assuming that the relays correctly decode the sources, the achievable DMT for each source of the repetition-coded CDF is

$$d(r) = 2N\left(1 - \frac{2L+1}{L}r\right) \qquad 0 \leq r \leq \frac{L}{2L+1}. \quad (11)$$

The achievable DMT for each source of the superposition-coded CDF is

$$d(r) = 3N\left(1 - \frac{2L+2}{L}r\right) \qquad 0 \leq r \leq \frac{L}{2L+2}. \quad (12)$$

Proof: See Appendix C. ■

B. Relay Selection

Because multiple antennas are equipped at D, the system diversity gain is dramatically increased over that of single-antenna systems. Full DMT thus cannot be attained in general source-relay channel conditions by the use of only two relays due to lower diversity provided by the source-relay links. Therefore, here we present a relay selection scheme to select two relays in a multi-relay scenario in order to achieve the full DMTs. More specifically, we assume there exist K ($K \geq 2$) single-antenna terminals (mobiles) in the network that can work as potential relays for the two sources. If a relay pair (R_α, R_β), in which $\alpha, \beta \in \{1, \ldots, K\}$ and $\alpha \neq \beta$, within the K potential relays can be found such that R_α can correctly decode S_1 and R_β can correctly decode S_2, the two relays are used to assist the sources. Otherwise, the sources communicate with the destination without the help of relays. If there exist more than one pair of such relays, one pair is *randomly* chosen. Note that to minimize the system complexity, we do not consider any specific selection criterion regarding the quality of relay-destination links (e.g. choosing the relays which have the best relay-destination links) or using more than one relay to simultaneously forward each source's codeword to achieve even higher diversity. It is assumed that the sources have no information about whether their codewords can be retransmitted by relays so that the transmission of the two frames from the two sources always takes $2L+1$ ($2L+2$) time slots for the repetition-coded (superposition-coded) CDF. The DMT performance is summarized as follows.

Corollary 4: The use of the relay selection scheme in a K-relay scenario achieves the following DMT for the repetition-coded CDF

$$d(r) = \min\{N+K, 2N\}\left(1 - \frac{2L+1}{L}r\right) \quad 0 \leq r \leq \frac{L}{2L+1}, \quad (13)$$

and for the superposition-coded CDF

$$d(r) = \min\{N+K, 3N\}\left(1 - \frac{2L+2}{L}r\right) \quad 0 \leq r \leq \frac{L}{2L+2}. \quad (14)$$

Proof: Following the analysis in Corollary 1, for the case in which the destination is equipped with N antennas, we have $P^o_{s,d} \doteq \rho^{-N(1-r')}$, $P^o_{sr,d} = \rho^{-2N(1-r')}$ ($r' = \frac{2L+1}{L}r$) for the repetition-coded CDF, and $P^o_{sr,d} = \rho^{-3N(1-r')}$ ($r' = \frac{2L+2}{L}r$) for the superposition-coded CDF. Since we have K potential relays in the network, it can be seen that $P(A_1) = P(A_2) \doteq \rho^{-K(1-r')}$, $P(A_1 \cap A_2) \doteq \rho^{-2K(1-r')}$, and $P(A_3) \doteq K\rho^{-(2K-2)(1-r')}(1 - \rho^{-(1-r')})^2 \doteq \rho^{-(2K-2)(1-r')}$. Therefore, the overall system outage probability can be expressed by

$$\begin{aligned} P_{out} &\doteq \rho^{-N(1-r')}\rho^{-\min\{K,2K-2\}(1-r')} \\ &\qquad + \rho^{-2N(1-r')}\left(1 - \rho^{-\min\{K,2K-2\}(1-r')}\right) \\ &\doteq \rho^{-\min\{N+\min\{K,2K-2\},2N\}(1-r')} \end{aligned} \quad (15)$$

for the repetition-coded CDF. For the superposition-coded CDF,

$$P_{out} \doteq \rho^{-\min\{N+\min\{K,2K-2\},3N\}(1-r')} \quad (16)$$

Since $K > 1$, $\min\{K, 2K-2\} = K$. The proof is complete. ■

Corollary 4 implies that if $K \geq N$ ($K \geq 2N$), the adaptive repetition-coded (superposition-coded) CDF always attains full DMT. This observation shows that if the number of potential relays is larger than some threshold (related to the number of antennas at D), the system DMT performance is the same as the case in which perfect source-relay transmission is assumed. Moreover, since $\min\{N+K, 3N\} \geq \min\{N+K, 2N\}$, for large L and $K > N$, the superposition-coded CDF always attains better DMT performance than the repetition-coded CDF.

IV. Discussion

Throughout this paper, we have focused precisely on a two-source scenario. The analysis can be directly used in a network in which the communication between a single source and its intended destination is assisted by two relays. Applying superposition coding in the relays or equipping multiple antennas at the destination can be used to further increase the diversity performance of the successive relaying protocol proposed in [7]. The higher diversity performance can be achieved under general rather than sufficiently good source-relay channel conditions. For example, when the destination has N antennas, if there exist $K \geq N+1$ potential relays (which are isolated to each other) in the network, the DMT $d(r) = 2N(1 - \frac{L+1}{L}r)$ can be attained. Similarly, if the interference between each relay pair out of $K \geq 2N+1$ potential relays is sufficiently strong, the DMT $d(r) = 3N(1 - \frac{L+2}{L}r)$ can be achieved by making use of the inter-relay interference. Extending such two scenarios to a more generalized M-source network ($M \geq 1$) is also straightforward.

For the superposition-coded and multiple-access CDF, when each relay retransmits the sum of two codewords, we have assumed that each codeword is transmitted with an individual power scaling factor. For the finite-SNR region, how transmit power is allocated to the two codewords according to different channel conditions or statistics may have an influence on the system error and capacity performance. However, such scaling factors have no consequence for the infinite-SNR DMT in Rayleigh fading environments, which is the case considered in this paper.

In this paper, we have considered information theoretic i.i.d. Gaussian random codes [19]. In practice, transmitting a combination of two messages in the superposition-coded CDF (and also the multiple-access CDF) can be realized by simply requiring each relay to retransmit the sum of the modulated symbols of the two messages, which is similar to *superposition modulation* as discussed in [20]. Another simple method for the relays to retransmit simultaneously both messages is similar to *code superposition* [21] such that the transmit signal of each relay is the XORed version of the two messages. It has been shown in [21] that code superposition brings error performance improvement over superposition modulation. In addition, if one of the two messages is correctly decoded at the destination, it is as if the relay transmits the other message with full power. Therefore, we conjecture that its performance is upper bounded by that of the system model in (4) when $\gamma_i = \gamma_d = 1$. Since the values of γ_i and γ_d have no impact on the infinite-SNR DMT performance, it is conjectured that such an approach also attains the full DMT (5). In recent years, a similar coding strategy has been commonly considered in the context of *network coding* [22]. Therefore, it would be interesting to consider combining results from network coding with our protocols as a future work.

For all the adaptive protocols, we have assumed that the sources have no advance information about whether the relays can decode their codewords so that the overall transmission time of the $2L$ codewords is fixed for each protocol. Of course, if limited feedback containing such information from relays or destination to sources can be exploited before the transmission process, the sources may adjust their transmission rates to enhance the probability of correct decoding at relays. Alternatively the network can change the overall transmission time to avoid the multiplexing loss induced by the extra time slots if the relays are not used.

The aim of this paper is to study how to efficiently exploit the advantages of multi-source multi-relay cooperative network structure to increase link reliability using simple coding strategy in relays without losing spectral efficiency. Hence, we have considered two extreme scenarios so that the interference between relays does not degrade the system DMT performance. The study of cancelling or using inter-relay interference in practical systems is beyond the scope of this paper. In addition, we have used a relay selection scheme to show that full DMT is achievable in general source-relay channel conditions. Exact criteria by which the two relays can be efficiently selected are not considered in this paper, but this is an interesting topic for future work.

V. Conclusions

In this paper, several relaying transmission protocols for a multi-user multi-relay scenario are presented such that the diversity performance of a previously proposed repetition-coded CDF protocol can be further improved. We have seen that the diversity performance of such protocols is not limited by the quality of source-relay links and the multiplexing performance does not suffer from the half-duplex limitation at relays. Since our protocols are repetition coding based (i.e. each relay simply re-encodes the source information to the same codeword as the source), improving diversity gain without losing multiplexing gain is accomplished in a very simple way, which highlights the important advantages of the proposed protocols.

Appendix A
Proof of Theorem 2

For a symmetric $2L$-user multiple-access SIMO system described in (4), following the capacity calculation in [23], there are $(2^{2L} - 1)$ source transmission rate constraints for a given realization of the channel, which can be expressed as,

$$|\mathcal{S}|r' \log\rho \leq \log\left(\det\left(\mathbf{I} + \rho\sum_{l\in\mathcal{S}}\mathbf{h}_l\mathbf{h}_l^H\right)\right), \quad \forall\mathcal{S} \subseteq \{1,\dots,2L\} \tag{17}$$

where $\mathbf{h}_l$ denotes the lth column of $\mathbf{H}$. For each constraint there is a probability of not meeting it. The outage probability is the highest one among all these probabilities [15]. Therefore, there are $(2^{2L} - 1)$ diversity-multiplexing tradeoffs corresponding to all those conditions and the lowest curve within the range of multiplexing gain is the achievable tradeoff curve for the system.

To characterize the DMT for each constraint, we consider an $(m+2) \times m$ MIMO channel

$$\mathbf{r} = \sqrt{\rho}\mathbf{G}_m \mathbf{s} + \mathbf{w} \tag{18}$$

where $\mathbf{s} = [s_1\ s_2\ \dots\ s_m]^T$ denotes an $m \times 1$ unit power transmit vector, $\mathbf{r}$ and $\mathbf{w}$ are the $(m+2)\times 1$ receive and AWGN vectors respectively, and $\mathbf{G}_m$ is an $(m+2)\times m$ channel fading matrix constituted by the first $m+2$ rows and m columns of $\mathbf{H}$ in (4). For infinite SNR, the task of finding the smallest diversity gain achieved by each constraint in (17) is the same as finding the smallest diversity gain achieved by the system (18) for every $1 \le m \le 2L$ [13].

When $m = 1$, the system model in (18) is a 1×3 SIMO system so that $d(r) = 3(1 - r')$.

When $m > 1$, the proof follows the DMT calculation for the ISI channel in [24]. Assume each codeword s_i is chosen from a QAM constellation[4] and ML decoding is applied. Defining $\mathbf{g} = [h_{S_1}\ h_{R_1}\ h_{S_2}\ h_{R_2}]^T$, it can be proved that the error probability can be upper bounded by $P_e \le c \cdot \bar{\lambda}^{-4}\rho^{-4(1-r')}$, where c is a constant, $\bar{\lambda} = \inf_{\mathbf{g}\in\mathcal{C}^4} \lambda_{\min}\left(\frac{\mathbf{G}_m}{||\mathbf{g}||}\right)$, $\mathcal{C}^4$ is the 4-dimensional complex space, and $\lambda_{\min}(\mathbf{X})$ denotes the minimum singular value of $\mathbf{X}$.

When m is even and odd, define the following matrices for these two cases:

$$\mathbf{A}_e = \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_1 & a_2 & 0 \\ a_3 & 0 & 0 & \xi_1^2 \\ 0 & \xi_2^3 & a_4 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \xi_{m-2}^{m-1} & a_m & 0 \\ 0 & 0 & 0 & \xi_{m-1}^m \\ 0 & a_m & 0 & 0 \end{bmatrix}, \mathbf{A}_o = \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_1 & a_2 & 0 \\ a_3 & 0 & 0 & \xi_1^2 \\ 0 & \xi_2^3 & a_4 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ a_m & 0 & 0 & \xi_{m-2}^{m-1} \\ 0 & \xi_{m-1}^m & 0 & 0 \\ 0 & 0 & 0 & a_m \end{bmatrix} \tag{19}$$

in which $\xi_\alpha^\beta = \gamma_i a_\alpha + \gamma_d a_\beta$, and a_i is defined as that in the proof of Theorem 3.4 in [24]. Then using a similar method as the proof of Lemma 4.1 in [24], it can be proved that $\bar{\lambda} > 0$. As $\bar{\lambda}$ is not a function of ρ, it can be concluded $P_e \dot{\le} \rho^{-4(1-r')}$.

Because the overall system diversity gain is dominated by the smallest diversity gain for all m, it thus is equivalent to that of the case in which $m = 1$ and expressed by (5).

Appendix B
Proof of Theorem 3

Consider an $(m+1) \times 2m$ MIMO channel

$$\mathbf{G} = \begin{bmatrix} g_1^1 & g_1^2 & 0 & 0 & \cdots & 0 & 0 \\ g_1^3 & g_1^4 & g_2^1 & g_2^2 & \cdots & 0 & 0 \\ 0 & 0 & g_2^3 & g_2^4 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & g_m^1 & g_m^2 \\ 0 & 0 & 0 & 0 & \cdots & g_m^3 & g_m^4 \end{bmatrix}. \tag{20}$$

[4]The use of QAM constellations is only for purposes of the proof. When codewords are chosen from Gaussian random codebooks, the performance would be at least as good as the case where codewords are chosen from QAM constellations.

Since the matrix $\left(\mathbf{I} + \rho\mathbf{G}\mathbf{G}^H\right)$ is a tridiagonal matrix, using the determinant calculation formula of tridiagonal matrices in [25], it can be proved that its determinant is lower bounded by

$$\det\left(\mathbf{I} + \rho\mathbf{G}\mathbf{G}^H\right) \dot{\ge} 1 + \sum_{i=1}^{m}\sum_{j=1}^{4} \rho|g_i^j|^2 + \prod_{i=1}^{m}(\rho|g_i^1|^2 + \rho|g_i^2|^2) + \prod_{i=1}^{m}(\rho|g_i^3|^2 + \rho|g_i^4|^2). \tag{21}$$

We consider a codeword set $\mathbf{s} = [\mathbf{s}_1\ \mathbf{s}_2\ \dots\ \mathbf{s}_p]^T$, where $\mathbf{s}_j = [s_1^j\ s_2^j]^T$ or $\mathbf{s}_j = [s_1^j\ 0]^T$ or $\mathbf{s}_j = [0\ s_2^j]^T$ and there are q codewords in total. Denote the channel matrix for such a codeword set as $\mathbf{H}_{p,q}$. According to (21), we can have

$$\begin{aligned} &\det\left(\mathbf{I} + \rho\mathbf{H}_{p,q}\mathbf{H}_{p,q}^H\right) \\ &\dot{\ge} 1 + \sum_{i=1}^{4}\rho|h_i|^2 + (\rho|h_3|^2)^{t_4}(\rho|h_4|^2)^{t_5} \\ &\quad + \left(\rho|h_1|^2 + \rho|h_2|^2\right)^{t_1}\left(\rho|h_1|^2\right)^{t_2}\left(\rho|h_2|^2\right)^{t_3} \end{aligned} \tag{22}$$

where $t_1 + t_2 + t_3 = p$, $2t_1 + t_2 + t_3 = q$, $t_4 + t_5 = p$, $t_4 = t_5$ if p is even, and $|t_4 - t_5| = 1$ if p is odd. Define v_i as the exponential order of $|h_i|^2$. Following the analysis in [4], the diversity of the system can be calculated by

$$d(r) = \min_{v_1,v_2,v_3,v_4\in O}(v_1 + v_2 + v_3 + v_4) \tag{23}$$

where $O = \{v_1, v_2, v_3, v_4 | \min\{t_2 v_1 + (p - t_2)v_2, t_4 v_3 + (p - t_4)v_4\} \ge p - qr'\}$.

Consider only one codeword (i.e. $p = q = 1$), $d(r) = 2(1 - r')$. Then for the case in which $p = 1$ and $q = 2$, it can be proved that $d(r) \ge 2(1 - r')$. For other cases, it can be proved that $d(r) \ge (v_1+v_2+v_3+v_4)|_{q=1,p=1} = 2(1-r')$ when $r' \le \frac{3}{10}$, and $d(r) \ge (v_1 + v_2 + v_3 + v_4)|_{q=3,p=6} = \frac{7}{2}(1 - 2r')$ when $\frac{3}{10} \le r' \le \frac{1}{2}$. Since the exact determinant expression of the matrix $\left(\mathbf{I} + \rho\mathbf{G}\mathbf{G}^H\right)$ is difficult to obtain in general, the DMT (7) calculated from (22) is only a lower bound. However, it is easy to see that $d(r) = 2(1 - r')$ when $r' \le \frac{3}{10}$ is tight since the diversity upper bound of each codeword is also $d(r) = 2(1 - r')$. Due to limited space, here we omit the detailed proof, which can be found in [26].

Appendix C
Proof of Corollary 3

The proof can follow a similar method as that in the proof of Theorem 2. More specifically, for the repetition-coded protocol, we consider an $(m+1)N \times m$ MIMO channel with the same form of (18) except that $\mathbf{r}$ and $\mathbf{w}$ are two $(m+1)\times 1$ vectors and

$$\mathbf{H}_m = \begin{bmatrix} \mathbf{h}_{S_1} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{h}_{R_1} & \mathbf{h}_{S_2} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{h}_{R_2} & \mathbf{h}_{S_1} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{h}_{S_\alpha} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{h}_{R_\alpha} \end{bmatrix} \tag{24}$$

where $\mathbf{h}_a$ denotes the channel fading vector between nodes a and D, and $\alpha = mod(m+1, 2) + 1$. The task thus to is find the smallest diversity gain achieved for every $1 \leq m \leq 2L$.

When $m = 1$, $d(r) = 2N(1 - r')$. When $m > 1$, following the analysis in the proof of Theorem 2 and defining matrices $\mathbf{A}_e$ and $\mathbf{A}_o$ for even and odd m respectively

$$\mathbf{A}_e = \begin{bmatrix} \mathbf{D}_1 & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{D}_1 & \mathbf{D}_2 & \mathbf{O} \\ \mathbf{D}_3 & \mathbf{O} & \mathbf{O} & \mathbf{D}_2 \\ \mathbf{O} & \mathbf{D}_3 & \mathbf{D}_4 & \mathbf{O} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{O} & \mathbf{D}_{m-1} & \mathbf{D}_m & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{D}_m \end{bmatrix}, \mathbf{A}_o = \begin{bmatrix} \mathbf{D}_1 & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{D}_1 & \mathbf{D}_2 & \mathbf{O} \\ \mathbf{D}_3 & \mathbf{O} & \mathbf{O} & \mathbf{D}_2 \\ \mathbf{O} & \mathbf{D}_3 & \mathbf{D}_4 & \mathbf{O} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{D}_m & \mathbf{O} & \mathbf{O} & \mathbf{D}_{m-1} \\ \mathbf{O} & \mathbf{D}_m & \mathbf{O} & \mathbf{O} \end{bmatrix}. \tag{25}$$

where matrix $\mathbf{D}_i$ is an $N \times N$ diagonal matrix in which all the main diagonal entries equal to a_i, it can be proved that $P_e \dot{\leq} \rho^{-4N(1-r')}$. Because the overall system diversity gain is dominated by the smallest diversity gain for all m, it thus is equivalent to that of the case in which $m = 1$.

Similarly, for the superposition-coded CDF, by replacing h_a in (18) with $\mathbf{h}_a$, replacing a_i in (19) with $\mathbf{D}_i$, it can be proved that $P_e \dot{\leq} \rho^{-4N(1-r')}$ when $m > 1$. The overall DMT is dominated by the case in which $m = 1$ and is equivalent to $d(r) = 3N(1 - r')$.

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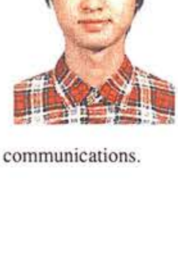

Chao Wang received his B.E. degree from University of Science and Technology of China (USTC), Hefei, China, in 2003 and his MSc degree (with distinction) from The University of Edinburgh, Edinburgh, UK, in 2005. He is currently a Ph.D. candidate at The University of Edinburgh and participates in the Delivery Efficiency Core Research Programme of the Virtual Centre of Excellence in Mobile and Personal Communications. His current research projects include multiple-input multiple-output (MIMO) wireless systems and cooperative communications.

Yijia Fan received his BEng degree in electrical engineering from Shanghai Jiao Tong University (SJTU), Shanghai, P.R. China, in July 2003, and PhD degree from the Institute for Digital Communications, University of Edinburgh, March, 2007. His PhD project was fully funded by Engineering and Physical Sciences Research Council (EPSRC), UK. He is currently a postdoctoral research associate in Department of Electrical Engineering, Princeton University. His research interests include signal processing and information theory and their applications in future wireless networks.

John S. Thompson received his BEng and PhD degrees from the University of Edinburgh in 1992 and 1996, respectively. From July 1995 to August 1999, he worked as a postdoctoral researcher at Edinburgh, funded by the UK Engineering and Physical Sciences Research Council (EPSRC) and Nortel Networks. In September 1999, he was appointed as a lecturer at the School of Engineering and Electronics at the University of Edinburgh. In October 2005, he was promoted to the position of reader. His research interests currently include signal processing algorithms for wireless systems, antenna array techniques and multihop wireless communications. He has published approximately 150 papers to date including a number of invited papers, book chapters and tutorial talks, as well as co-authoring an undergraduate textbook on digital signal processing. He is currently editor-in-chief of the IET Signal Processing journal and was a technical programme co-chair for the IEEE International Conference on Communications (ICC) 2007, held in Glasgow in June 2007.

H. Vincent Poor (S'72, M'77, SM'82, F'87) received the Ph.D. degree in EECS from Princeton University in 1977. From 1977 until 1990, he was on the faculty of the University of Illinois at Urbana-Champaign. Since 1990 he has been on the faculty at Princeton, where he is the Michael Henry Strater University Professor of Electrical Engineering and Dean of the School of Engineering and Applied Science. Dr. Poor's research interests are in the areas of stochastic analysis, statistical signal processing and their applications in wireless networks and related fields. Among his publications in these areas are the recent books *MIMO Wireless Communications* (Cambridge University Press, 2007) and *Quickest Detection* (Cambridge University Press, 2009).

Dr. Poor is a member of the National Academy of Engineering, a Fellow of the American Academy of Arts and Sciences, and a former Guggenheim Fellow. He is also a Fellow of the Institute of Mathematical Statistics, the Optical Society of America, and other organizations. In 1990, he served as President of the IEEE Information Theory Society, and in 2004-07 he served as the Editor-in-Chief of the *IEEE Transactions on Information Theory*. He is the recipient of the 2005 IEEE Education Medal. Recent recognition of his work includes the 2007 IEEE Marconi Prize Paper Award, the 2007 Technical Achievement Award of the IEEE Signal Processing Society, and the 2008 Aaron D. Wyner Award of the IEEE Information Theory Society.

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On the Diversity-Multiplexing Tradeoff of Concurrent Decode-and-Forward Relaying

Chao Wang, John S. Thompson
Institute for Digital Communications
University of Edinburgh
Edinburgh, EH9 3JL, UK
Email: {chao.wang, john.thompson}@ed.ac.uk

Yijia Fan, H. Vincent Poor
Department of Electrical Engineering
Princeton University
Princeton, NJ, 08544, USA
Email: {yijiafan, poor}@princeton.edu

***Abstract*—In this paper, the diversity-multiplexing tradeoff (DMT) behavior of a novel concurrent decode-and-forward (DF) relaying cooperative diversity transmission protocol is analyzed. A two-source two-relay one-destination scenario is considered in which concurrent transmission among the network nodes is used by combining the two sources' two classic DF relaying steps. Through the DMT analysis, it is shown that the proposed protocol can effectively recover the multiplexing loss induced by the classic DF relaying protocol, while still obtaining diversity gain. The system model is further extended to a generalized M-source network.**

I. Introduction

Cooperative diversity protocols [1]-[8] have attracted much interest in recent years because of their diversity improvement for relay networks. Relaying schemes consist of two steps: the source broadcasting message to both the relays and the destination (broadcasting step), and the relays forwarding the source message to the destination (relaying step). Due to the half-duplex limitation (relays cannot receive and transmit simultaneously), for the classic relaying transmission protocols (e.g. [1] [2]), the two steps often take two time-division-multiple-access (TDMA) time slots. For decode-and-forward (DF) relaying, repetition coding (the relay simply repeats its received message) is often applied and discussed due to its low complexity. Although diversity gain can be obtained to enhance the link reliability, the repetition-coded classic protocol loses spectral efficiency for the high signal-to-noise ratio (SNR) region and suffers from *multiplexing loss* (obtaining lower multiplexing gain than TDMA direct source-destination transmission) because of its inefficient use of two time slots to transmit one signal packet.

In order to recover the multiplexing loss, the protocols proposed in [6] and [7] use two amplify-and-forward (AF) relays to help the source in turn. Such an idea is applied for DF relaying in [8], where the scheme is called successive relaying. Reference [9] extends the single-source network in [8] into a two-user scenario, in which the scheme is called concurrent DF relaying. In this paper, we mainly study the diversity-multiplexing tradeoff (DMT) [10] behavior of the concurrent DF relaying protocol. The major difference between this paper and reference [9] is that [9] assumes a single-antenna setup and perfect source-relay channels, while in this paper we extend the network to a multiple antenna (at the destination only) scenario and relax the perfect source-relay channel assumption so that an adaptive protocol can be used. Specifically, we consider a five-node network with two sources, two relays and one common destination and permit the destination to be equipped with N antennas. We utilize concurrent transmissions [11] among the network nodes by combining one source's broadcasting step (time slot 1) with the other source's relaying step (time slot 2) and use $2L+1$ time slots to finish the transmission of L codewords from each source to the destination. Through DMT analysis, we show that our concurrent DF relaying protocol can improve the maximal multiplexing gain for each source from $\frac{1}{4}$ for the classic DF relaying protocol to $\frac{L}{2L+1}$, while still increasing the maximal diversity gain over TDMA direct source-destination transmission without the help of relays (from N to $2N$ for the perfect source-relay link assumption, or to $N+1$ for the adaptive protocol). Furthermore, when L is large, the maximal multiplexing gain for the concurrent DF relaying protocol approaches $\frac{1}{2}$, which matches the result for TDMA direct transmission. The multiplexing loss induced by the classic DF relaying protocol is thus fully recovered. Finally, we further extend our two-source system model to a generalized M-source network. The DMT performance results confirm the benefits of the M-source concurrent DF relaying protocol in terms of multiplexing performance while still providing significant diversity gain.

II. Protocol Description

We extend the work in [9] to a five-node network with two single-antenna sources (denoted as S_1 and S_2), two single-antenna *half-duplex* DF relays (denoted as R_1 and R_2), and one N-antenna destination (denoted as D). The transmitted messages from each source are divided into different frames, each containing L codewords denoted as x_i^j, $i=1,2$, $j=1,\ldots,L$. The two

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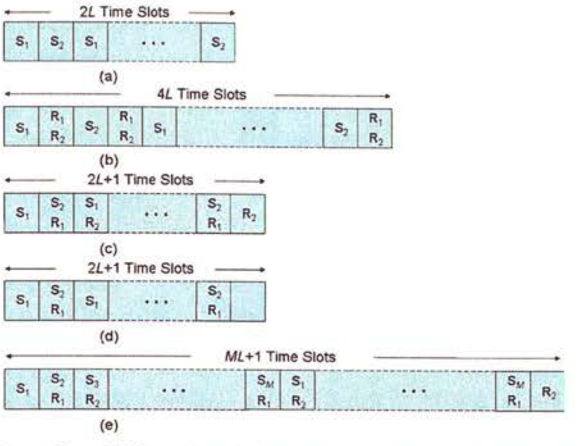

Fig. 1. Time-division channel allocations for (a) TDMA direct source-destination transmission, (b) classic DF relaying protocol, (c) concurrent DF relaying protocol with perfect source-relay links, (d) adaptive concurrent DF relaying protocol when only S_1 is helped by R_1, and (e) M-source concurrent DF relaying protocol with perfect source-relay links. The terminals displayed in each time slot denote the transmitters in that time slot.

sources use two independent Gaussian random codebooks, which are known by both relays. Each codeword x_i^j is independently chosen from the associated Gaussian random codebook. We assume a slow, flat, block Rayleigh fading environment. The fading channel coefficient for each channel realization is independent and identically distributed (i.i.d.). All transmitters are assumed to transmit with equal power.

Define the diversity gain d (i.e. the rate at which the outage probability P_{out} decays) and multiplexing gain r (i.e. the rate at which the transmission rate R scales with respect to $\log \rho$) as [10]

$$d = -\lim_{\rho\to\infty} \frac{\log P_{out}(\rho)}{\log \rho} \quad and \quad r = \lim_{\rho\to\infty} \frac{\log R(\rho)}{\log \rho} \qquad (1)$$

where ρ indicates the average receive SNR. Denote by $d^*_{m,n}(r)$ the optimal DMT achieved by an $m \times n$ multiple-input multiple-output (MIMO) system, i.e. [10]

$$d^*_{m,n}(r) = (m-r)(n-r). \qquad (2)$$

We assume the system is *symmetric* [12], which means the two sources have identical multiplexing gains r. For conventional TDMA direct source-destination transmission, the achievable DMT for each source is $d^*_{1,N}(2r)$. The maximal diversity gain is N and the maximal multiplexing gain is $\frac{1}{2}$ since the L codewords from each source are transmitted to the destination during $2L$ time slots as displayed in Fig. 1 (a).

The classic DF relaying protocol for a multiple-relay network was proposed by Laneman and Wornell [2] and a practical example, where two relays are available in the network, is studied in [3]. For such a protocol, each codeword's transmission is divided into two time slots due to the half-duplex limitation. During the first time slot (broadcasting step), the source broadcasts the codeword to the relays and the destination. During the second time slot (relaying step), the relays decode, re-encode (utilizing distributed space-time codes), and retransmit the codewords to the destination. The destination combines the codewords received from both time slots and performs decoding to recover the transmitted source information. Clearly, as displayed in Fig. 1 (b), $4L$ time slots are used to transmit the $2L$ codewords from the two sources. If the source-relay channels are sufficiently good such that the relays can always successfully decode the source codewords, the DMT for each source of the classic protocol is equivalent to $d^*_{1,3N}(4r)$. Although a maximal diversity gain $3N$ can be obtained, the classic protocol induces *multiplexing loss* as the maximal achievable multiplexing gain is reduced to $\frac{1}{4}$ because of the inefficient use of two time slots to transmit only one codeword. If the source-relay links are not good enough, an adaptive protocol called selection relaying [1] requires that the relays are used to help the sources only if they can correctly decode the source codewords. The adaptive protocol obtains the DMT $d^*_{1,N+2}(4r)$, which implies the maximal diversity gain is $N+2$, while the maximal multiplexing gain is still $\frac{1}{4}$.

To overcome the multiplexing limitation of the classic DF relaying protocol, we have proposed a concurrent DF relaying protocol in [9]. Instead of using both relays to help each source's transmission, we require S_1 and S_2 to be served by R_1 and R_2 respectively. We utilize concurrent transmission among the network nodes by combining one source's relaying step (time slot 2) with the other source's broadcasting step (time slot 1) and let one source and one relay communicate with the common destination simultaneously (except in the first and the last time slots) until the $2L$ codewords are finished transmitting during $2L+1$ time slots. The specific steps divided by each transmission (reception) time slot for the transmission of the two frames are described as follows:

Time slot 1: S_1 broadcasts x_1^1 to both R_1 and D; S_2 and R_2 remain silent.

Time slot 2: R_1 forwards x_1^1 to D. S_2 transmits x_2^1. R_2 listens to S_2 while being interfered by x_1^1 from R_1. D receives x_1^1 from R_1 and x_2^1 from S_2.

Time slot 3: R_2 forwards x_2^1 to D. S_1 transmits x_1^2. R_1 listens to S_1 while being interfered by x_2^1 from R_2. D receives x_2^1 from R_2 and x_1^2 from S_1.

This process repeats until the $(2L)$th time slot.

Time slot $2L+1$: R_2 decodes, re-encodes and retransmits x_2^L, the last codeword from S_2, to D.

After all the $2L$ codewords are received via both direct and relay links, D performs joint decoding to recover the information transmitted by the two sources. The time-division channel allocation and the transmission schedule for this protocol are illustrated in Fig. 1 (c) and Fig. 2, respectively.

The major issue of our concurrent DF relaying protocol is the interference generated among relays when one relay is listening to its associated source, while the other relay is forwarding its

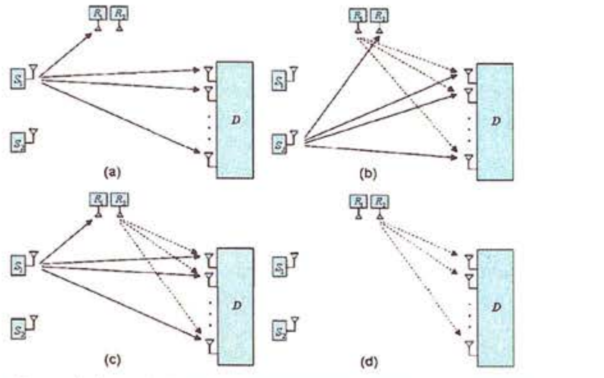

Fig. 2. Transmission schedule for the concurrent DF relaying protocol in (a) time slot 1, (b) time slot $2i$, $i = 1, \ldots, L$, (c) time slot $2i+1$, $i = 1, \ldots, L-1$, and (d) time slot $2L+1$. Solid lines and dashed lines denote the broadcasting step (time slot 1) and relaying step (time slot 2) of each source's classic DF relaying process, respectively

source message to the destination. In this paper, we focus on two specific situations. One situation is the *isolated-relay scenario* [7], which means the two relays are far away from each other and the quality of the inter-relay link is then much worse than those of the links between the relatively close sources and their individual relays. In this case, the interference between the relays is negligible when compared with the source-relay transmissions and thus can be ignored. The other situation is the *strong interference link scenario* [13], in which the quality of the inter-relay channel is much better than those of the other channels (i.e. the source-relay, relay-destination, source-destination channels). One practical example of this scenario is where the two relays are located very close to each other. In this case, we require each relay to decode the interference signal first. After that, the relay subtracts the interference signal from the received signal and then decodes the desired signal. The good quality of the inter-relay channel will guarantee that the relay can correctly decode the interference before decoding the desired source codeword with very high probability. Therefore, the interference between relays will not limit the system outage probability.

Obviously, the quality of the source-relay links also affects the network outage performance. In this paper, we divide our analysis into two parts. Firstly, we assume the source-relay links are sufficiently good such that the source messages are always correctly decoded by the relays. In this case, the system outage probability is not limited by the source-relay channel conditions.

Secondly, if the source-relay links are not good enough, we consider an adaptive protocol, i.e. a relay is activated to help its associated source only if it can correctly decode its source codewords, i.e.

$$\log(1+\rho|h_{S_i,R_i}|^2) \geq R \tag{3}$$

where h_{S_i,R_i} denotes the channel fading coefficient between S_i and R_i ($i = 1, 2$) and R denotes the source transmission rate. Otherwise, the relay remains *silent* when the other source

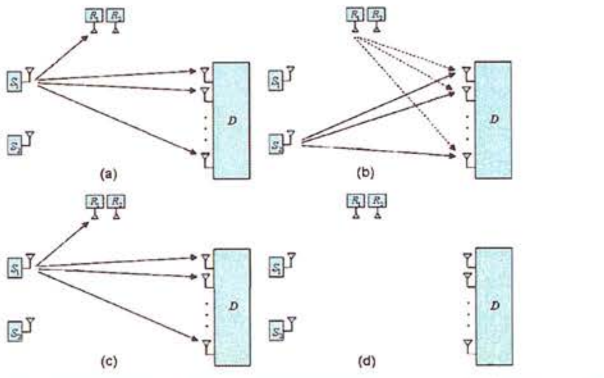

Fig. 3. Transmission schedule for the adaptive concurrent DF relaying protocol when only S_1 is helped by the relay R_1 in (a) time slot 1, (b) time slot $2i$, $i = 1, \ldots, L$, (c) time slot $2i+1$, $i = 1, \ldots, L-1$, and (d) time slot $2L+1$.

transmits. We assume that the sources are not aware of whether their relays are activated to assist them. Then, the transmission of the $2L$ codewords *always* takes $2L+1$ time slots. Specifically, if (3) holds for both $i = 1, 2$, both relays can decode their source codewords. This case is the one displayed in Fig. 1 (c). If (3) holds for neither relay, the adaptive protocol acts as TDMA direct transmission except that during the $(2L+1)$th time slot, the destination will not receive information from any transmitter. If (3) only holds for one relay (e.g. R_1), only relay R_1 is activated, while R_2 keeps silent for all the $2L+1$ time slots. The time-division channel allocation and the transmission schedule for this example are illustrated in Fig. 1 (d) and Fig. 3, respectively.

III. Diversity-Multiplexing Tradeoff Analysis

If the assumption of sufficiently good source-relay links can be satisfied and the source messages are assumed to be successfully decoded and retransmitted by the relays to the destination for all the $2L+1$ time slots, our concurrent DF relaying protocol mimics a $2L$-user multiple access single-input multiple-output (SIMO) channel. The associated input-output channel relation for the relay network can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{4}$$

where the equivalent channel matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_{S_1,D} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{h}_{R_1,D} & \mathbf{h}_{S_2,D} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{h}_{R_2,D} & \mathbf{h}_{S_1,D} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{h}_{R_1,D} & \mathbf{h}_{S_2,D} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h}_{R_2,D} \end{bmatrix}, \tag{5}$$

$\mathbf{0}$ denotes an $N \times 1$ zero vector, $\mathbf{h}_{a,D}$ is the $N \times 1$ channel fading vector between node a and the destination, $\mathbf{x} = [x_1^1\ x_2^1\ x_1^2\ x_2^2\ \ldots\ x_1^L\ x_2^L]^T$ is the $2L \times 1$ transmit signal vector, $\mathbf{y} = [\mathbf{y}_1^T\ \mathbf{y}_2^T\ \ldots\ \mathbf{y}_{2L+1}^T]^T$, $\mathbf{y}_i$ is the $N \times 1$ receive signal vector at the ith time slot, and $\mathbf{n}$ is the $(2L+1)N \times 1$ complex circular additive white Gaussian noise (AWGN) vector at the destination. Unlike conventional multiple access SIMO channels, the dimensions of $\mathbf{H}$, $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{n}$ are expanded in the time domain rather than the space domain. In terms of the DMT, we have the following theorem.

Theorem 1: In a symmetric scenario, on assuming that the source codewords are correctly decoded by the relays, the achievable DMT for each source of the concurrent DF relaying protocol (i.e. the system model in (4) with channel matrix (5)) is equivalent to $d^*_{1,2N}(\frac{2L+1}{L}r)$ and can be expressed as

$$d(r) = 2N\left(1 - \frac{2L+1}{L}r\right)^+ \tag{6}$$

where $(x)^+$ means $\max\{0, x\}$.

Proof: For such a multiple-access SIMO system, following the capacity calculation in [14], there are $(2^{2L}-1)$ source transmission rate constraints for a given realization of the channel, which can be expressed as,

$$R \le \log\left(\det\left(\mathbf{I} + \rho\mathbf{h}_k\mathbf{h}_k^H\right)\right) \tag{7}$$

$$2R \le \log\left(\det\left(\mathbf{I} + \rho\mathbf{h}_{k_1}\mathbf{h}_{k_1}^H + \rho\mathbf{h}_{k_2}\mathbf{h}_{k_2}^H\right)\right) \tag{8}$$

$$\cdots$$

$$2LR \le \log\left(\det\left(\mathbf{I} + \rho\mathbf{H}\mathbf{H}^H\right)\right) \tag{9}$$

where $\mathbf{h}_k$ denotes the kth column of $\mathbf{H}$. We define an outage event as occurring when any inequality from (7) to (9) is not satisfied. The proof thus can be made by finding the minimal diversity order within all outage dominated events, considering each constraint from (7) to (9).

We assume that each source codeword is chosen from its source's Gaussian random codebook of length l and is transmitted with data rate R. Since each source uses $2L+1$ time slots to transmit L different codewords, the average transmission rate is $\overline{R} = \frac{L}{2L+1}R$ bits per channel use (BPCU). We assume the average transmission rate changes as $\overline{R} = r\log\rho$ with respect to ρ, and it is easy to see $R = \frac{2L+1}{L}r\log\rho$. Obviously, (7) leads to the achievable diversity gain of (6) since (7) represents the rate constraint of a $1\times 2N$ SIMO channel. It can also be proved that the diversity gain induced by any rate constraint from (8) to (9) is no less than $d^*_{1,2N}(\frac{2L+1}{L}r)$. Therefore, the achievable DMT for our concurrent DF relaying protocol is expressed as (6). Due to limited space, here we omit the detailed proof, which can be found in [15]. ■

From (6) we can see that if the source-relay links are sufficiently good, our concurrent DF relaying protocol achieves the maximal multiplexing gain $\frac{L}{2L+1}$ for each user. Compared with the maximal multiplexing gain $\frac{1}{4}$ for the classic protocol, our scheme has a smaller multiplexing loss induced by the half-duplex operation in the relays. When L is chosen as a large value, the maximal multiplexing gain can approach $\frac{1}{2}$, which is achieved by TDMA direct transmission. The multiplexing loss

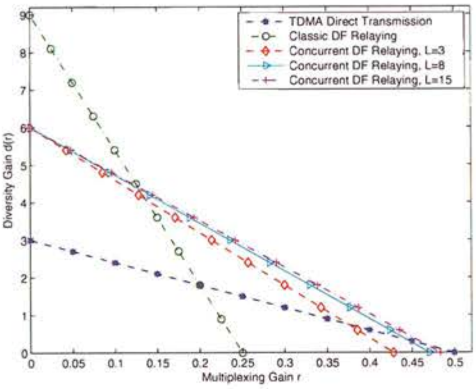

Fig. 4. Diversity-multiplexing tradeoff performance of the concurrent DF relaying protocol, $N = 3$

is thus *fully* recovered. Since each codeword is transmitted via $2N$ independent paths, the maximal diversity gain achieved by this protocol is $2N$. Although the diversity improvement of the concurrent DF relaying is not as much as that of the classic protocol, when compared with TDMA direct transmission, our protocol also increases the diversity gain significantly. Fig. 4 displays an example ($N = 3$) of the DMT comparison.

If the quality of the source-relay links are not good enough to guarantee perfect source-relay transmissions, the adaptive protocol is utilized. As discussed in Section II, if neither relay is activated, the adaptive protocol acts as TDMA direct transmission except that $2L+1$ time slots are used to finish the transmission. If both relays are activated, the equivalent channel matrix of the adaptive protocol is expressed by (5). If only one relay is activated, e.g. only S_1 is assisted by R_1, the equivalent channel matrix can be written as

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_{S_1,D} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{h}_{R_1,D} & \mathbf{h}_{S_2,D} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{h}_{S_1,D} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{h}_{R_1,D} & \mathbf{h}_{S_2,D} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \end{bmatrix}. \tag{10}$$

The rate constraints (7)-(9) can still be applied to this case except the matrix $\mathbf{H}$ is defined in (10). Therefore, the achievable DMT of the adaptive protocol can be summarized in the following theorem.

Theorem 2: In a symmetric scenario, the achievable DMT for each source of the adaptive concurrent DF relaying protocol is equivalent to $d^*_{1,N+1}(\frac{2L+1}{L}r)$ and can be expressed by

$$d(r) = (N+1)\left(1 - \frac{2L+1}{L}r\right)^+. \tag{11}$$

Proof: There are four situations that need to be considered: 1) both relays are in outage; 2) only the relay R_1 is in outage;

3) only the relay R_2 is in outage; and 4) neither of them is in outage. We denote the outage probability in R_1 and R_2 as $P^O_{R_1}$ and $P^O_{R_2}$. $P^O_{D_1}$, $P^O_{D_2}$, $P^O_{D_3}$ and $P^O_{D_4}$ denote the outage probabilities in the *destination* for the four different situations, respectively. The overall outage probability then can be expressed as

$$\begin{aligned} P^O &= P^O_{R_1}(1-P^O_{R_2})P^O_{D_2} + (1-P^O_{R_1})P^O_{R_2}P^O_{D_3} \\ &\quad + P^O_{R_1}P^O_{R_2}P^O_{D_1} + (1-P^O_{R_1})(1-P^O_{R_2})P^O_{D_4} \end{aligned} \tag{12}$$

The system diversity gain is thus dominated by the lowest diversity gain induced by each term in (12).

The diversity gain induced by $P^O_{R_1}$ and $P^O_{R_2}$ is $(1-\frac{2L+1}{L}r)^+$ since the sources and relays are single-antenna terminals. It is also clear that for TDMA direct source-destination transmission, $P^O_{D_1}$ leads to diversity gain $N(1-\frac{2L+1}{L}r)^+$. From *Theorem 1* we know $P^O_{D_4}$ leads to diversity gain $2N(1-\frac{2L+1}{L}r)^+$. We can also prove that diversity gain $N(1-\frac{2L+1}{L}r)^+$ can be achieved by $P^O_{D_2}$ and $P^O_{D_3}$. Therefore, for the high SNR regime, the overall outage probability P^O is dominated by the term with diversity gain $(N+1)(1-\frac{2L+1}{L}r)^+$. The proof is thus complete. A more detailed proof can be found in [15]. ∎

Theorem 2 implies that the adaptive protocol obtains the maximal diversity gain $N+1$ (due to the fact that only one channel between each source and its relay exists) and the maximal multiplexing gain $\frac{L}{2L+1}$. The multiplexing improvement over the classic adaptive DF relaying protocol is obvious. Compared with TDMA direct transmission, if N is large, the maximal diversity gain of the concurrent DF relaying protocol is almost same as that of TDMA direct transmission and thus the advantage of our adaptive protocol may not be significant. However, if the destination is equipped with a small number of antennas (e.g. $N=1$ or 2), the diversity improvement is clear and if we choose large L, our scheme will have almost no multiplexing loss. The direct transmissions between sources and the destination thus benefit from the help of the relays. An example of the DMT comparison ($N=2$) is illustrated in Fig. 5.

IV. M-Source Concurrent DF Relaying

The two-source system model can be easily extended to a large network with M single-antenna sources, two single-antenna relays and one N-antenna destination, as displayed in Fig. 6. The basic idea is that the M sources communicate with the common destination using TDMA and the two relays take turns helping each source until the transmission of the L codewords from each source is finished. Regarding the time used to complete the transmission of the ML codewords, TDMA direct transmission uses ML time slots, the classic protocol uses $2ML$ time slots, and the concurrent DF relaying protocol uses $ML+1$ time slots. For example, if M is an even number, on assuming all the source-relay links are sufficiently good and the source codewords are correctly decoded by the relays, the specific transmission steps can be described as follows:

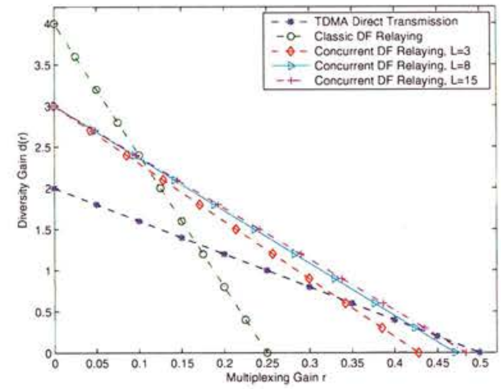

Fig. 5. Diversity-multiplexing tradeoff performance of the adaptive concurrent DF relaying protocol, $N=2$

Time slot 1: S_1 broadcasts x^1_1 to R_1 and D.

Time slot 2: R_1 forwards x^1_1. S_2 broadcasts x^1_2 to R_2 and D.

Time slot 3: R_2 forwards x^1_2. S_3 broadcasts x^1_3 to R_1 and D.

The transmission proceeds similarly until the Mth time slot.

Time slot $M+1$: R_2 forwards x^1_M. S_1 broadcasts x^2_1 to R_1 and D.

Time slot $M+2$: R_1 forwards x^2_1. S_2 broadcasts x^2_2 to R_2 and D.

The progress repeats until the (ML)th time slot.

Time slot $ML+1$: R_2 decodes, re-encodes and retransmits x^L_M, the last codeword from S_M, to D.

Thus, the M-source concurrent DF relaying protocol mimics an ML-user multiple access SIMO channel with input-output relation

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{13}$$

where the equivalent channel matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_{S_1,D} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{h}_{R_1,D} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{h}_{S_M,D} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{h}_{R_2,D} & \mathbf{h}_{S_1,D} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h}_{R_1,D} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{h}_{R_1,D} & \mathbf{h}_{S_M,D} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h}_{R_2,D} \end{bmatrix}, \tag{14}$$

$\mathbf{x} = [x^1_1\ x^1_2\ \ldots\ x^1_M\ x^2_1\ x^2_2\ \ldots\ x^L_M]^T$ is the $ML \times 1$ transmit signal vector, $\mathbf{y} = [\mathbf{y}^T_1\ \mathbf{y}^T_2\ \ldots\ \mathbf{y}^T_{ML+1}]^T$ is the $(ML+1)N \times 1$ receive signal vector. The time-division channel allocation and the transmission schedule are illustrated in Fig. 1 (e) and Fig. 6 respectively. For the adaptive protocol, the analysis is

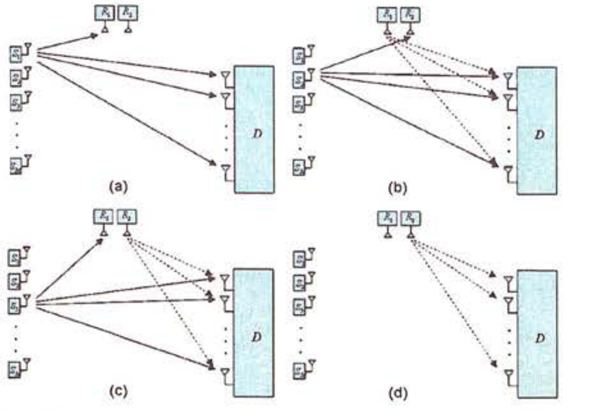

Fig. 6. Transmission schedule for the M-Source concurrent DF relaying protocol in (a) time slot 1, (b) time slot $(Mi+2)$, $i=0,\ldots,L-1$, (c) time slot $(Mi+3)$, $i=0,\ldots,L-1$, and (d) time slot $ML+1$.

straightforward. Therefore, in terms of the DMT, we summarize the results as the following corollaries to *Theorem 1* and *2*.

Corollary 1: In a symmetric scenario, on assuming that the source codewords are correctly decoded by the relays, the achievable DMT for each source of the M-source concurrent DF relaying protocol (i.e. the system model in (13) with channel matrix (14)) is equivalent to $d^*_{1,2N}(\frac{ML+1}{L}r)$ and can be expressed as

$$d(r) = 2N\left(1-\frac{ML+1}{L}r\right)^+. \quad (15)$$

Corollary 2: In a symmetric scenario, the achievable DMT for each source of the adaptive M-source concurrent DF relaying protocol is equivalent to $d^*_{1,N+1}(\frac{ML+1}{L}r)$ and can be expressed as

$$d(r) = (N+1)\left(1-\frac{ML+1}{L}r\right)^+. \quad (16)$$

Corollary 1 and *Corollary 2* imply that, compared with the classic DF relaying protocol for the M-source network, our concurrent DF relaying protocols can improve the maximal achievable multiplexing gain from $\frac{1}{2M}$ to $\frac{L}{ML+1}$. Furthermore, if ML is a large number, the maximal multiplexing gain approaches $\frac{1}{M}$ (the maximal multiplexing gain for TDMA direct transmission). In this way, the multiplexing loss is fully recovered and the requirement of L being very large is relaxed. Obviously, when $M=1$, the protocol is the successive relaying protocol proposed in [8], and when $M=2$, the protocol is the concurrent DF relaying protocol discussed in the previous two sections.

V. CONCLUSION

We have studied the diversity-multiplexing tradeoff behavior of the concurrent DF relaying cooperative diversity transmission protocol. The DMT performance indicates that the proposed protocol can effectively compensate the multiplexing loss of the classic DF relaying protocol and still improve the diversity gain over TDMA direct transmission.

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Superposition-Coded Concurrent Decode-and-Forward Relaying

Chao Wang*, Yijia Fan†, Ioannis Krikidis*, John S. Thompson* and H. Vincent Poor†
*Institute for Digital Communications, University of Edinburgh, Edinburgh, UK
†Department of Electrical Engineering, Princeton University, Princeton, USA

Abstract—**In this paper, a superposition-coded concurrent decode-and-forward (DF) relaying protocol is presented. A specific scenario, where the inter-relay channel is sufficiently strong, is considered. Assuming perfect source-relay transmissions, the proposed scheme further improves the diversity performance of previously proposed repetition-coded concurrent DF relaying, in which the advantage of the inter-relay interference is not fully extracted.**

I. Introduction

The exploitation of cooperation among users has been studied in recent years as a means for improving diversity performance for single-antenna wireless systems. Due to the half-duplex limitation, standard cooperative diversity protocols (e.g. [1] [2]) usually require two time-division-multiple-access (TDMA) time slots to finish each signal codeword's transmission. Although diversity gain can be improved over conventional TDMA direct source-destination transmission, standard cooperation protocols result in lost spectral efficiency, especially in the high signal-to-noise ratio (SNR) region.

To overcome the multiplexing limitation of standard protocols, an advanced successive relaying protocol (independently proposed by [3], [4], and [5] in different contexts) has been considered such that two relays take turns helping the source to mimic a full-duplex relay. The single-source single-antenna network studied in [5] has been extended to a two-source multiple-antenna (at the destination only) scenario in [6] and [7], in which the scheme is termed concurrent decode-and-forward (DF) relaying. For such a protocol, a two-source two-relay one-destination cooperation network has been considered. The two sources' standard DF relaying steps are combined so that the degrees of the freedom of the channel are efficiently used and the multiplexing loss induced by standard protocols can be effectively recovered.

The major issue with concurrent DF relaying is that the interference generated among the two relays significantly affects the system diversity-multiplexing tradeoff (DMT) performance. In [7], two specific scenarios (i.e. the *isolated-relay* and *strong-interference* scenarios) are examined to investigate the impact of the inter-relay interference. However, for both scenarios, reference [7] requires the relays to use repetition coding to retransmit their source messages. In this paper, we argue that such an assumption is not very efficient for the strong-interference scenario because the advantage of the inter-relay interference, which is also useful information, is not fully extracted. Specifically, for the strong-interference scenario, instead of requiring each relay to forward its own source's codeword, we permit it to use superposition coding to transmit both sources' codewords. In this way, the achievable diversity gain can be further improved with the sacrifice of only one extra transmission time slot. When the signal frame length L is large, the multiplexing loss induced by this extra transmission time is negligible.

The rest of this paper is organized as follows. In Section II, we briefly review the DMT behavior of the repetition-coded concurrent DF relaying protocol and present the superposition-coded concurrent DF relaying protocol for a two-source network. The system model is generalized to an M-source network in Section III. Finally, we offer simulation results and discussions in Section IV.

II. Two-Source Concurrent DF Relaying

We first study a five-node network with two single-antenna sources S_1 and S_2, two single-antenna *half-duplex* DF relays R_1 and R_2, and one N-antenna destination D. The transmitted messages from each source are divided into different frames, each containing L codewords denoted as x_i^j, $i = 1, 2$, $j = 1, \ldots, L$. Two independent Gaussian random codebooks are used by the two sources and are known by both relays. Each codeword x_i^j is *independently* chosen from the associated Gaussian random codebook and has unit average power. A slow, flat, block Rayleigh fading environment is assumed, where the channel remains static for one coherence interval (two frame periods) and changes independently in different coherence intervals. Moreover, we assume a uniform power allocation scheme, i.e. the total transmit power in each transmission time slot remains the same and each terminal transmits with equal power.

A. Repetition-Coded Concurrent DF Relaying

For such a two-relay scenario, due to the half-duplex operation of the relays, for each source codeword, the *space-time-coded standard DF relaying* protocol [8], which is a practical example of the protocol proposed by [2], requires each source to broadcast the codeword to both relays and the destination in the first time slot (broadcasting step). The relays then retransmit the codeword (using a distributed Alamouti space-time block code) to the destination in the second time slot (relaying step), as shown in Fig. 1 (b). Assuming the source messages are correctly decoded by the relays, the standard protocol can provide significant diversity gain improvement

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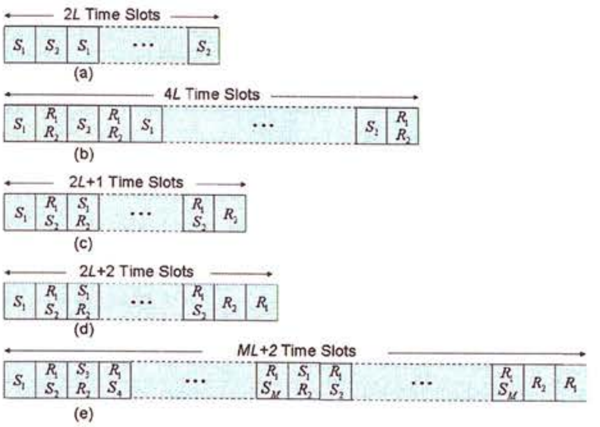

Fig. 1. Time-division channel allocations for (a) TDMA direct transmission, (b) space-time-coded standard DF relaying, (c) repetition-coded concurrent DF relaying, (d) superposition-coded concurrent DF relaying for the two-source network, and (e) superposition-coded concurrent DF relaying for the M-source network (M is even). The terminals displayed in each time slot denote the transmitters in that time slot.

over TDMA direct source-destination transmission. However, to finish the transmission of the $2L$ codewords from the two sources to the destination, $4L$ time slots must be used. Compared with TDMA direct transmission displayed in Fig. 1 (a), which needs only $2L$ time slots, the standard protocol loses spectral efficiency, especially for the high SNR region.

In order to compensate for the multiplexing gain reduction induced by the standard protocol, for concurrent DF relaying [6] it is assumed that each source is individually assisted by one relay (i.e. S_1 and S_2 are supported by R_1 and R_2 respectively) and one source's broadcasting step is combined with the other source's relaying step. As displayed in Fig. 1 (c), except in the first and the last time slots, one relay and one source always communicate with the destination simultaneously so that only $(2L+1)$ time slots are needed to finish the transmission of the $2L$ codewords.

It is clear that the interference generated among relays can significantly degrade the system capacity and diversity performance. However, the two relays may be *isolated* [4], which means the quality of the inter-relay link is much worse than those of the source-relay links. In this case, the inter-relay interference is trivial compared with source-relay transmissions and thus can be ignored. Since the relays are assumed to simply repeat their source codewords after decoding them, we refer to this transmission scheme as the *repetition-coded concurrent DF relaying* throughout the paper.

Define the diversity gain d and multiplexing gain r as those in [9] and assume the system is *symmetric* [10], where the two sources have identical multiplexing gains r. Assuming the source-relay links are sufficiently strong such that the relays can always perfectly decode their source messages, the DMT achieved by each source for the repetition-coded concurrent DF relaying protocol can be expressed by [7]

$$d(r) = 2N\big(1 - \frac{2L+1}{L} r\big). \tag{1}$$

The repetition-coded concurrent DF relaying significantly improves the diversity performance over TDMA direct transmission (with DMT $d(r) = N(1-2r)$) except for a multiplexing loss $\frac{1}{2} - \frac{L}{2L+1} = \frac{1}{4L+2}$. Such multiplexing loss decreases as L increases and can be neglected for large frame length L. However, compared with the space-time-coded standard DF relaying (with DMT $d(r) = 3N(1-4r)$), the repetition-coded concurrent DF relaying obtains smaller diversity gain when $0 \le r \le \frac{L}{8L-2}$ since each codeword is only forwarded by one relay.

B. Superposition-Coded Concurrent DF relaying

A *strong-interference scenario* [11], where the channel between the two relays is sufficiently stronger than the source-relay links, is also studied in [7]. In this case, each relay is required to decode the interference signal first and subtract it from the received signal before decoding the desired signal. The good quality of the inter-relay channel guarantees that each relay can correctly decode the interference before decoding its desired source codeword with very high probability. Therefore, the interference between relays does not limit the system DMT performance. However, for such a strong-interference scenario, reference [7] still assumes that each relay only forwards its own source message (the desired signal). In fact, since the interference signal is the transmitted codeword from the other source, in this paper, we argue that we can make use of the interference signal to further improve the system diversity gain. Specifically, we permit the relays to use superposition coding [11] to retransmit both sources' messages, i.e. instead of retransmitting its desired source codeword, each relay transmits the sum of the interference codeword and the desired codeword. To guarantee every codeword to be transmitted via three independent paths, $(2L+2)$ time slots are used to finish the transmission of the $2L$ codewords from the two sources. The transmission of the two frames can be described as follows:

Time slot 1: S_1 broadcasts x_1^1 to both R_1 and D; S_2 and R_2 remain silent.

Time slot 2: R_1 forwards x_1^1 to D and S_2 transmits x_2^1. R_2 listens to S_2 while being interfered by x_1^1 from R_1. D receives x_1^1 from R_1 and x_2^1 from S_2.

Time slot 3: R_2 forwards $(x_2^1 + x_1^1)$ to D. S_1 transmits x_1^2. R_1 listens to S_1 while being interfered by $(x_2^1 + x_1^1)$ from R_2. D receives $(x_2^1 + x_1^1)$ from R_2 and x_1^2 from S_1.

Time slot 4: R_1 forwards $(x_1^2 + x_2^1)$ to D. S_2 transmits x_2^2. R_2 listens to S_2 while being interfered by $(x_1^2 + x_2^1)$ from R_1. D receives $(x_1^2 + x_2^1)$ from R_1 and x_2^2 from S_2.

This process repeats until the $(2L)$th time slot.

Time slot $2L+1$: R_2 retransmits $(x_2^L + x_1^L)$ to R_1 and D.

Time slot $2L+2$: R_1 decodes, re-encodes and retransmits x_2^L to D.

Unlike the repetition-coded case, from the 3rd to the $(2L+1)$th time slot, the interference signal received by each relay is not only the other relay's desired source codeword, but also the codeword transmitted by the relay itself during the previous time slot. Because each relay has full knowledge of its own transmitted codeword, it can subtract its previously transmitted codeword from the received signal before decoding without

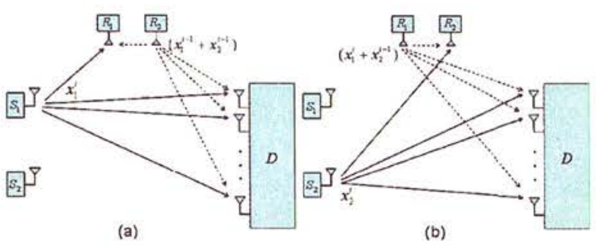

Fig. 2. Transmission schedule for the superposition-coded concurrent DF relaying protocol (from time slot 3 to time slot $2L$) in (a) time slot $2i-1$, and (b) time slot $2i$, $i = 2, \dots, L$. Solid lines and dashed lines denote the broadcasting step (time slot 1) and relaying step (time slot 2) of each source's standard DF relaying process respectively.

any difficulty. After all the $2L$ codewords are received, D performs joint decoding to recover the source information. We refer to this protocol as the *superposition-coded concurrent DF relaying* and its time-division channel allocation and the transmission schedule (from the 3rd time slot to the $2L$th time slot) are illustrated in Fig. 1 (d) and Fig. 2 respectively.

Assuming perfect source-relay transmissions, the proposed protocol mimics a $2L$-user multiple access single-input multiple-output (SIMO) channel (except that the dimensions of the signals are expanded in the time domain):

$$\mathbf{y} = \sqrt{\rho}\ \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{2}$$

in which the equivalent channel matrix is

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_{S_1} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \frac{\mathbf{h}_{R_1}}{\sqrt{2}} & \frac{\mathbf{h}_{S_2}}{\sqrt{2}} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \frac{\mathbf{h}_{R_2}}{\sqrt{4}} & \frac{\mathbf{h}_{R_2}}{\sqrt{4}} & \frac{\mathbf{h}_{S_1}}{\sqrt{2}} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\mathbf{h}_{R_1}}{\sqrt{4}} & \frac{\mathbf{h}_{R_1}}{\sqrt{4}} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \frac{\mathbf{h}_{R_1}}{\sqrt{4}} & \frac{\mathbf{h}_{S_2}}{\sqrt{2}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \frac{\mathbf{h}_{R_2}}{\sqrt{2}} & \frac{\mathbf{h}_{R_2}}{\sqrt{2}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h}_{R_1} \end{bmatrix}, \tag{3}$$

where $\mathbf{h}_a$ is the $N \times 1$ channel fading vector between node a and the destination, $\mathbf{0}$ denotes an $N \times 1$ all zero vector, $\mathbf{y} = [\mathbf{y}_1^T\ \mathbf{y}_2^T\ \dots\ \mathbf{y}_{2L+2}^T]^T$, $\mathbf{y}_i$ is the $N \times 1$ receive signal vector at the ith time slot, $\mathbf{x} = [x_1^1\ x_2^1\ x_1^2\ \dots\ x_2^L]^T$ is the $2L \times 1$ transmit signal vector, $\mathbf{n}$ is a $(2L+2)N \times 1$ unit power complex circular additive white Gaussian noise (AWGN) vector at the destination, and ρ means the average received SNR. It is worth noting that the scaling factors $\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{4}}$ come from the uniform power allocation assumption and have no consequence for the system infinite-SNR DMT performance. In terms of the achievable DMT, we have the following theorem.

Theorem 1: In a symmetric scenario, on assuming that the source codewords are correctly decoded by the relays, the achievable DMT for each source of the superposition-coded concurrent DF relaying protocol (i.e. the system model in (2)) is given by

$$d(r) = 3N\big(1 - \frac{2L+2}{L} r\big). \tag{4}$$

Proof: For a symmetric $2L$-user multiple-access SIMO system described in (2), following the capacity calculation in [12], there are $(2^{2L} - 1)$ source transmission rate constraints for a given realization of the channel:

$$R \le \log\left(\det\left(\mathbf{I} + \rho \mathbf{h}_k \mathbf{h}_k^H\right)\right), \tag{5}$$

$$2R \le \log\left(\det\left(\mathbf{I} + \rho \mathbf{h}_{k_1} \mathbf{h}_{k_1}^H + \rho \mathbf{h}_{k_2} \mathbf{h}_{k_2}^H\right)\right), \tag{6}$$

$$\vdots$$

and

$$2LR \le \log\left(\det\left(\mathbf{I} + \rho \mathbf{H}\mathbf{H}^H\right)\right), \tag{7}$$

where $\mathbf{h}_k$ denotes the kth column of $\mathbf{H}$. The system diversity gain is thus the smallest diversity gain calculated by all the constraints from (5) to (7).

Consider an $(m+2)N \times m$ multiple-input multiple-output (MIMO) channel (each codeword s_i has multiplexing gain $r' = \frac{2L+2}{L} r$ so that the average transmission rate $\bar{R} = \frac{L}{2L+2} r' \log \rho = r \log \rho$)

$$\begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \mathbf{r}_3 \\ \vdots \\ \mathbf{r}_{m+1} \\ \mathbf{r}_{m+2} \end{bmatrix} = \sqrt{\rho} \begin{bmatrix} \mathbf{g}_1 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{g}_2 & \mathbf{g}_3 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{g}_4 & \mathbf{g}_4 & \mathbf{g}_1 & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{g}_2 & \mathbf{g}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{g}_{k_1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{g}_{k_2} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{g}_{k_3} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ \vdots \\ s_m \end{bmatrix} + \mathbf{n}, \tag{8}$$

where $k_1 = 1$, $k_2 = 2$, and $k_3 = 4$ when m is odd and $k_1 = 3$, $k_2 = 4$, and $k_3 = 2$ when m is even. For infinite SNR, the task of finding the smallest diversity gain obtained by each constraint from (5) to (7) is the same as finding the smallest diversity gain achieved by the system (8) for every $1 \le m \le 2L$ [6].

When $m = 1$, the system model in (8) is a $1 \times 3N$ SIMO system. The achievable DMT is clearly $d(r) = 3N\left(1 - r'\right) = 3N\left(1 - \frac{2L+2}{L} r\right)$. When $m > 1$, applying a method similar to that used for the DMT calculation for the ISI channels in [13], it is not difficult to show that $d(r) = 4N\left(1 - r'\right)$. Because the overall system diversity gain is dominated by the smallest one for all m, it thus is (i.e. the case where $m = 1$) the same as the right hand side of (4). Due to limited space, here we omit the detailed proof, which can be found in [14]. ■

Theorem 1 indicates that superposition-coded concurrent DF relaying obtains the maximal diversity gain $3N$ and maximal multiplexing gain $\frac{L}{2L+2}$. This means that the diversity performance of the repetition-coded concurrent DF relaying is further improved by making use of the inter-relay interference. Therefore, unlike the repetition-coded case, where the achievable diversity gain is larger than that of the space-time-coded standard protocol only in the high r region, superposition-coded concurrent DF relaying strictly outperforms the standard protocol within the range of all possible multiplexing gains (except for the worst case $L = 1$, where the two protocols have identical performance). Although there exists a

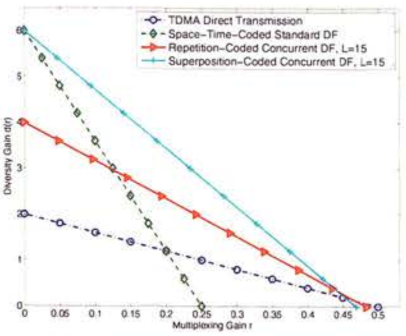

Fig. 3. DMT performance for different protocols with $N = 2$.

slight difference for the maximal achievable multiplexing gain $\frac{L}{2L+1} - \frac{L}{2L+2} = \frac{L}{(2L+1)(2L+2)}$ between the repetition-coded and superposition-coded concurrent DF relaying protocols (due to the extra transmission time slot), when L is large this difference is negligible and the maximal multiplexing gains for both protocols approach $\frac{1}{2}$. The multiplexing loss induced by the standard protocol is fully compensated in both protocols. Fig. 3 displays an example ($N = 2$, $L = 15$) of the DMT comparison.

Throughout this paper, we assume that the source-relay transmissions are perfect so that the system diversity gain is not limited by the quality of source-relay links. Making use of the inter-relay interference can thus further improve the diversity performance over the simple repetition-coded protocol. One may argue that, in practical systems, such good source-relay links may not be able to be guaranteed and the system DMT performance may be affected by any weak source-relay link. In fact, in a general cooperation network, there usually exist multiple terminals which can act as potential relays. If the number of potential relays is very large, the probability of selecting at least one relay pair such that one relay can correctly decode one source and the other relay can correctly decode the other source is sufficiently high. In this case, the system DMT performance behaves the same as the case in which the transmissions between the sources and their relays are always successful. Therefore, our assumption is actually not uncommon in reality. The impact of using relay selection schemes in multiple-relay scenarios on the system DMT performance is currently under investigation.

III. M-Source Concurrent DF Relaying

The two-source system model can also be extended to a large network with M single-antenna sources, two single-antenna relays and one N-antenna destination, as has been done for the repetition-coded case in [7]. The basic idea is that the M sources communicate with the common destination using TDMA and the two relays take turns helping each source until the transmission of the L codewords from each source is finished. Therefore, $ML + 2$ time slots are used to complete the transmission of the ML codewords from the M sources. Assuming perfect decoding at the relays, the time-division channel allocation is illustrated in Fig. 1 (e) (where M is even) and in terms of the achievable DMT, we have the following corollary to *Theorem 1*.

Corollary 1: In a symmetric scenario, on assuming perfect source-relay transmissions, the achievable DMT for each source of the superposition-coded M-source concurrent DF relaying protocol is given by

$$d(r) = 3N\left(1 - \frac{ML+2}{L}r\right). \tag{9}$$

Corollary 1 implies that, compared with repetition-coded concurrent DF relaying for the M-source network, which needs $(ML + 1)$ time slots and obtains DMT $d(r) = 2N\left(1 - \frac{ML+1}{L}r\right)$, the superposition-coded protocol improves the maximal achievable diversity gain from $2N$ to $3N$, but reduces the maximal achievable multiplexing gain from $\frac{L}{ML+1}$ to $\frac{L}{ML+2}$. However, if ML is large, the maximal multiplexing gain difference is negligible and both gains approach $\frac{1}{M}$ (the maximal multiplexing gain for TDMA direct transmission) so that the multiplexing loss is fully recovered and the requirement of L being large is relaxed. Clearly, when $M = 1$, the system model is the single-source scenario studied in the content of the successive relaying protocol proposed in [5]. This means that superposition coding can also be used in successive relaying to further increase diversity performance and thus (9) offers a generalized result.

IV. Simulation Results and Discussions

In this section, we compare our two-source superposition-coded concurrent DF relaying scheme with other schemes discussed in Section II in terms of error probability through Monte-Carlo simulations. The source messages are assumed to be always correctly decoded by the relays. In our simulations, we consider the signal frame lengths $L = 1$ and $L = 2$ for the repetition-coded and superposition-coded concurrent DF relaying protocols, respectively. For this choice, both schemes obtain the maximal multiplexing gain $\frac{1}{3}$. These two cases are actually the worst cases for both schemes. (Recall that when $L = 1$, the superposition-coded concurrent DF relaying has the same DMT performance as the space-time-coded standard protocol and we therefore do not consider this case.) And following the analysis in Section II, when $L > 1$ ($L > 2$), the performance of the repetition-coded (superposition-coded) concurrent DF relaying would be even better than those shown in the following simulations.

Fig. 4 displays the outage probabilities comparison for different schemes when multiplexing gain $r = \frac{1}{6}$ (i.e. the transmission rates are not fixed and scale with SNR). Following the analysis in Section II, it can be seen that the DMT curves for the standard protocol and the repetition-coded concurrent DF relaying intersect, which means the two protocols have the same diversity gains. Clearly, this diversity gain is further improved by the use of the superposition coding in the relays. Such a diversity performance can be seen by comparing the slopes of the high-SNR outage probability curves for different schemes.

We also study the error performance for uncoded symbols for different schemes. For a fair comparison, we consider 4-

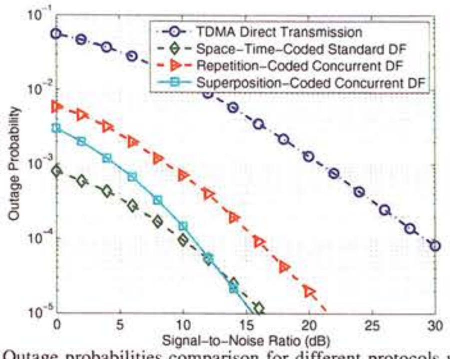

Fig. 4. Outage probabilities comparison for different protocols with $N = 2$ and multiplexing gain $r = \frac{1}{6}$.

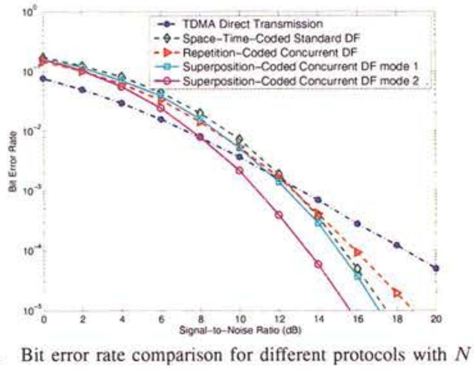

Fig. 5. Bit error rate comparison for different protocols with $N = 2$.

QAM, 8-QAM and 16-QAM modulation for TDMA direct transmission, concurrent DF relaying and the standard protocol respectively so that all schemes have identical average transmission rates at two bits per channel use (BPCU). For decoding at the destination, a maximal ratio combining (MRC) receiver is used for TDMA direct transmission and the standard protocol, and a maximum likelihood sequence detector (MLSD) receiver is used for the concurrent DF relaying protocols. Moreover, we consider two different ways to use superposition coding in the relays. The first one (denoted as mode 1 in Fig. 5) is similar to superposition modulation [15] and we require each relay to retransmit the direct sum of its desired signal and the interference. The second one is similar to code superposition [16] (denoted as mode 2). In this case, each codeword transmitted by the relays represents the XORed version of the two signals.

From Fig. 5, it can be seen TDMA direct transmission has the worst high-SNR performance. Although repetition-coded concurrent DF relaying improves the error performance due to the signal protection by the relays, it performs worse than space-time-coded standard DF relaying since each codeword is only forwarded by one relay. Clearly, superposition-coded concurrent DF relaying has the same diversity order as the standard protocol. Furthermore, mode 2 superposition coding outperforms mode 1 by nearly 1.7 dB, which confirms the advantage of code superposition analyzed in [16]. This observation suggests interesting future work in applying network coding techniques in our approach.

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